Laterally loaded fixed-head Piles in a layered soil system

by

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Introduction

ANALYSES of the behaviour of piles subjected to lateral loads have generally employed the theory of subgrade reaction. A more satisfactory analysis for the behaviour of a single pile as well as pile groups has been recently developed in which the soil is assumed to be an elastic continuum, Spillers and Stoll (1964), Poulos (1971, 1975) and George and Char (1974). In these analyses the elastic modulus of soil is considered either constant or increasing linearly with depth. Davisson and Gill (1963) has studied analytically the effect of a layered soil system on laterally loaded piles using the concept of subgrade reaction. The analysis has been restricted to infinitely long piles, i.e. flexible piles and the conclusions are valid for long piles only.

In the design and analysis of the pile foundations, engineers are likely to be encountered with a stratified soil strata having different soil properties. It is also recognised that uniform soil deposit offers less resistance to a laterally loaded pile near the ground surface and the stiffening of the surficial soil reduces the deflection of a laterally loaded pile.

Pise (1979) has studied the influence of a two-layer soil system on a free-head pile subjected to lateral load and moment, using the elastic theory In this paper the author presents an analysis and results for a laterally loaded fixed-head pile embedded in a two-layer soil system using elastic theory. The analysis is similar in principle to that suggested by Spillers and Stoll (1964) and the results are presented in a more general and non-dimensional from involving geometrical parameters of a pile, elastic properties of the soil and the pile and the thickness of the surface layer. The results presented cover wide ranges of practical utility and it is believed that they will provide guidelines to the design and analysis of pile foundations to the practising engineers whose main concern is the minimum deflection and maximum bending moment occurring in a pile,

Theoretical Analysis

General

Figure 1 show the free body diagram of a pile and soil mass, and the variation of elastic modulus used in the analysis. The elastic modulus of the surface layer E_t is expressed in terms of elastic modulus of the under layer E_b. The ratio E_t/E_b is termed the layer cofficient C which is assigned

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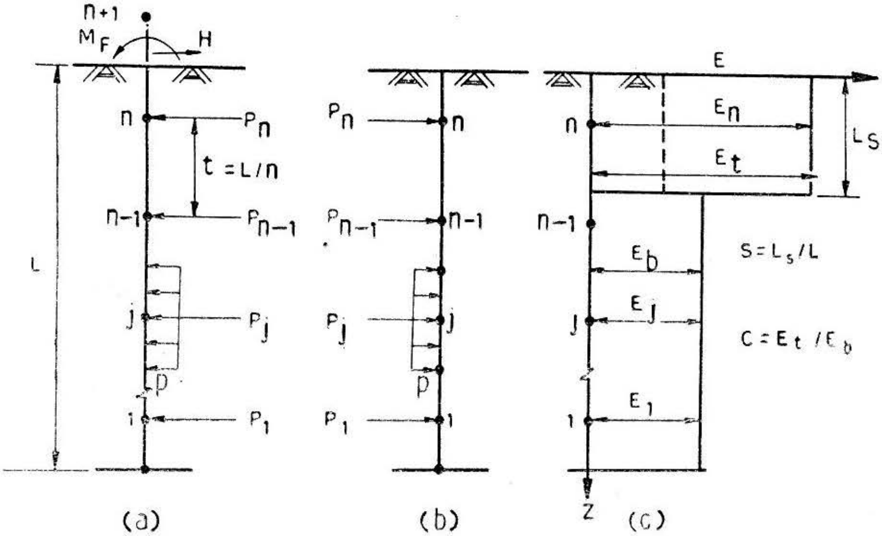

FIGURE 1 (a) Free body diagram of a Pile
(b) Free Body diagram of soil Mass
(c) Elastic Modulus Variations of soil used

values from 0.5 to 10. The thickness of the surface layer L_s is defined in terms of the embedded length L of the pile. The ratio L_s/L is termed the layer thickness ratio S. Surface layer thicknesses from 5 per cent to 40 per cent are studied.

