Short Communication

An investigation on mass ratio and eccentric moment

by

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Introduction

Foundations are subjected to vibrations due to several causes such as high speed machines. earthquakes, explosions, pile driving and wind. In all these cases, dynamic loads also act on the foundations in addition to the static loads. Therefore, the design principles of a machine foundation are different from that of the conventional foundation as it involves vibrations due to to unbalanced force caused by the machine. After satisfying the stability and settlement criteria in the design of a machine foundation, one has to ensure further that, the amplitude of motion of the machine foundation-soil system is kept within the permissible limits and also to avoid resonance condition.

Research work carried out at I.I.T. Madras has shown that prediction of response of a machine foundation, resting on soil surface and subjected to either vertical horizontal or rocking mode of vibrations, can be satisfactorily made from a single field vibration test. Analysis of test data on vertical vibrations conducted at Vicksburg and Eglin sites (Fry, 1963), the test data of Novak's (Novak, 1970), the field tests conducted at I.I.T,, Madras, for vertical vibrations (Subrahmanyam, 1971), the field tests conducted at I.I.T., Madras for horizontal viberations (Rama Sastri, 1975), and the field tests conducted by the author at I.I.T., Madras for rocking mode of vibrations (Sreekantiah, 1978) have shown that the observed values of resonant frequency and peak amplitude of motion of a machine foundation resting on soil surface and subjected to either vertical or horizontal or rocking mode of vibrations can satisfy many contact pressure (or contact shear) distributions each with corresponding values of shear modulus, G, and Poisson's ratio, μ. In this paper, an attempt has been made to find the influence of mass ratio and eccentric moment on the dimensionless amplitude and frequency factor with respect to three contact pressure distributions, namely, rigid base, uniform and parabolic.

Dimensionless parameters

The dimensionless parameters used in the analysis of vertical, horizontal and rocking modes of vibrations are as follows.

Dimensionless frequency factor

$$a_o = \omega . r_o . \sqrt{\frac{\rho}{G}} \qquad ...(1)$$

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where,

ω = circular frequency of the system in radians/unit time.

ρ = mass density of soil

G = shear modulus

r_o = radius of footing.

Dimensionless amplitude

$$A_1 = \frac{m A_z}{M_e l} \text{ (Richart and Whitman. 1967) } \quad ...(2)$$

where,

A_1 = dimensionless amplitude in vertical motion.

A_z = amplitude of vertical vibrations

$M_e l$ = eccentric moment

m = mas of machine and foundation block.

Dimensionless amplitude

$$A_2 = \frac{m A_x}{M_e l} \quad ...(3)$$

where,

A_2 = dimensionless amplitude in horizontal motion

A_x = amplitude of horizontal vibrations.

Dimensionless amplitude

$$A_3 = \frac{m A_\varphi \; r_o}{M_e l} \text{ (Sreekantiah, 1978) } \quad ...(4)$$

where,

A_3 = dimensionless amplitude in rocking motion

A_φ = amplitude in rocking motion

r_o = radius of footing.

Dimensionless mass ratio

$$b = \frac{m}{\rho r_o^3} \quad ...(5)$$

where,

b = dimensionless mass ratio in translation

r_o = radius of footing

m = mass of machine and foundation block

ρ = mass density of soil.

Dimensionless inertia ratio

$$b' = \frac{I}{\rho r_{o5}} \quad ...(6)$$

where,

b' = dimensionless mass ratio in rotation

I = mass moment of inertia of the machine and foundation block about the axis of rotation.

Field vibration tests

Field vibration tests were conducted at I.I.T., Madras, for vertical, horizontal and rocking modes of vibration on footings of various sizes and masses and for different eccentric moments. The footings are resting on soil surface and subjected to either vertical or horizontal or rocking modes of vibrations. The salient features of the field tests are summarised as follows :

Vertical vibrations

The soil at site (I.I.T., Madras) is silty sand with an average mass density of 1.75 x 10^{-6} kg. sec^2/cm^4. The average moisture content was 6% and the water table was about 3 m below the ground surface. The footings are circular and the diameters of the footings are 76.2 cm, 91.4 cm and 106.7 cm and their weights range from 177 kg to 353.8 kg. The footings are resting on soil surface and subjected to veritical vibrations. The mass of the oscillator and foundation block has been varied from 74.4 kg. sec^2/m to 116.8 kg. sec^2/m. The eccentric moment has been varied from 0.0025 kg. sec^2 to 0.0203 kg. sec^2 (Subrahmanyam, 1971). The value of shear modulus, G, has been evaluated at 180 kg/cm^2 based on essentially constant value of the dynamic soil modulus, $k_s = \left(\frac{4G}{1-\mu}\right)$, at site with $\mu = 0.3$ (silty sand).

