Plane Strain Behaviour of Granular Medium

by

T. Ramamurthy*

P.C. Rawat**

V.K. Tokhi***

Introduction

PLANE strain condition is often encountered in Geotechnical Engineering while seeking solutions to problems of earth dams, earth retaining structures and foundations. More often solutions to these problems are evolved on the basis of data obtained from axisymmetric triaxial compression or extension tests. Only a few laboratories have developed equipment to test soil under plane strain condition. It is therefore necessary to make a comparative study of strength and deformation in axisymmetric triaxial and plane strain compression tests with the ultimate objective of evolving a relationship between these parmeters. The conventional triaxial test may then continue to be used for some more time due to its simplicity of design, fair degree of versatility and easy commercial availability. It, however, lacks the the facility of application of general stress condition in which $\sigma_1' \neq \sigma_2' \neq \sigma_3'$ and independent measurement of uninterfered deformation of specimen under the applied stress system.

This paper very briefly describes Universal Triaxial Apparatus to test soil under general stress systems including plane strain. Results of consolidated drained tests conducted in this apparatus under axisymmetric compression and plane strain compression are described. A comparison of stress-strain curves, modulus values, peak strengths, axial and lateral strains at failure, volumetric strains and volumetric strain rates is made. Effect of anisotropy of consolidation on these properties is discussed.

Expressions have been developed to predict the plane strain strength at any relative density from strength in axisymmetic condition. An expression is presented to predict the strain ratio (ϵ_3/ϵ_1) in plane strain at failure.

Test Equipment

Many investigators have attempted to test soil under plane strain condition. The essential differences between the apparatus used by various investigators are with respect to the shape of specimen and the

---

* Professor, Civil Engineering Department, Indian Institute of Technology, New Delhi-110 016, India

** Senior Engineer, Ocean Engineering Division, Engineers India Ltd., Hindustan Times House, New Delhi-110 001. India

*** Assistant Professor, Civil Engineering Department, M.A. College of Technology, Bhopal-462 007, India

This paper was received in March, 1981 and is open for discussion till the end of February 1982.

method of preventing deformation in the intermediate principal stress direction. The specimens have been usually rectangular prism or cube/-cubodidal in shape. Plane strain condition is maintaied with a rigid system normally consisting of rigidly connected solid plates at fixed distance (equal to the width of the specimen) or with a flexible system which depends on skillful monitoring of the magnitude of intermediate principal stress such that no lateral deformation occurs in that direction. In both the systems a little deformation in the intermediate stress direction is practically inevitable because of the elastic extension of mechanism connecting the rigid system, reduction in thickness of rubber membrane enclosing the specimen and when sandwithces of rubber membrane smeared with high vacuum silicone grease are used; and also when not being able to prevent any deformation on the plane faces in the flexible system. Some investigators like Wood (1958), Cornforth (1964), Henkel and Wade (1966), Hambly and Roscoe (1969), Lee (1970) and Nagaraj and Somashekar (1979) have used rectangular prism shaped specimen along with a rigid system to impose plane strain condition. Ko and Scott (1967), Green (1971), Arther and Menzies (1972), Lade and Duncan (1973), Green and Reades (1975), Shankariah (1977) and Rawat and Ramamurthy (1978) have used flexible system to maintain plane strain condition on the specimen.

The equipment used in the present investigation (a slight modification of the equipment originally developed by Ramamurthy 1970) uses a cubical specimen of 76 mm side, contained in a thin rubber membrane. Vertical load is applied through top and bottom rigid lubricated platens and is measured by a stiff proving ring. Vertical deformatian is measured with the help of a dial gauge. The lateral pressure on the two pairs of opposite faces is applied through fluid contained in specially fabricated and lubricated rubber bags. The flexible rubber bag applying higher lateral pressure is strengthened with prismatic sponge pieces along the edges. This arrangement prevents interference with adjacent bags even at large strains and prevents distortion of vertical edges of the specimen. The flexible bags develop uniform deformation on all four edges of the specimen. The lateral pressures applied through the bags are measured with the help of two separate Bourden pressure gauges. The lateral displacements are measured at the mid-height of each of the faces through steel rods embedded within the rubber bags.

Figure 1 shows the general set up of the apparatus. A detailed description of the apparatus has been published elsewhere (Rawat 1976, and Rawat and Ramamurthy 1978).

Material Tested

The material used in this investigation was uniform Ottawa sand all of which passes through B.S. sieve No. 18 and was retained on B.S. sieve No. 52. The effective size of the sand was 0.43 mm with a uniformity coefficient of 1.28. The maximum and the minimum porosities were 41 and 32 per cent respectively. Specific gravity of particles was found to be 2.66.

Test Programme

All tests reported in this paper were conducted on saturated sand in Universal Triaxial Apparatus on cubical specimens consolidated and then

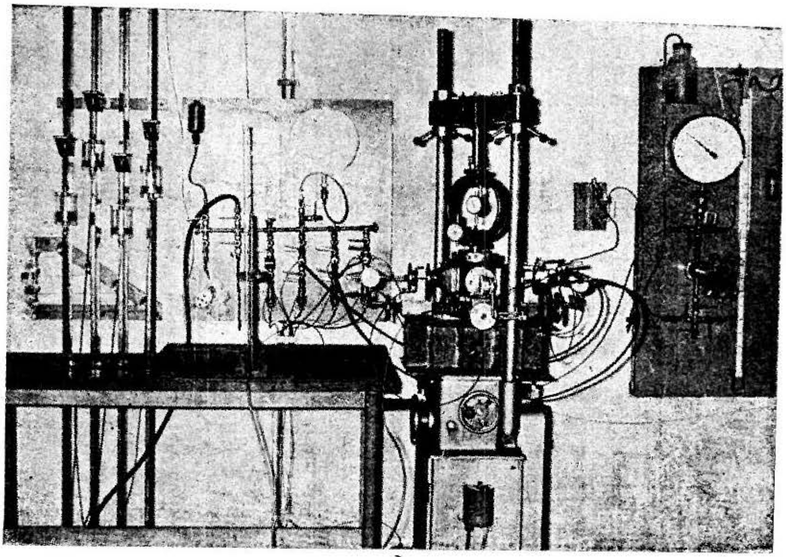

FIGURE 1 General set up of the universal triaxial apparatus

sheared at deformation rate of 0.2 per cent per minute under fully drained conditions. Specimens were either isotropically consolidated or anisotropically consolidated with lateral to axial stress ratio of 0.5.

In each test a sandwitch of two 0.35 mm thick rubber membranes smeared with high vacuum silicone grease was used at the top and bottom to minimize friction acting at the end platens. The specimen was enclosed in a thin rubber membrane and sealed at the top and bottom loading platens. The outer faces of the membrane enclosing the specimen and also of the rubber bags were smeared with high vacuum silicone grease to reduce friction drag on the vertical faces of the specimen and also to enable the bags to slip through their guides with least friction.

Each specimen was prepared by taking fresh sample of pre-boiled and cooled sand filled into a rubber membrane enclosed in a sand former in three layers. Each layer was rodded with a spatula to get the requisite initial porosity. Detailed procedure for the preparation of the specimen was described in an earlier publication (Rawat and Ramamurthy 1978).

Specimens consolidated keeping $\sigma'_1 = \sigma'_2 = \sigma'_3 = 2.05$ Kg/cm² are referred to as isotropically consolidated specimens and those with $\sigma'_1 = 4.10$ Kg/cm² and $\sigma'_2 = \sigma'_3 = 2.05$ Kg/cm² (i.e. $K = 0.5$) as anisotropically consolidated specimens.

In plane strain tests the deformation in one of the lateral directions (x-direction) was prevented by continuously varying the height of mercury pot and building up the requisite intermediate principal stress. In spite of great care a little deformation in the intermediate principal stress direction occurred. It is believed that this small deformation in the intermediate principal stress direction will not affect the plane strain strength significantly, as is evident from the following example.