The soil in soil in the respective loyers is assumed to be an ideal, homogeneous, isotropic and elastic material. The displacement is assumed to be continuous, across the interface of two layers. Although, Mindlin's (1936) equations are valid for elastic, isotrophic and homogeneous soil mass, they have been successfully employed to predict the deformations in non-uniform soil media by Poulos (1973, 1979) and George and Char (974). For convenience and economy Mindlin's (1936) equation has been used to predict the horizontal displacements in the soil media albeit approximately.

The pile displacements have been obtained from the equation of flexure of a beam expressed in finite difference form. The pile is assumed to be a thin rectangular vertical strip of width d, length L and constant flexibility E^pI_p. For the purpose of carrying out analysis, the pile is divided into n elements each of equal length, $t = L/n$, as shown in Figure 1. Each element is acted upon by an unifarm horizontal stress p which is approximated to concentrated force P acting at the centre of the element.

Soil Displacements

According to the Mindlin's equation (1936) the horizontal displacement $\bar{y}_{ij}$, of the soil at a point, i, along a vertical line adjacent to the pile surface at a depth, $\bar{y}_i$, due to horizontal load, P_j, located at a depth, D_j, from the ground surface and acting on the element, j, can be written as,

$$\bar{y}_{ij} = \frac{P_j}{16\pi\,(1-v_s)G_j}\quad\frac{(3-4v_2)}{[Z,-D_j]}+\frac{1+2(1-v_s)\,(1-2v_s)}{(Z_i+D_j)}+\frac{2D_j\;\;Z_i}{(Z_i+D_j)^3}\quad\ldots(1)$$

where G_j = shear modulus of soil adjacent to the pile element j, v_s = Poisson's ratio of soil.

$$G_j = \frac{E_j}{2(1+v_s)} \quad ...(2)$$

where E_j = Young's modulus of soil adjacent to pile element j. The displacement y_i at the point i at a depth Z_i below the ground surface due to horizontal forces acting on all elements of the pile is,

$$y_j = \sum_{j=1}^{n} y_{ij} \quad ...(3)$$

The term $1/[Z_i - D_j]$ becomes singular in Equation 1 at $Z_i = D_j$. Its value is taken equal to $2/a$ at $Z_i = D_j$ as taken by Spillers and Stoll (1964), where

$$a = \left(\frac{t\ d}{\pi}\right)^{1/2} \quad ...(4)$$

It is seen from Equation 3 that the displacement y_i of the i-th point is a function of P_j and G_j. The Poisson's ratio v_s for both the layers of soil is taken as constant.

Pile Displacements

Expressing the beam equation in finite difference form, for any point i, on the pile

$$E_p I_p \left(\frac{y_{i-1} - 2y_i + y_{i+1}}{t^2}\right) = M_i \quad ...(5)$$

or

$$y_{i-1} - 2y_i + y_{i+1} = \frac{M_i t^2}{E_p I_p} \quad ...(6)$$

where M_i= bending moment in the pile at point i; and y_i= deflection of the pile at point i.

For a fixed-head floating pile, shear and moment are equal to zero at the pile tip. The bending moment M_i can be written in terms of point forces acting on the centre of the elements as

$$M_i = \sum_{j=2}^{i=1} P_j (i-j)t \quad ...(7)$$

From Equation 6 and 7

$$y_{i-1} - 2y_i + y_{i+1} = \frac{t^3}{E_p I_p} \sum_{j=1}^{i=1} P_j (i-j) \quad ...(8)$$

Considering an imaginary element $(n+1)$ extending above the top of the pile head, the rotation at ground surface, θ_g, of a pile head can be expressed as,

$$\theta_g = \frac{y_n - y_{n+1}}{t} \quad ...(9)$$

For a fixed-head pile $\theta_g=0$, and from Equation 8

$$y_n=y_{n+1} \qquad ...(10)$$

Equilibrium condition for horizontal forces for a fixed-head floating pile is

$$\sum_{j=1}^{n} P_j=H \qquad (...11)$$

where $H=$ applied lateral load.