Horizontal vibrations

The soil at site (I.I.T., Madras) is silty sand with an average mass density of 2.02×10^{-6} kg. sec^2/cm^4. The average moisture content was 6% and the water table was about 3.5 m below the ground surface R.C.C. square and rectangular footings were used and their sizes varied from 60×60×15 cm, 75×75 x 15 cm, 90×90×15 cm, and 100×150×15 cm. The footings had their weights ranging from 140 kg to 584 kg. The footings are resting on soil surface and subjected to horizontal vibrations. The mass of the oscillator and foundation block has been varied from 0.411 kg. sec^2/cm to 1.465 kg. sec^2/cm. The eccentric moment has been varied from 0.0025 kg. sec^2 to 0.0408 kg. sec^2 (Rama Sastri, 1975). The value of G has been evaluated at 98 kg/cm^2 based on the essentially constant value of the dynamic soil shear modulus. $k_{sh} = \frac{8G}{(2-\mu)}$, at site with $\mu = 0.3$.

Rocking Vibrations

The soil at site (I.I.T., Madras) is silty sand which was fairly homogeneous to a large depth. The average mass density of soil was 1.95×10^{-6} kg. sec^2/cm^4. The average moisture content was 9 per cent and the water table was about 3.5 m below the ground surface. R.C.C. square fottings were used and the size of the footings were 60×60×30 cm, 60×60×45 cm, 90×90×30 cm, 90×90×45 and 120×120×45 cm. The footings are resting on soil surface and subjected to horizontal vibrations which generate coupled modes of rocking and sliding vibrations. The mass of the oscillator and foundation block has been varied from 0.565 kg. sec^2/cm to

2.284 kg. sec²/cm and the mass moment of inertia, I, of the machine and foundation block about the axis of rotation has been varied from 954 kg. sec². cm to 5820 kg. sec². cm. The eccentric moment has been varied from 0.00510 kg. sec² to 0.03935 kg. sec². The value of the lever arm, H, measured from the base of the footing to the horizontally excited force, varies from 51.5 cm to 79.3 cm. The magnitude of the moment, $M_e 1H$, varies from 0.26265 kg. sec². cm to 3.12045 kg. sec². cm (Sreekantiah, 1978). The value of G has been evaluated at 94.5 kg/cm² based on the essentially contant value of the dynamic soil rocking modulus, $k_{sr} = \frac{8G}{3(1-\mu)}$, at site, with $\mu = 0.3$.

Analysis of test data

The field vibration tests conducted at I.I.T., Madras, for each mode of vibration have been used to study the effect of mass ratio and eccentric moment on the dimensionless amplitude and frequency factor with respect to three contact pressure (or contact shear) distributions. The analysis consists of obtaining the dimensionless amplitudes and frequency factors in each mode of vibrations for the tests which satisfy either rigid base or uniform or parabolic contact pressure (or contact shear) distribution. The results are presented in the form of curves for each mode of vibrations in Figs. 1 to 6.

Discussion

Vertical Vibrations

The curves showing the relationship between A_1 and b, for different values of eccentric moment, for the case of parabolic contact pressure