A specimen with a porosity of 38.76 per cent was tested in plane strain and its strength was found to be 39.0°. Another specimen was tested in general compression with $\sigma'_2 > \sigma'_3$ but in this the specimen deformed by 0.4 per cent in the σ'_2 stress direction. Its porosity was 38.22 per cent and its strength was found to be 38.6°. The stress path followed in this case was approximate-plane strain. Stress-strain curves for these two specimens are shown in Figure 2 in which it is seen that the two curves for σ'_z/σ'_y, ϵ_y/ϵ_z and $\triangle V/V$ against axial strain are practically the same. There is small difference in the curves for σ'_x/σ'_y. This test shows that a small departure from true plane strain condition in a test does not materially alter the stress-volumetric strain characteristics. Marchi *et al* (1969) have made similar obseration that the strength of an imperfect

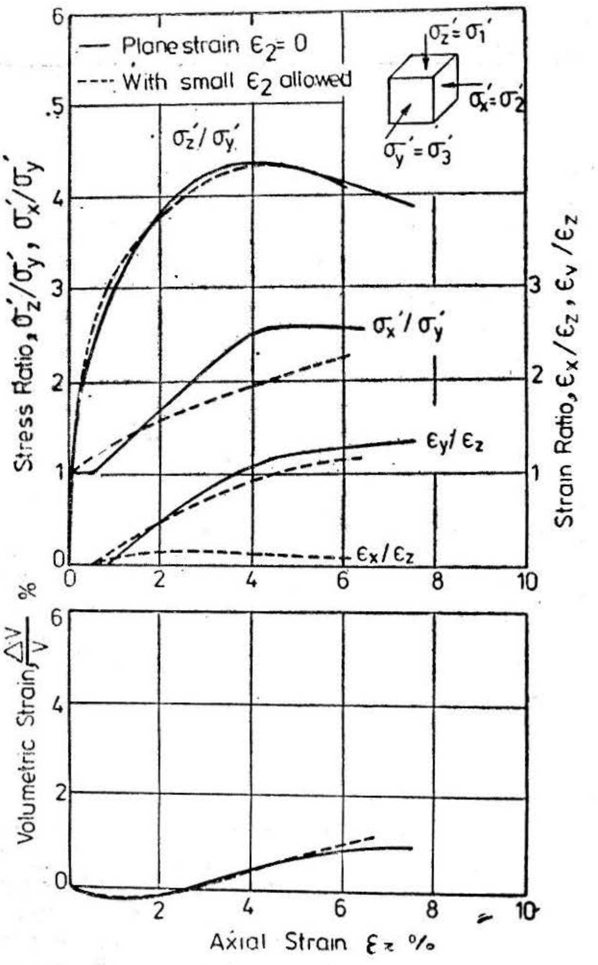

FIGURE 2 Effect of small deformation in intermediate principal stress direction on plane strain behaviour

plane strain specimen in which a small amount (less than about 0.4 per cent) of longitudinal strain is allowed does not differ materially from the strength of a perfect plane strain specimen.

Test Results and Discussion

Results of axisymmerric compression and plane strain compression tests under isotropic and anisotropic conditions are presented. Since all the tests are conducted in fully drained conditions all the stresses reported are effective stress. Initial porosity has been used as basis for comparison of test results.

Stress-Strain Curves

Figures 3, 4 and 5 show the stress-stain curves for equal or nearly equal initial porosities for specimens that are isotropically consolidated and tested under axisymmetric compression together with stress-strain curves for specimens tested in plane strain after isotropic and anisotropic consolidation. Comparing isotropically consolidated specimens under axisymmetric and plane strain compression, it is observed that the initial tangent modulus is of the same order in both cases. Examining the σ_x' / σ_y' (i.e. σ_2' / σ_3') curves it is observed that the minor principal

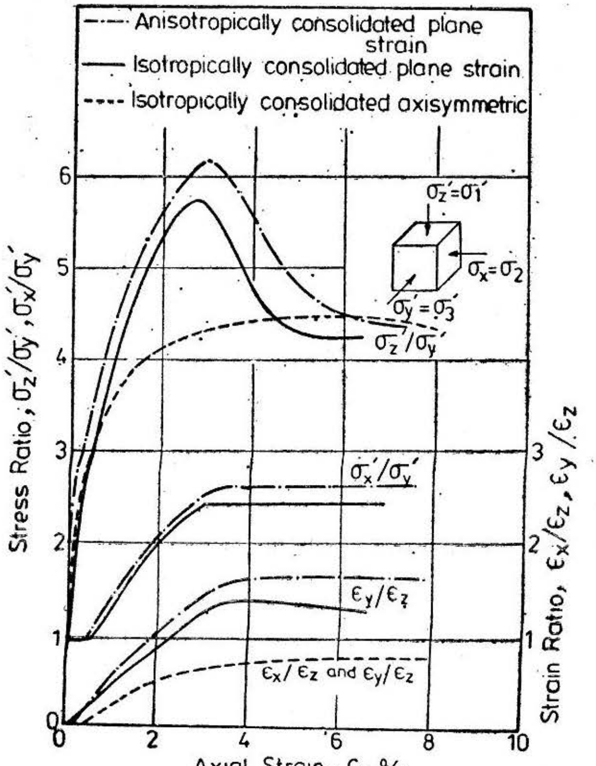

FIGURE 3 **Stress-strain curve for isotropically consolidated axisymmetric specimen and istropically/anisotropically consolidated plane strain specimens—dense sand**

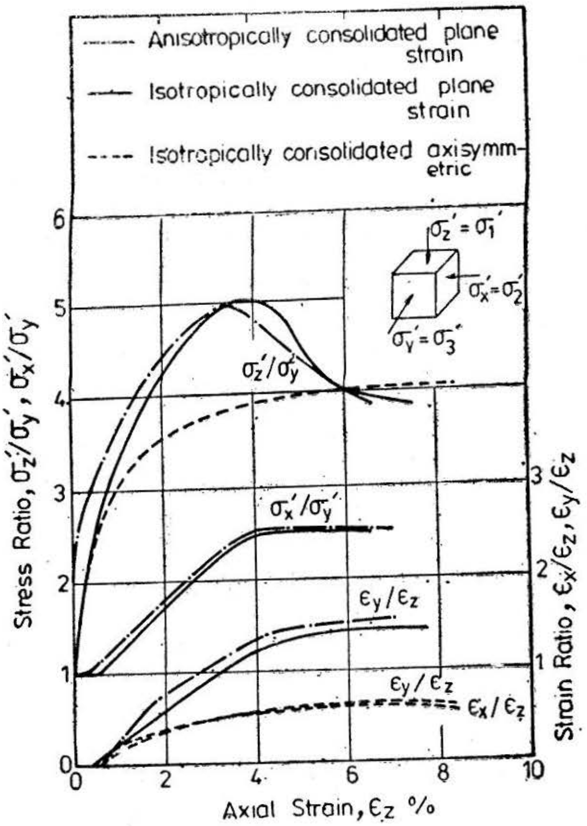

FIGURE 4 Stress-strain curve for isotropically consolidated axisymmetric specimen and isotropically/anisotropically consolidated plane strain specimens—medium dense sand

stress (σ'_y) is almost equal to intermediate principal stress (σ'_x) until a small axial strain is reached (of the order of 0.75 per cent). Hence up to this axial strain the effect of intermediate principal stress is not reflected on the stress-strain curves and therefore the two curves up to this axial strain are not much different. When σ'_x is beginning to be more than σ'_y, the ratio σ'_z/σ'_y has values between 2.6 and 3.1 which approximately corresponds to K_o-condition. If a tangent is drawn to the stress-strain curve at stress ratio corresponding to the above, then the modulus for plane strain compression is higher than for axisymmetric compression.

When stress-strain curves for isotropically consolidated axisymmetric specimens are compared with anisotropically consolidated plane strain specimens, it is found that the initial tangent modulus for the two test conditions is of the same order; but after a small axial strain of 0.5 to 0.75 per cent, tangent modulus of the specimen tested under plane strain is substantially higher than for specimen tested in axisymmetric condition.

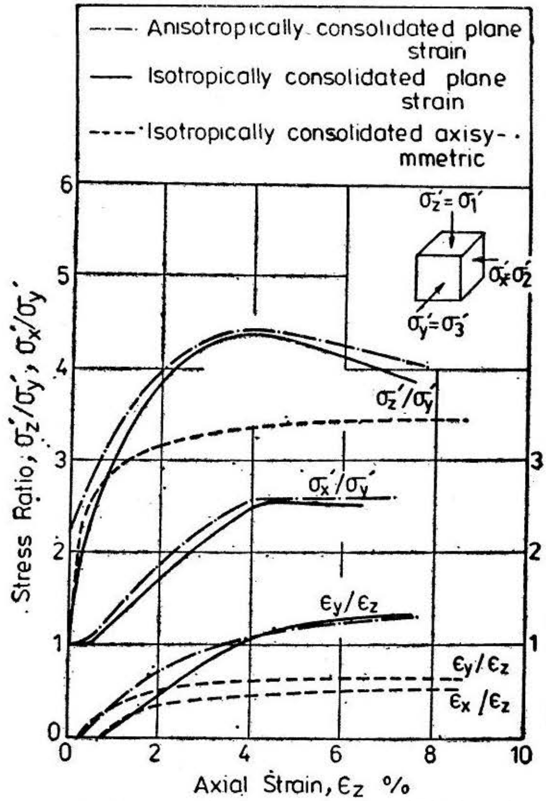

FIGURE 5 **Stress-strain curve for isotropically consolidated axisymmetric specimen and isotropically/anisotropically consolidated plane strain specimens—loose sand**

In this case (in relation to specimens compared in the previous paragraph) σ'_x starts rising above σ'_y at a slightly smaller axial strain.