Taking $y_n=y_{n+1}$, $(n-1)$ equations can be formulated from Equation 8 for the points $i=2$ to n in terms of displacements and forces. Assuming elastic conditions prevailing within the soil, the soil and pile displacements may be equated at the element centres. The values of soil displacements $\bar{y}_i$ as predicted from Equation 3 may be substituted for the pile displacements $\bar{y}_i$ in the above (n_{-1}) equations. The solution of the (n_{-1}) simultaneous equations with the Equation 11 give the values of the unknown forces P_j and hence the unknown displacements, bending moments at elements centres for points $i=1$ to n.

Fixed-Head Moment and Ground Displacement of a Pile Head

The fixed-head moment, M_F, is expressed as

$$M_F=M_n - \frac{H\,t}{2} \qquad ...(12)$$

Writing Equation 6 for a point on the pile head at ground surface,

$$y_n-2y_g+y_{n+1}=\frac{M_F\,t^2}{4\,E_p\,I_p} \qquad ...(13)$$

where $y_g=$ ground displacement of a fixed-head pile.

From Equationl 10 and 13,

$$y_g=y_n-\frac{M_F\,t^2}{8\,E_p\,I_p} \qquad ...(14)$$

The pile head displacement and fixed-head moment are found to depend markedly on length to diameter ratio, L/d, stiffness of the pile relative to the soil, layer thickness ratio S, layer coefficient, C and lateral load H. Analogous to the expressions given by Poulos (1971, 1973), following equations are conveniently written for a laterally loaded pile embedded in a layered soil system.

$$y_g=I_{yF}\,\frac{H}{E_b\,L} \qquad ...(15)$$

$$K_R = \frac{E_p\,I_p}{E_b\,L^4} \qquad ...(16)$$

$$\text{and } M_F= m_f\,H\,L \qquad ...(17)$$

where $I_{yF}=$ displacement influence factor for a fixed-head pile; $K_R=$ pile flexibility factor with reference to E_b; and $m_f=$ fixed-head moment coefficient.

Analysis of the Results

In the present analysis the pile was divided into 20 elements. From set of values of K_R, H, E_b, L, d, C, S and ν_s=0.5, twenty simultaneous equations involving P_j were formulated. The solution of these equations for P_j was obtained on the computer. The ground displacement and fixed-head moment were evaluated and in turn I_{yF} and m_f were calculated. L/d was assigned values of 10, 25, 50 and 100 and $K_R=10^{-5}$ to 10^0.

Discussion of Typical Results

Piles of different L/d ratio behaved qualitatively alike. Typical results of L/d=10 and 100 have been considered here for discussion.

Values of I_{yF} are plotted for L/d= 10 and 100 for various layer coefficients for S= 0.2 in Figure 2(*a*). K_R, L/d, and C influence the behaviour of a laterally loaded fixed head pead pile. I_{yF} i. e. in turn displacement increases as either K_R decreased or C decreases. I_{yF} is higher for piles having larger L/d ratio and lower C values of soil.

Figure 2(*b*) shows m_f versus K_R for typical cases. m_f i. e. in turn fixed-head moment in a pile increases as either K_R increases or C decreases and m_f is higher for piles having larger L/d ratio and lower C values of soil.

Figures 3 and 4 shows I_{yF} versus layer coefficient for few typical cases. The curves intersect at a layer coefficient of unity; furthermore, the curves for a layer thickness ratio of zero are horizontal lines passing through the points of intersection. The magnification or damping of the deflection as C is less than or more than unity, respectively, is clearly seen. There appears to be little gain with respect to damping of deflection for layer coefficient exceeding 8; and points of diminishing returns are reached at approximately 6 for all piles irrespective of L/d ratio and K_R values. From Figures 5 and 6, it is seen that with respect to damping of displacements, points of diminishing returns are reached approximately at layer thickness ratios of 0.05, 0.10 and 0.20 for piles having $K_R=10^{-5}$, 10^{-3} and 10^{-1} respectively irrespective of L/d ratio.