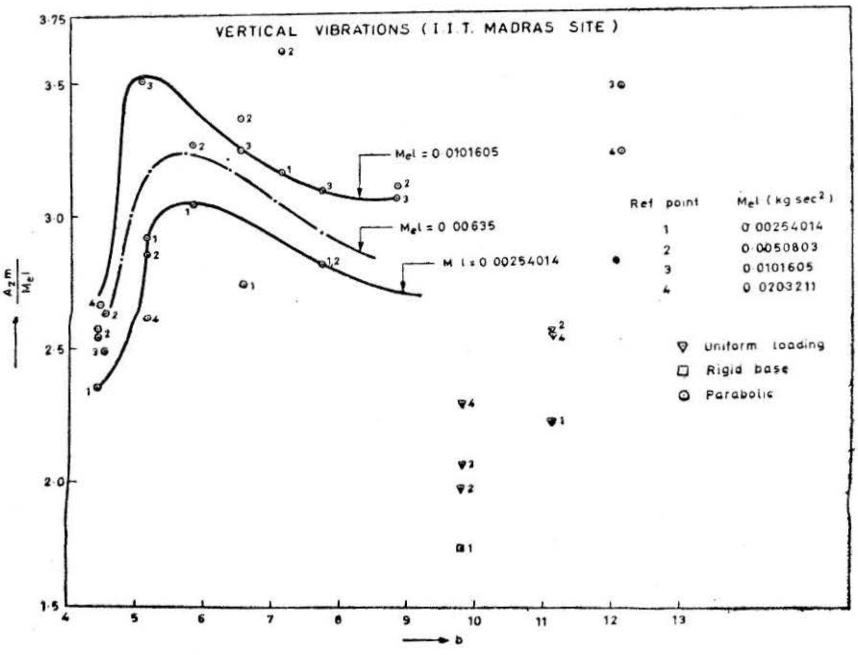

FIGURE 1 Relationship between $\frac{A_z m}{M_e 1}$ and b

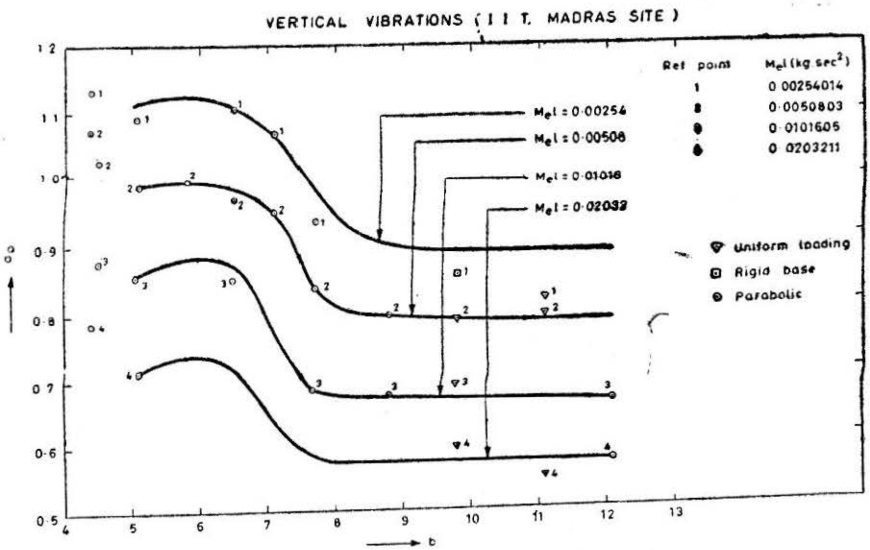

FIGURE 2 Relationship between a_0 and b

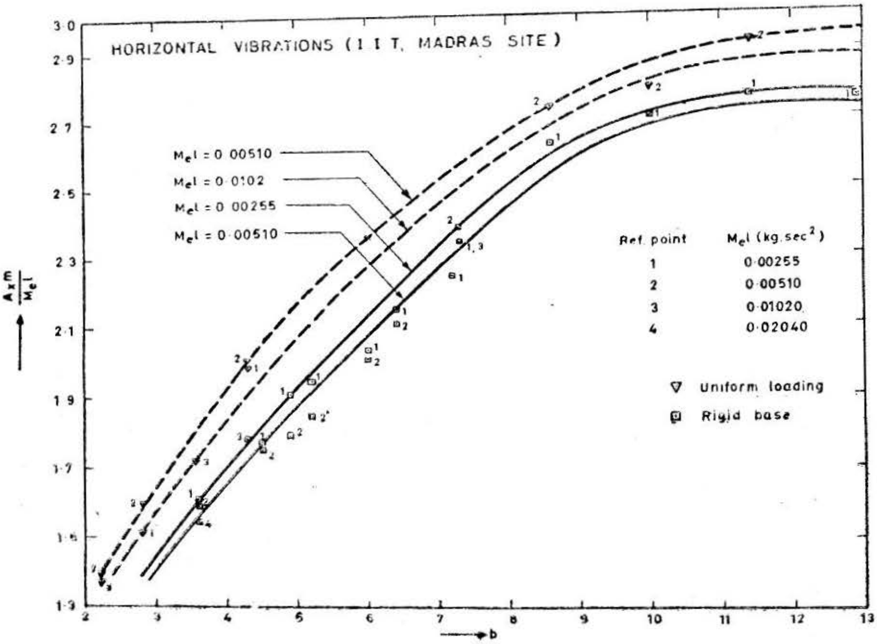

FIGURE 3 Relationship between $\frac{A_x m}{M_e 1}$ and b

distribution are presented in Figure 1. Owing to insufficient data it was not possible to draw curves showing the variation of M_e 1 for the case of uniform and rigid base contact pressure distributions.