When the comparison is limited to plane strain specimens that are isotropically and anisotropically consolidated it is found that their stress-*strain curves run almost parallel to each other from* $\sigma'_z / \sigma'_y = 2.0$ up to a little less than the failure strain; the peak strength of anisotropically consolidated plane strain specimen being slightly higher. The curves for σ'_x / σ'_y versus axial strain also run nearly parallel with higher value of σ'_x / σ'_y *for the anisotropically consolidated specimens.*

A comparison of secant modulus at failure strain reveals that the secant modulus of plane strain specimen is much higher than that of the axisymmetric specimen. Anisotropy of consolidation stresses has very little effect on the secant modulus at failure in either axisymmetric or plane strain condition as is evident from Table 1.

Another characteristic feature of stress-strain curves for plane strain specimens as compared to axisymmetric specimens is that the former reach

TABLE 1

Values of Secant Modulus at Failure

Initial Porosity n, percent	Type of test and consolidation condition	E_{sec} Kg/cm²	Initial Porosity n, per cent	Type of test and consolidation condition	E_{sec} Kg/cm²
33.80	AXS (*I*)	134.2	33.80	AXS (*A*)	132.9
33.80	PS (*I*)	371.3	33.80	PS (*A*)	364.1
36.10	AXS (*I*)	089.9	36.66	AXS (*A*)	087.9
36.61	PS (*I*)	224.4	36.65	PS (*A*)	244.8
38.29	AXS (*I*)	078.9	38.13	AXS (*A*)	066.6
38.76	PS (*I*)	174.3	39.09	PS (*A*)	174.3

Note: AXS Axisymmetric Compression
PS Plane Strain Comprsssion
(I) Isotropic consolidation
(A) Anisotropic consolidation

their peak principal stress ratio (σ'_z / σ'_y) at a smaller axial strain followed by a rapid decrease suggesting work softening behaviour as a result of constraint on deformation in the intermediate principal strain direction.

Relationship Between Triaxial and Plane Strain Strength

Experimental results of peak strength in axisymmetric and plane strain compression are shown in Figure 6. It is noted that the difference $(\phi'_p - \phi'_c)$ increases with denseness of the soil. The difference in the present investigation is about 6° in dense state and 4° in loose state. Cornforth (1964) and Green and Reades (1975) have also made similar observations though magnitude of $(\phi'_p - \phi'_c)$ was different in their cases.

Some investigators like Finn and Mittal (1963) and Parry (1971) have predicted plane strain strength from triaxial compression strength. Rowe (1969) has suggested a range of values to be expected in plane strain corresponding to triaxial compression strength in two conditions only namely in the looset ($D = 1$) and the densest ($D = 2$) conditions. Ramamurthy and Tokhi (1981) have predicted plane strain strength from the axisymmetric compression test based on the observation that the plot of $(\sigma'_1 - \sigma'_3)$ and $\sigma'_m \left[= \frac{1}{3} (\sigma'_1 + \sigma'_2 + \sigma'_3) \right]$ at failure for axisymmetric and plane strain tests are nearly parallel and are located quite close to each other and can be represented by a common straight line, as shown in

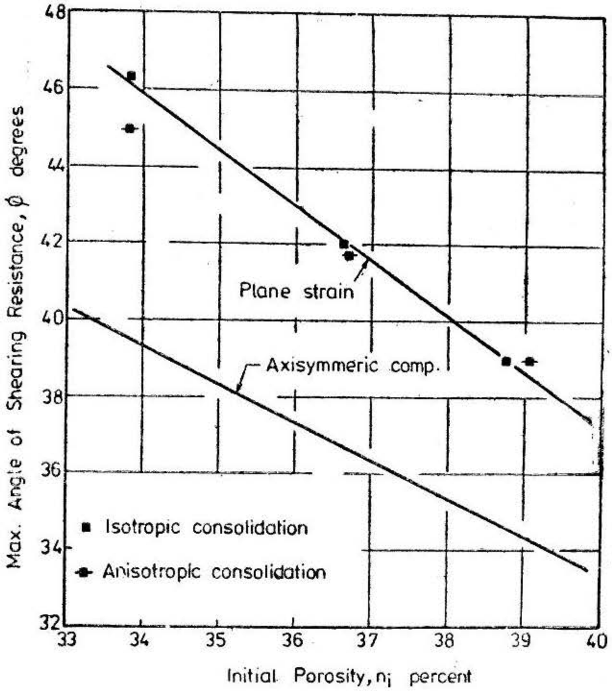

FIGURE 6 Variation of peak angle of shearing resistance with porosity for axisymmetric and plane stain specimens

Figure 7, such that,

$$\left(\frac{\sigma_1' - \sigma_3'}{\sigma_1' + 2\sigma_3'}\right)_c = \left(\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_2' + \sigma_3'}\right)_p \qquad \ldots (1)$$

subscript, c refers to axisymmetric and p to plane strain condition.

$$\left(\frac{\sigma_1' + \sigma_3'}{\sigma_1' - \sigma_3'} + \frac{\sigma_3'}{\sigma_1' - \sigma_3'}\right)_c = \left(\frac{\sigma_1 + \sigma_3'}{\sigma_1' - \sigma_3'} + \frac{\sigma_2' - \sigma_3'}{\sigma_1' - \sigma_3'} + \frac{\sigma_3'}{\sigma_1' - \sigma_3'}\right)_p$$

Using Mohr-Coulomb criteria

$$\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = (\sin \phi)_{c,\ p}$$

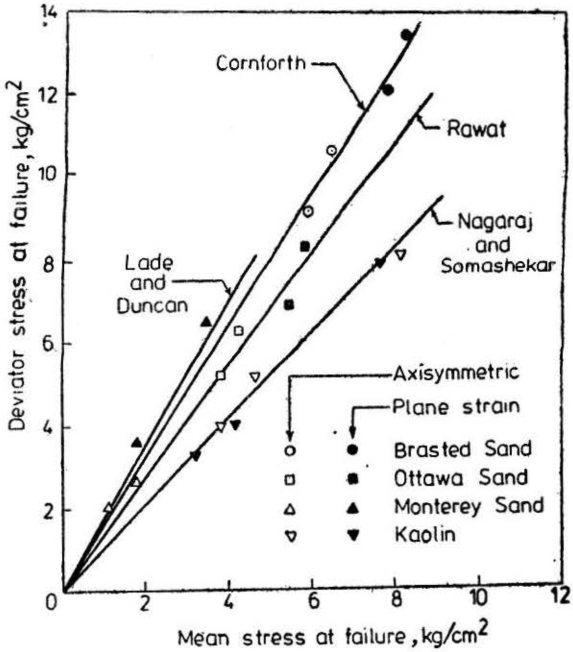

FIGURE 7 **Deviator stress versus mean stress at failure for axisymmetric and plane strain compression**

and

$$b = \frac{(\sigma_2' - \sigma_3')}{(\sigma_1' - \sigma_3')}$$

$$\frac{1}{\sin \phi_c} + \frac{1}{2}\left(\frac{1}{\sin \phi_c} - 1\right) = \frac{1}{\sin \phi_p} + b_p + \frac{1}{2}\left(\frac{1}{\sin \phi_p} - 1\right)$$

Rearranging

$$\frac{1}{\sin \phi_c} - \frac{1}{\sin \phi_p} = \frac{2}{3}\, b_p \qquad \dots (2)$$

This is a general equation connecting strength in axisymmetric and plane strain tests with the value of b_p. Figure 8 shows the plot of Equation 2 for two selected values of ϕ_c'. Also included in the figure are experimental results of a few investigators. It would be noted that the agreement between the observed and predicted strength values is good but the agreement in the values of b_p is not to the same extent. The possible reasons for deviation in b_p values are presented later in the paper.

In order to make prediction of plane strain strength from ϕ_c', the magnitude of b_p must be known or assumed. Bishop (1966) has suggested the following relationship

$$\sigma_2' = \frac{1}{2}\,(\sigma_1' + \sigma_3')\cos^2 \phi_p' \qquad \dots (3)$$

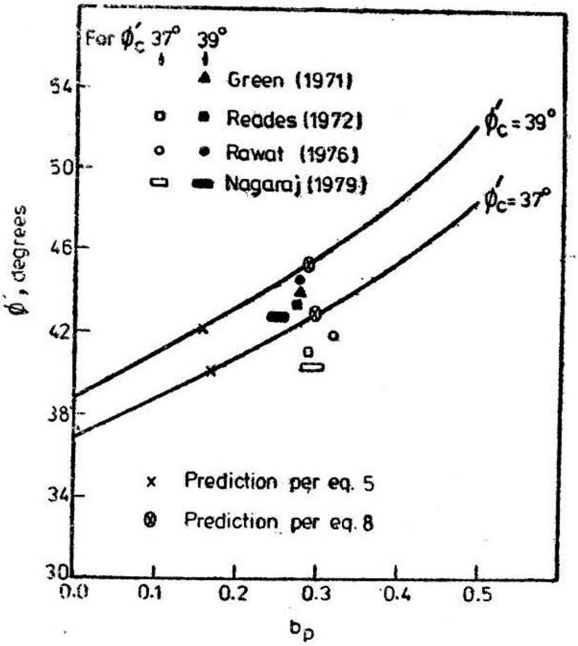

FIGURE 8 **Relationship between axisymmetric strength, plane strain strength and b_p at failure**

Equation 3 can be substituted in Equation 2 from which,

$$\sin \phi'_p + 3 \left(\frac{1}{\sin \phi'_c} - \frac{1}{\sin \phi'_p} \right) = 1 \qquad \text{... (4)}$$

From equation 3 it follows,

$$b_p = \frac{1 - \sin \phi'_p}{2} \qquad \text{... (5)}$$

Similarly, Green (1971) has suggested an expression for σ'_2 in plane strain,

$$\sigma'_2 = \sqrt{\sigma'_1 \sigma'_3} \qquad \text{... (6)}$$

Substituting Equation 6 in Equation 2,

$$\frac{\sin \phi'_p}{2 + \cos \phi'_p} = \frac{\sin \phi'_c}{3 - \sin \phi'_c} \qquad \text{... (7)}$$

From equation 6 it follows

$$b_p = \frac{1}{2}\left(1 - \frac{1 - \cos \phi'_p}{\sin \phi'_p}\right) \qquad \text{... (8)}$$

Equations 4 and 7 cannect the axisymmetric compression strength to plane strain strength while Equations 5 and 8 give the associated values of b_p. Figure 9 shows the plot of Equations 4 and 7 and Figure 10 shows the plot of Equations 5 and 8. Both these figures include the experimental results of several investigators. It will be readily noted that agreement of predicted and observed plane strain strength is quite satisfactory, with some scatter of results. From a critical examination of the data for σ_2' of several investigators from plane strain tests it seems that Equation 5 is more appropriate for loose sands or normally consolidated clays that show small dilation at failure while Equation 8 is appropriate for dense sands that dilate significantly at failure. Assuming a linear variation of ϕ_p' predicted from equations 4 and 7 with initial relative density, following equation is suggested for computing the predicted plane strain strength at any desired relative density (I_D, expressed as a fraction).

$$\phi'_{p(I_D)} = 1.42\,(\phi'_{p^7} - \phi'_{p^4})\,(I_D - 0.25) + \phi'_{p^4} \qquad \dots (9)$$

Where ϕ'_{p^7} and ϕ'_{p^4} indicate the predicted plane strain strength corresponding to Equations 7 and 4 respectively. Figure 11 shows the comparison of experimental and predicted ϕ_p' at various relative densities and it is

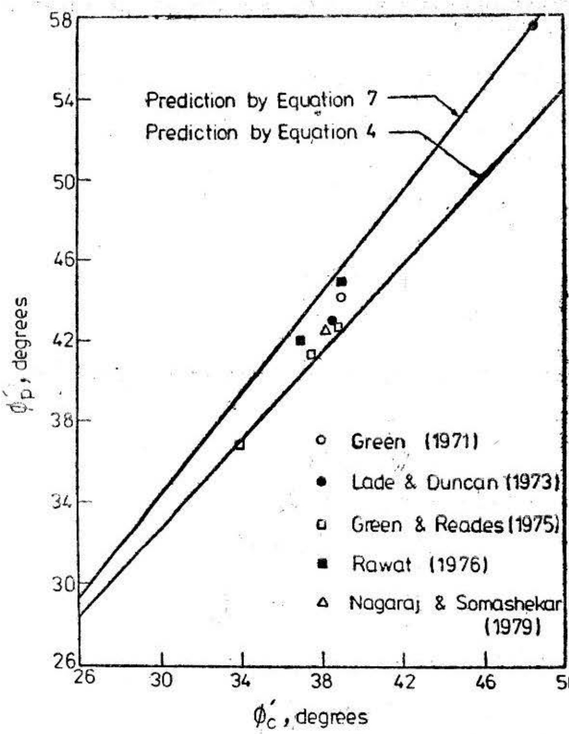

FIGURE 9 Prediction of plane strain strength with axisymmetric compression and comparison with experimental results

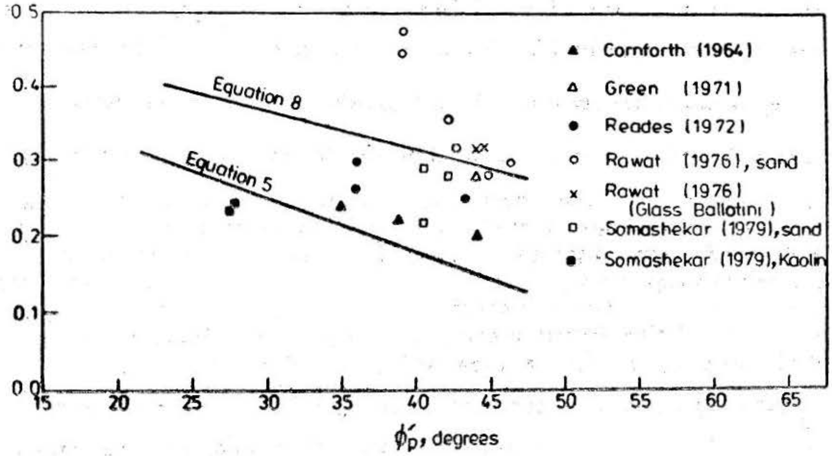

FIGURE 10 **Prediction of b_p [$= (\sigma'_2 - \sigma'_3)/(\sigma'_1 - \sigma'_3)$] and comparison with experimental results**

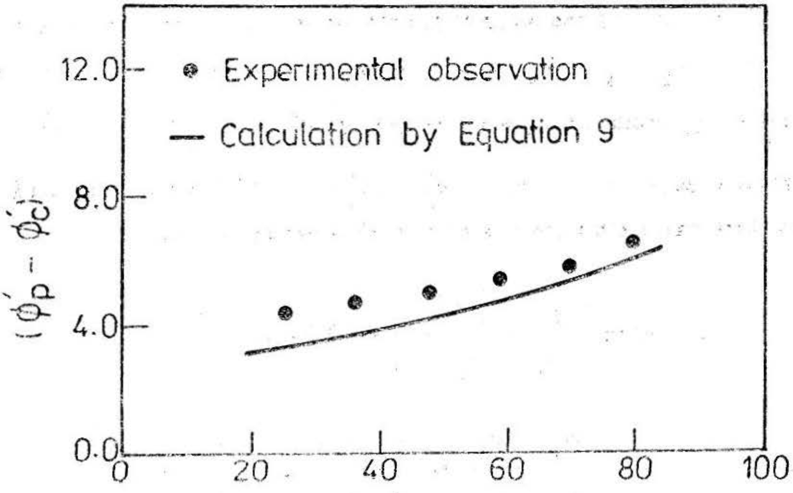

FIGURE 11 **Variation of plane strain strength with relative density and comparison with predicted value**

observed that the predicted values agree reasonably well for dense state but some deviation exists in loose state.