From the above curves, it has been observed that for a particular value of M_e 1, A_1 reaches a peak value corresponding to $b = 5.5$ and then it

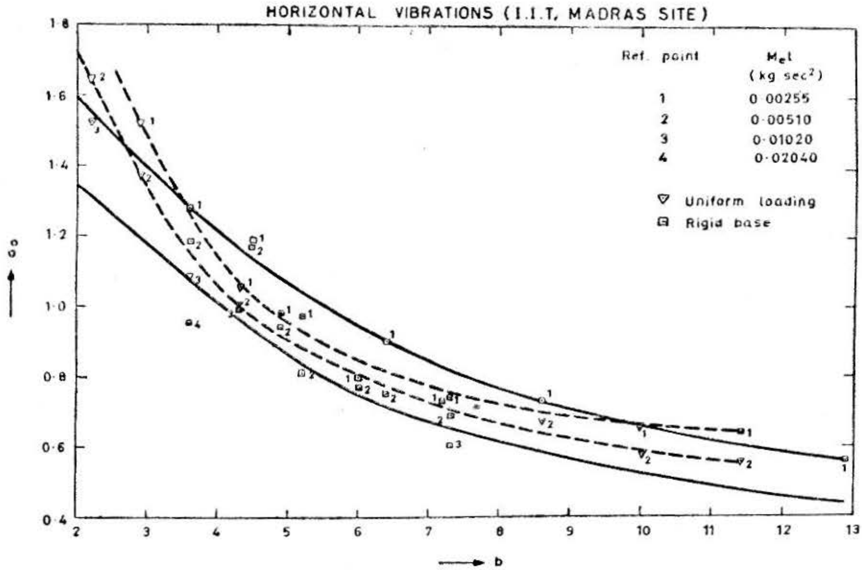

FIGURE 4 Relationship between a_0 and b

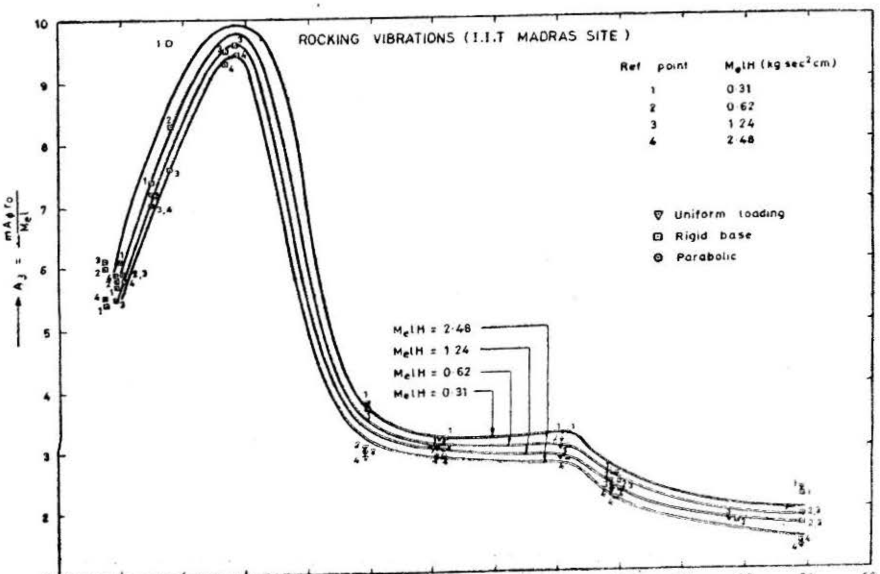

FIGURE 5 Relationship between A_3 and b'

decreases steeply with further increase in b. Further it has been found that as eccentric moment increases A_1 also increases.

The curves showing the relationship between a_0 and b for different values of M_e 1 for the case of parabolic contact pressure distribution are presented in Figure 2. For the case of uniform and rigid base contact pressure distributions, owing to insufficient data it was not possible to draw curves showing the variation of M_e 1.

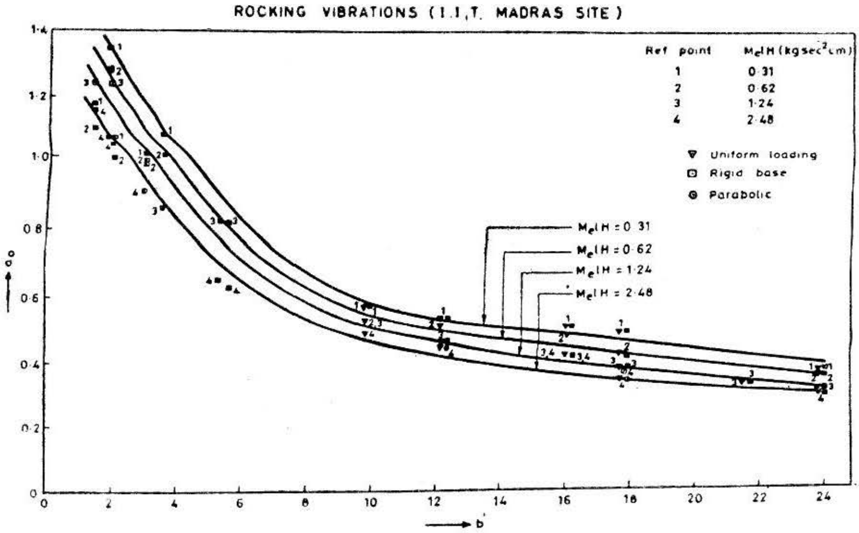

FIGURE 6 Relationship between a_0 and b'

From the above curves, it has been observed that, for a particular value of M_e 1, a_0 reaches a peak value corresponding to $b = 6$, then it decreases upto a value corresponding to $b = 8.5$, then there is not much variation in a_0 with further increase in b. Further, it has been found that as eccentric moment increases, frequency factor a_0 decreases.

Horizontal Vibrations

The curves showing the relationship between A_2 and b, for different values of eccentric moment, for the case of uniform and rigid base contact shear distributions are presented in Figure 3. It has been observed from the curves that, for a particular value of eccentric moment, A_2 increase as b increases and further as M_e 1 increases A_2 decreases. The curves showing the relationship between a_0 and b, for different values of eccentric moment, for the case of uniform and rigid base contact shear distributions have been presented in Figure 4. It has been observed from the curves that, for a particular value of eccentric moment, a_0 decreases with increase in b and further as M_e 1 increases, a_0 decreases.

Rocking Vibrations

The curves showing the relationship between A_3 and b', for different values of the magnitude of the moment (M_e $1H$), for case of uniform and rigid base contact pressure distributions have been presented in Figure 5. These curves show that, for a particular value of M_e $1H$, A_3 increases and a peak formed around $b' = 5.5$, then it decreases with further increase in b' and after reaching a value of $b' = 10$, this decrease is not much significant. Further, it has been found that, as M_e $1H$ increases A_2 decreases. For the case of parabolic contact pressure distribution, owing to insufficient data it was not possible to draw curves showing the variation of M_e $1H$. The curves showing the relationship between a_0 and b', for different values of M_e $1H$, for the case of uniform and rigid base contact pressure distributions have been presented in Figure 6. These curves show that, for a particular value of M_e $1H$, a_0 decreases uniformly with corresponding increases in b' and further it has been observed that as M_e $1H$ increases, a_0 decreases. For the case of parabolic contact pressure distribution, owing to insufficient data, it was not possible to draw curves showing the variation of M_e $1H$.

Conclusions

Based on the above investigation, the following conclusions have been drawn.

1. The effect of mass ratio is to increase the dimensionless amplitude in both vertical and horizontal vibrations, but the effect of inertia ratio is to decrease the dimensionless amplitude after it reaches a maximnm value in rocking vibrations.
2. The effect of eccentric moment is to increase the dimensionless amplitude in vertical vibrations and to decrease the dimensionless amplitude in horizontal vibrations, but the effect of the magnitude of the moment is to decrease the dimensionless amplitude in rocking vibrations.
3. The effect of mass (or inertia) ratio is to decrease the frequency factor in all the modes of vibrations.
4. The effect of eccentric moment (or magnitude of moment) is to decrease the frequency factor in all the modes of vibrations.
5. The effect of contact pressure distribution on the dimensionless amplitude in vertical vibrations is the highest in parabolic and higher in uniform with reference to that in the rigid base. The effect of contact shear distribution on the dimensionless amplitude in horizontal vibrations is higher in uniform than in the rigid base. However, this effect of contact pressure distribution is not much significant in rocking vibrations.
6. The effect of contact pressure distribution on the dimensionless frequency in vertical vibrations is the highest in rigid base and higher in uniform with reference to that in the parabolic. This effect of contact pressure distribution is not much significant in rocking vibrations.

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