Figure 10 shows that agreement of experimental and predicted b_p values is not so good. The reason is not immediately known, but it may be that the range of variation of b_p is rather small, being between 0.2 to 0.4 approximately and thus a relatively small error in estimating of b_p causes large error in predicted value of ϕ'_p; for example from Figure 8 it is seen that an error of only 0.04 in the estimation of b_p would change the predicted ϕ'_p value by 1°. Similar magnitude of error in the measurement of σ'_1 is much less serious. Hence greater scatter in b_p value is to be expected and can result from different degree of accuracy with which σ'_2 can be measured in the different equipments used by different investigators. Another point of interest is that most of the investigators experimenting with sand have observed value of b_p closer to that given by Equation 8 rather than Equation 5. However, the prediction of plane strain strength according to Equation 9 is satisfatory. This equation could be applied for soils with relative density greater than 25 per cent. When soil shows only slight dilation at failure even at high relatlve density, then the value of ϕ'_p given by Equation 4 seems to be appropriate. On the contrary if the soil shows volumetric contraction at high relative density then plane strain strength may be smaller than even that predicted by Equation 4.

Value of $\sigma'_3 / (\sigma'_1 + \sigma'_3)$ and b

While analysing the plane strain results several investigators have used the ration $\sigma'_2 / (\sigma'_1 + \sigma'_3)$ and $b [= (\sigma'_2 - \sigma'_3) / (\sigma'_1 - \sigma'_3)]$ to denote the relative magnitude of σ'_2 with respect to σ'_1 and σ'_3. Sometimes Lode's parameter $\mu_l [= (2\sigma'_2 - \sigma'_1 - \sigma'_3) / (\sigma'_1 - \sigma'_3)]$ has also been used. In fact these parameters are related to each other as follows,

$$\mu_l = \frac{2\sigma'_2 - \sigma'_1 - \sigma'_3}{\sigma'_1 - \sigma'_3} = (2\,b-1) \qquad \ldots (10)$$

and

$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3} = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 + \sigma'_3} \cdot \frac{\sigma'_1 + \sigma'_3}{\sigma'_1 - \sigma'_3}$$

or

$$\frac{\sigma'_2}{\sigma'_1 + \sigma'_3} = 0.5 - (0.5 - b) \sin \phi' \qquad \ldots (11)$$

i.e.

$$b = 0.5 - \frac{1}{\sin \phi'} \left(0.5 - \frac{\sigma'_2}{\sigma'_1 + \sigma'_3} \right) \qquad \ldots (12)$$

Equations 11 and 12 connect $\sigma'_2 / (\sigma'_1 + \sigma'_3)$, b and ϕ'. Figure 12 presents

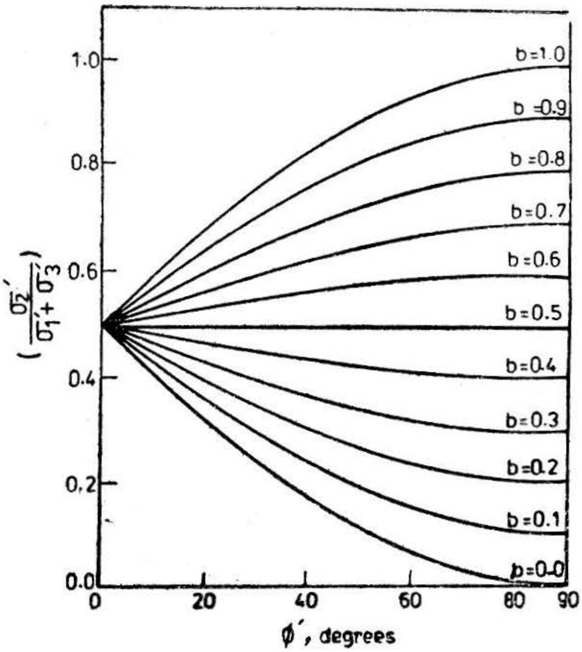

FIGURE 12 Relationship of ϕ', $\sigma_2' / (\sigma_1' + \sigma_3')$ and b

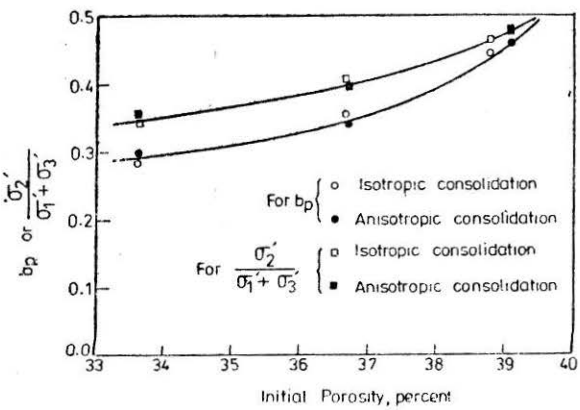

FIGURE 13 Variation of b_p and $\sigma_2' / (\sigma_1' + \sigma_3')$ with initial parosity

these equations in a convenient graphical form which if any two of $\sigma_3' / (\sigma_1' + \sigma_3')$, b or ϕ' are known the third can be obtained directly.

Figure 13 shows the plot of b and $\sigma_2' / (\sigma_1' + \sigma_3')$ versus initial porosity. It is seen that neither b nor $\sigma_2' / (\sigma_1' + \sigma_3')$ is significantly affected by anisotropy of consolidation stresses. The value of $\sigma_2' / (\sigma_1' + \sigma_3')$ varies between

0.345 and 0.474 and that of b between 0.281 to 0.461 over the entire range of porosity which correspouds to a range of relative density from 82.2 to 24.2 per cent. It appears at high porosity nearer to loosest $b_p \cong \sigma_2' /(\sigma_1' + \sigma_3')$. Cornforth (1964) has also reported the value of $\sigma_2' /(\sigma_1' + \sigma_3')$ to vary from 0.29 to 0.36 for the entire or range of porosity (relative density between 84 and 15 per cent).

Figure 14 shows a plot of $\sigma_2' /(\sigma_1' + \sigma_3')$ versus peak angle of shearing resistance as observed in the present investigation and by several other investigators for various soils. The figure suggests that this ratio is not constant but decreases with increase in ϕ' values. Plot of Equations 3 and 6 are also shown in this figure. It suggests that Equation 3 gives good agreement for low ϕ'_p values while for high ϕ'_p value Equation 6 gives better agreement. Low value of $\sigma_2' /(\sigma_1' + \sigma_3')$ seems to be associated with higher volumetric strains which may correspond to slightly dilating granular

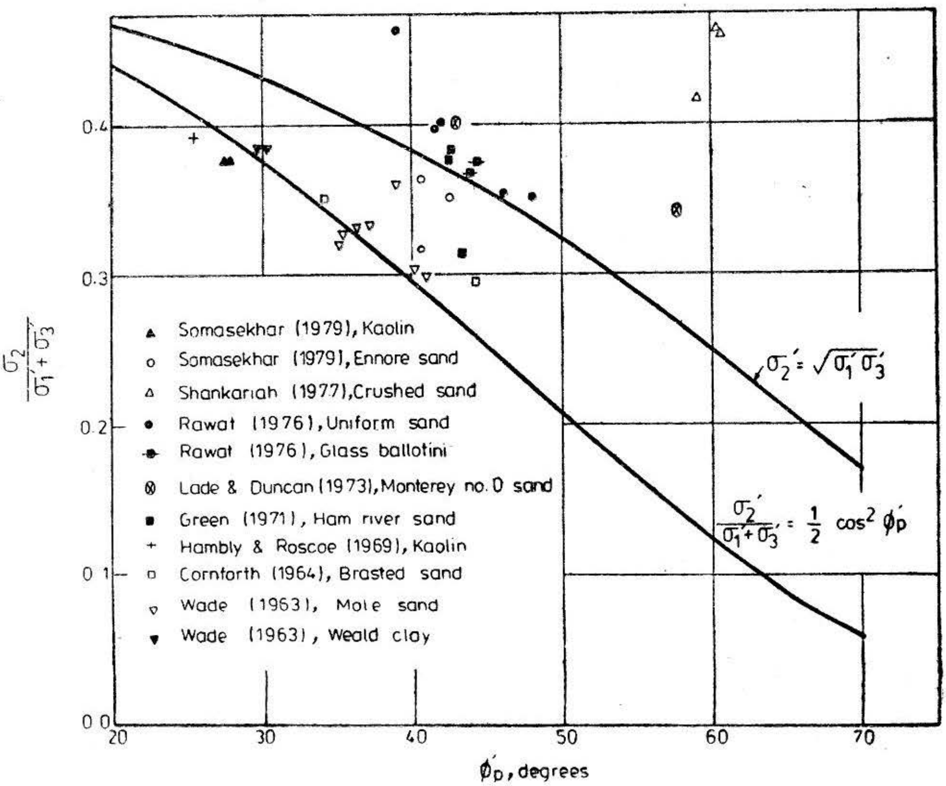

FIGURE 14 Variation of $\sigma_2' /(\sigma_1' + \sigma_3')$ with peak angle of shearing resistance

soil. High values of $\sigma_2' /(\sigma_1' + \sigma_3')$ are associated with granular medium showing small volumetric strains.

By adopting a critical state approach and assuming a constant value of $\sigma_2' /(\sigma_1' + \sigma_3')$ at 0.4, Parry (1971) has related plane strain strength with triaxial compression strength. From Figures 13 and 14 it is readily seen that $\sigma_3' /(\sigma_1' + \sigma_3')$ cannot be taken to be constant over a wide range of ϕ_p' value. This may be the reason that Parry's predicted ϕ_p' values are lower than generally observed peak plane strain strength.

Axial Strain

Figure 15 shows the axial strain to failure for the isotropically and anisotropically consolidated specimens tested under axisymmetric and plane strain compression. The failure strain decreases near linearly with decrease in porosity. Results of isotropically and anisotropically consolidated specimens can be represented by a common line suggesting that the anisotropy due to consolidation stresses does not have any significant effect on axial failure strain in either type of test.

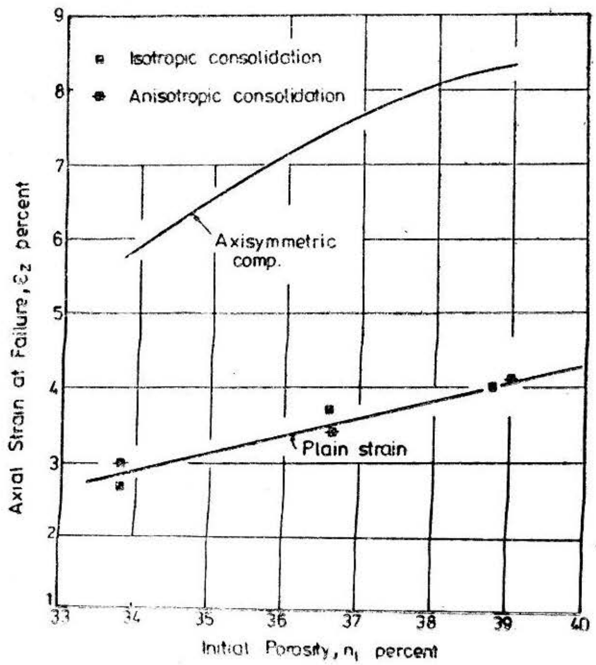

FIGURE 15 Variation of axial strain to failure with initial porosity for axisymmetric and plane strain specimens

Over the range of porosity used in this investigation axial strain at failure varies between 6.2 and 8.0 per cent for axisymmetric compression and from 2.7 to 4.1 per cent for specimens tested in plane strain compression. The ratio of axial strain at failure in plane strain to that in axisymmetric condition varies between 2/3 to 1/2 over the entrie range of porosity, higher ratio being observed for dense specimens. Cornforth (1964) reported a range of ½ to 1/3 while Nagaraj and Samashekar (1979) reported it to be between 0.5 to 0.6 for Kaolin and between 0.70 to 0.80 for Ennore sand. Only Green and Reades (1975) have found the ratio to be *nearly* constant at 1/3 for the entrie range of porosity. It is thus concluded that generally this ratio cannot be taken to be a constant over the entire porosity range. Finn and Mittal (1963) have estimated the 'upper bound' plane strain strength from conventional triaxial compression strength by assuming this ratio to be constant at 3/4 for equal stress ratios (and *also* assuming that Poisson's ratio at failure to be 0.5). The assumption of constant ratio of these axial strains at failure is not borne out by the experimental results presented above. Further, contrary to the experimental evidence, their theory predicts decreasing $(\phi'_p - \phi'_c)$ with increasing value of ϕ'_c.

Lateral Strain

Figure 16 shows the plot of initial porosity versus lateral strain at failure for all the specimens tested under axisymmetric and plane strain conditions. For both types of conditions of testing there is greater scatter for dense specimens than for loose specimens. Lateral strain (ϵ_3) in plane strain is always smaller than the lateral strain in axisymmetric compression. In axisymmetric tests the lateral strain varies between 4.3 to 4.7 per cent even though the extreme values are 4.15 and 5.15 per cent, the corresponding values for plane strain compression being 3.6 per cent for dense specimens and 4.5 per cent, for loose specimens.

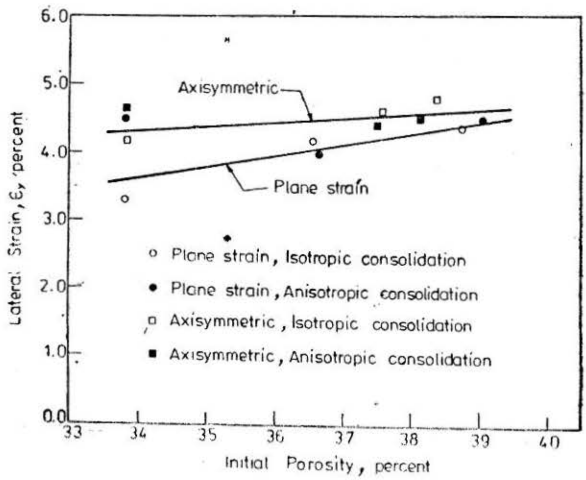

FIGURE 16 Variation of lateral strain at failure with initial porosity for axisymmetric and plane strain specimens

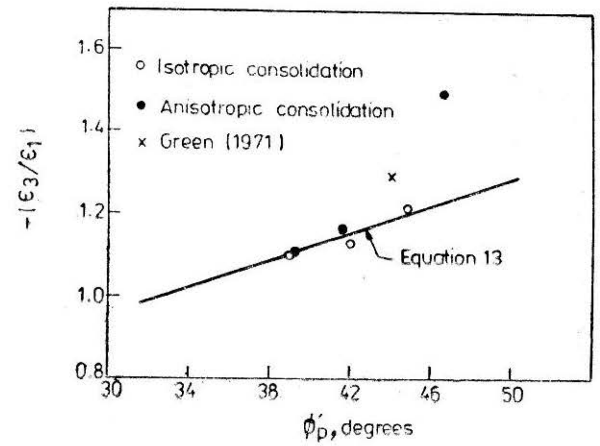

FIGURE 17 Relationship between plane strain strength and principal strain ratio $\left(\frac{\epsilon_3}{\epsilon_1}\right)$ at failure

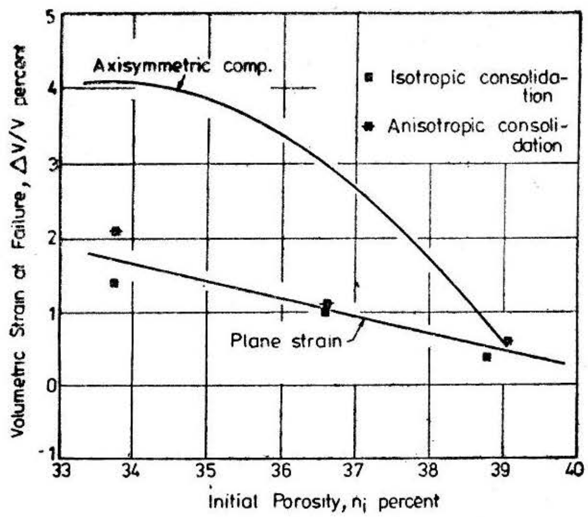

FIGURE 18 Variation of volumetric strain at failure with initial porosity for axisymmetric and plane strain specimens

The ratio of lateral strain to axial strain at failure is higher in plane strain compared to axisymmetric compression specimens. Following equation gives good agreement with observed values,

$$\left(\frac{\epsilon_3}{\epsilon_1}\right)_{pf} = \cos^2 \phi'_p - 1.71 \qquad \dots (13)$$

Figure 17 compares Equation 13 with observed values.

Volumetric Strains and Volumetric Strain Rates

Figure 18 shows the variation of volumetric strain $(\triangle V/V)$ at failure

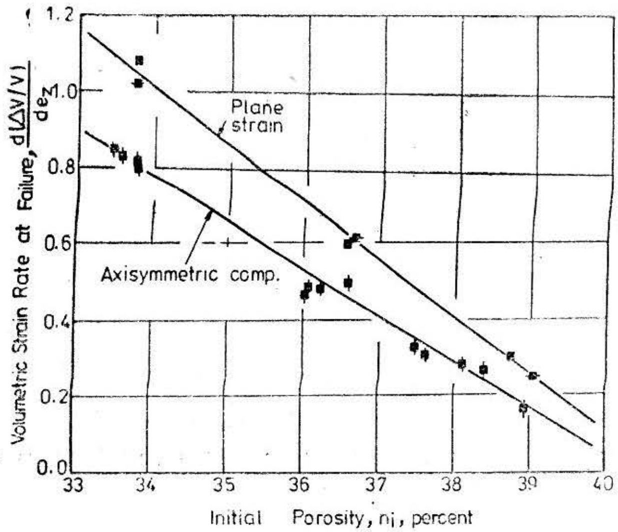

FIGURE 19 Variation of volumetic strain rate at faiture with initial porosity for axisymmetric and plane strain specimens

versus porosity for specimens tested in axisymmetric compression and plane strain compression. Anisotropy due to consolidation stresses does not appear to have significant effect on volumetric strains in either test condition. The difference in volumetric strain for axisymmetric and plane strain specimens is significant for dense and medium dense specimens but this difference is rather small for loose specimens.

The variation in volumetric strain rate, $\delta(\Delta V/V)/\delta\epsilon_1$ at failure versus initial porosity is shown in Figure 19. Volumetric strain rate in plane strain and axisymmerric specimens do not differ much in loose specimens (-0.16 for axisymmetric case and -0.21 for plane strain) while for dense specimens the difference is significant (-0.82 for axisymmetric case and -1.08 for plane strain).

The relationship between peak angle of shearing resistance and volumetric strain rate is shown in Figure 20. The value of angle of shearing resistance at zero volumetric strain rate(ϕ'_{cv}) is higher for plane strain compression by 4° over the axisymmetric case ; actual values being 32.6° and 36.6°. Wade (1963) and Cornforth (1964) have also found $(\phi'_{cv})_p$ to be higher than $(\phi'_{cv})_c$ by about 2°.

Octahedral Stresses and Strains

Ratio of $(\tau'_{oct} / \sigma'_{oct})_c / (\tau'_{oct} / \sigma'_{oct})_p$ has been plotted against porosity in Figure 21. It is clearly seen that anisotropy of consolidation stresses has no effect on this ratio since a common curve represents both the

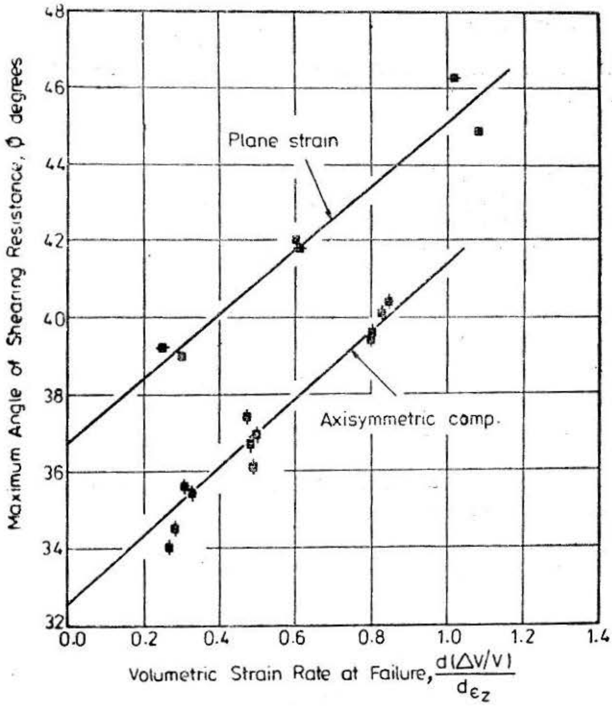

FIGURE 20 Relationship between peak angle of shearing resistance and volumetric strain rate at faiture for axisymmetric and plane strains specimens

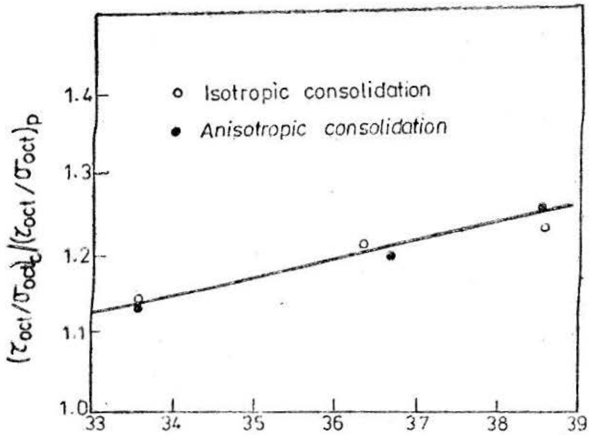

FIGURE 21 Variation of $(\tau'_{oct}/\sigma'_{oct})_c/(\tau'_{oct}/\sigma'_{oct})_p$ with initial porosity

conditions. The stress ratio $(\tau'_{oct}/\sigma'_{oct})_c / (\tau'_{oct}/\sigma'_{oct})_p$ increases slightly with porosity in this investigation and for the range of porosity used, this ratio is represented by a simple equation

$$\left(\frac{\tau'_{oct}}{\sigma'_{oct}}\right)_c \Big/ \left(\frac{\tau'_{oct}}{\sigma'_{oct}}\right)_p = 1.13+0.02\,(n-33.0) \qquad ...(14)$$

The ratio $(\tau'_{oct}/\sigma'_{oct})_c/(\tau'_{oct}/\sigma'_{oct})_p$ has always remained more than unity at all porosities. This is because in an actual test the $(\sigma'_{oct})_p$ will be larger than $(\sigma'_{oct})_c$ even if σ'_3 is the same in both the cases. In the above expression of octahedral stress ratios the respective value of octahedral normal stress is used and therefore this ratio exceeds one. It is also observed that $(\tau'_{oct})_p$ will always be greater than $(\tau'_{oct})_c$ for the same value of σ'_3.

Plot of octahedral shear strain versus initial porosity is shown in Figure 22. It is found that octahedral shear strain increases with increase in porosity in axisymmetric and plane strain compression and also that the anisotropy of consolidation stresses has no significant effect on γ_{oct}. The ratio $(\gamma_{oct})_c/(\gamma_{oct})_p$ is of the order of 1.8 over a wide range of porosity used in this experimentation.

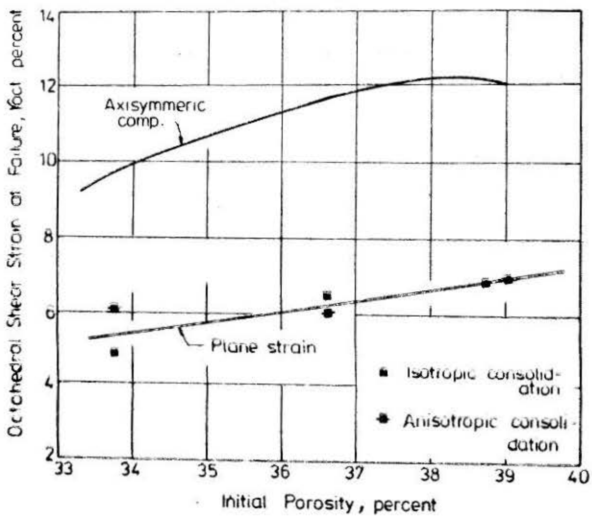

FIGURE 22 Variation of octahedral shear strain with initial porosity for axisymmetrie and plane strain specimens

Conclusions

From the axisymmetric and plane strain compression tests conducted on isotropically and anisotropically consolidated sand specimens with flexible vertical faces and sheared under fully drained conditions following conclusions can be drawn.

Strength of soil as expressed by Mohr-Coulomb angle of shearing resistance is larger by 4° to 6° in plane strain condition than in axisymmetric compression. The difference ($\phi'_p - \phi'_c$) increases with increasing ϕ'_c. Axial strain to failure in plane strain specimens is ½ to 2/3 of that in axisymmetric compression. Specimens tested in plane strain show a marked strain softening behaviour after the peak strength has been reached. Similar observations have also been made by other investigators.

Tangent modulus of axisymmetric and plane strain specimens are of the same order up to an axial strain that corresponds to stress ratio (σ'_1 / σ'_3) of about 2.5 to 3.0.

After this at larger strains, the secant modulus of plane strain specimen is higher than that of axisymmetric specimen. At failure strain, secant modulus in plane strain is 2.25 to 2.75 times the value in axisymmetric compression.

At failure strain, the anisotropy due to consolidation stresses does not significantly affect secant modulus.

Expressions have been presented to predict plane strain strength in terms of peak angle of shearing resistance from the axisymmetric strength for soils showing dilation at failure in axisymmetric triaxial tests. Another expression has been presented to compute the plane strain strength at any desired relative density.

The ratio $(\sigma'_{oct}/\sigma'_{oct})_c/(\sigma'_{oct}/\sigma'_{oct})_p$ has been found to vary over a narrow range irrespective of the porosity and the peak angle of shearing resistance. Anisotropy of consolidation stresses has little effect on this ratio.

At any given porosity, the lateral strain at failure for a plane strain specimen is lower than in axisymmetric test ; the ratio, $-\left(\frac{\epsilon_3}{\epsilon_1}\right)_f$, is however higher in plane strain. Ansiotropy of consolidation stresses has no effect on this ratio. An expression is presented to relate $\left(\frac{\epsilon_3}{\epsilon_1}\right)_{pf}$ to the peak angle of shearing resistance in plane strain.

Volumetric strain at failure for axisymmetric specimen is higher than in plane strain specimen, but the volumetric strain rate at failure for a plane strain specimen is higher.

The angle of shearing resistance at zero volumetric strain rate (ϕ'_{cv}) is higher by about 4° in plane strain condition than in axisymmetric case.

Octahedral shear strain in axisymmetric condition is higher than in plane strain condition. The ratio $(\gamma_{oct})_c/(\gamma_{oct})_p$ has a value of 1.8 (approximately) and does not seem to be affected by anisotropy of consolidation stresses.

References

ARTHUR, J.R.F. and MENZIES, B.K. (1972), "Inherent Anistropy in a sand." *Geotechnique,* Vol. 22, No. 1, pp.115-128.

BISHOP, A.W. (1966) "The Strength of Soils as Engineering Materials" (6th Rankine Lecture), *Geotechnique*, Vol. 16, No. 2 pp. 91-128.

CORNFORTH, D.H. (1964), "Some experiments on influence of Strain conditions on Strength of Sand,', *Geotechnique*, Vol. 14, No. 2, pp. 143-167.

FINN, W.D.M. and MITTAL, H.K., (1963). "Shear Strength of Soil in a General Stress Field", *Symposium on Laboratory Shear Testing of Soils,* ASTM, STP. No. 361 pp. 42-48.

FINN, W.D.M., WADE, N.H. and LEE K.L., (1967). "Volume Changes in Triaxial and Plane Strain Tests," *Journal of Soil Mechanics and Foundation Division* ASCE, Vol. 93, pp. 297-308.

GREEN, G.E., (1971). "Strength and Deformation of Sand Measured in an Independent Stress Control Cell," *Proceedings Roscoe Memorial Symposium,* Cambrige, pp. 285-323.

GREEN G.E. and READES, D.W. (1975). "Boundary Conditions, Anistropy and samples Shape Effects on Stress-Strain Behaviour of sand in Triaxial Compression and Plane Strain," *Geotechnique*, Vol. 25, No. 2, pp. 333-356.

HENKEL, D.J. and WADE, N.H. (1966). "Plane Strain Tests on a Saturated Remoulded Clay," *Journal of Soil Mechanics and Foundation Engineering Division*, ASCE, Vol. 92, SM-6, pp. 67-80.

HAMBLY, E.C. (1969). "A new True Triaxial Apparatus," *Geotechnique*, Vol. 19, No. 2, pp. 307-309.

HAMBLY, E.C. and ROSCOE, K.H. (1969). "Observation and prediction of Stresses and Strains During Plane Strain of 'Wet' Clays," *Proceedings 7th Internatianal Conference on Soil Mech. and Foundation Engineering*, Vol. 1, pp. 173-181.

KO, H.Y. and SCOTT, R.F. (1967). "A new Soil Testing Apparatus," *Geotechnique*, Vol. 17. No. 1, pp. 40-57.

LADE, P.V. and DUNCAN, J.M. (1973). "Cubical Triaxial Tests on a Cohesionless Soil," *Journal of Soil Mechanics and Foundation Division*, ASCE Vol. 99, SM-10, pp. 793-812.

LEE, K.L. (1970). "Comparison of Plane Strain and Triaxial Test on Sand," *Journal of Soil Mechanics and Foundation Division*, ASCE Vol. 96, SM 3 pp. 901-923.

MARACHI, N., CHAN, C.K., SEED, H.B. and DUNCAN, J.M. (1969). "Strength and Deformation Characteristics of Rockfill Materials," *Report TE-69-5 to State of California*, Department of Water Resources, University of California, Berkley.

NAGARAJ, T.S. and SOMASEKHAR, B.V. (1979). "Stress Deformation and Strength of Soils in Plane Strain," *Proceedings of 6th Asian Reg. Conf. on Soil Mech. and Foundation Engineering*, Singapore, Vol. 1, pp. 43-46.

PARRY, R.H.G. (1971). "A study of the Influence of the Intermediate Principal Stress on ϕ' Values using a Critical State Theory," *Proceedings of 4th Asian Regional Conference on Soil Mech. and Foundation Engineering* Bangkok, Vol. 1, pp. 159-165.

RAMAMURTHY, T. (1970). A Universal Triaxial Apparatus," *Journal of Indian National Society of Soil Mechanics and Foundation Engineering*, Vol. 9, No. 3, pp.251-269.

RAMAMURTHY, T. and RAWAT, P.C. (1973). "Shear Strength of Sand under General Stress Systems," *Proceedings of 8th International Conf. on Soil Mechanics and Foundation Engineering*, Moscow, Vol. 1, pp. 339-342.

RAMAMURTHY, T. and TOKHI, V.K. (1981). "Relation of Triaxial and Plane Strain Strengths," *Proceedings of 10th International Conference on Soil Mechanics and Foundation Engineering*, Stockholm, Vol. 1, pp. 755-758.

RAWAT, P.C. (1976). "Shear Behaviour of Cohesionless Materials under Generalized Conditions of Stress and Strain,"*Ph.D. Thesis*, I.I.T. Delhi.

RAWAT, P.C. and RAMAMURTHY, T. (1978). "Shear Behaviour of sand under Generalized Conditions of Stress and Strain," *Indian Geotechnical Journal*, Vol. 8, No. 4, pp. 235-269.

ROWE, P.W. (1969). "The Relation Between the Shear Strength of Sands in Triaxial Compression, Plane Strain and Direct Shear," *Geotechnique*, Vol. 19, No. 1, pp. 75-86.

SHANKERIAH, B, (1977). Behaviour of Anisotropic Granular Media Under General Stress Systems. *Ph.D. Thesis*, I.I.T. Delhi.

TOKHI, V.K. and RAMAMURTHY, T. (1980), "Evaluation of Strength Parameters for Plane Strain Problems," *Proceedings of GEOTECH-80, A Conference on Geotechnical Engineering*, Bombay, Vol. 1, pp. 45-49.

WOOD, C.C. (1958). "Shear Strength and Volume Change Characteristics of Compacted Soil under Conditions of Plane Strain," *Ph.D. Thesis*, London University.

WADE, N.H. (1963). Plane Strain Failure Characteristics of a Saturated Clay. *Ph.D. Thesis*, University of London.

Notations

b $= (\sigma'_2 - \sigma'_3)/(\sigma'_1 - \sigma'_3)$

D $= 1 - \dfrac{\delta(\Delta V/V)}{\delta \epsilon_1}$ Volumetric strain rate, expansion, negative

I_D = Relative density (density index) $= \dfrac{e_{max} - e}{e_{max} - e_{min}}$ where e_{max} and e_{min} are the maximum and the minimum void ratios and e is the desired void ratio

K = Stress ratio,

K_o = Coefficient of earth pressure at rest

n = Porosity, per cent

$\Delta V/V$ = Volumetric strain, expansion, negative

ϕ' = Angle of shearing resistance based on effective stresses

μ_l = Lode parameter $= (2\sigma'_2 - \sigma'_1 - \sigma'_3)/(\sigma'_1 - \sigma'_3)$

$\sigma'_1, \sigma'_2, \sigma'_3$ = Major, intermediate and minor principal effective stresses

$\epsilon_1, \epsilon_2, \epsilon_3$ = Strains in major, intermediate and minor principal stress directions

σ'_{oct} = Octahedral normal stress (effective)

$= 1/3\,(\sigma'_1 + \sigma'_2 + \sigma'_3)$

τ'_{oct} = Octahedral shear stress (effective)

$= 1/3\sqrt{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2}$

γ_{oct} = Octahedral shear strain

$= 2/3\sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2}$

Subscript f denotes at failure

p denotes plane strain compression test

c denotes axisymmetric compression test