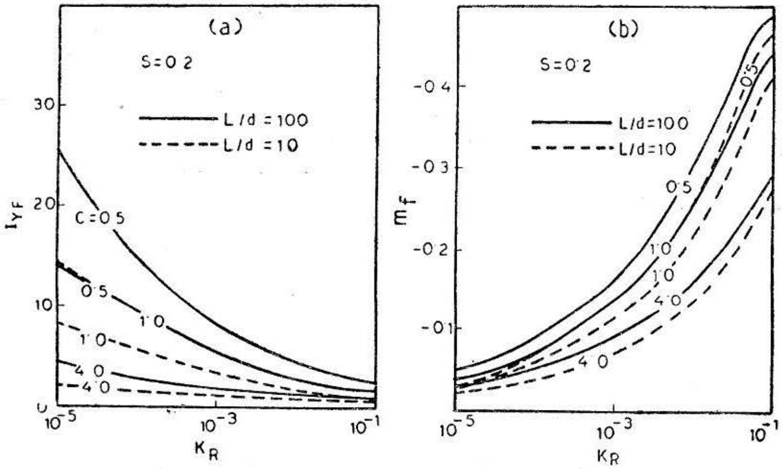

FIGURE 2 (*a*) **Influnce of** L/d **and** C **on** I_{YF} (*b*) **Influnce of** L/d **and** C **on** m_f.

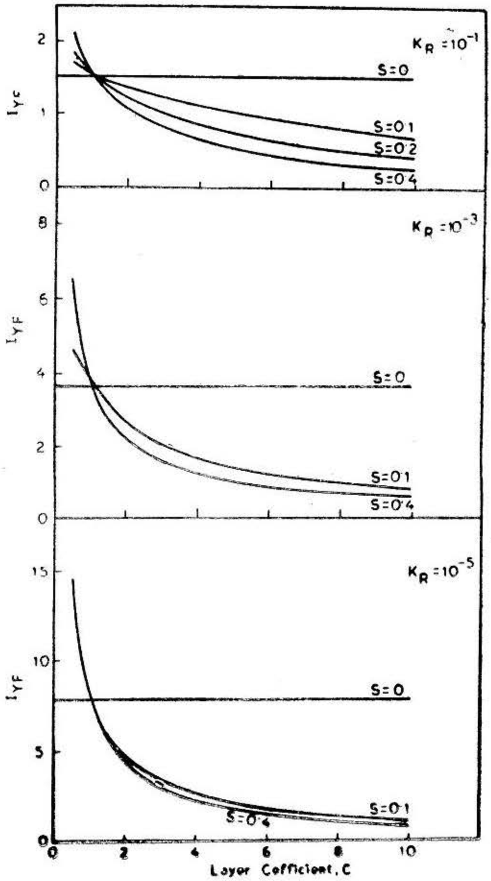

FIGURE 3 I_{YF} **Versus layes Coefficient** C, $(L/d=10)$

Figures 7 and 8 show m_f versus layer coefficient C for typical cases. The curves intersect at a layer coefficient of unity. Curves for a layer thickness ratio of zero are horizontal lines through the points of intersection. The magnification or damping of the fixedhead moments as C is less than or more than unity, respectively, is clearly seen. There appears to be little gain with respect to damping of fixed-head moment for layer coefficients exceeding 5 and 10 for piles having $K_R=10^{-5}$ and 10^{-3}, and $K_R=10^{-1}$ respectively and points of diminishing returns are reached at layer coefficients of 4 and 8 respectively for the respective K_R—values irrespective of L/d ratio. The curves of m_f against S are plotted in Figures 9 and 10 for few layer coefficients. It is seen that the curves pass through points of optimum magnification or reduction as C—value is less or more than unity respectively. For stiff surface layers $C>1$, points of diminishing

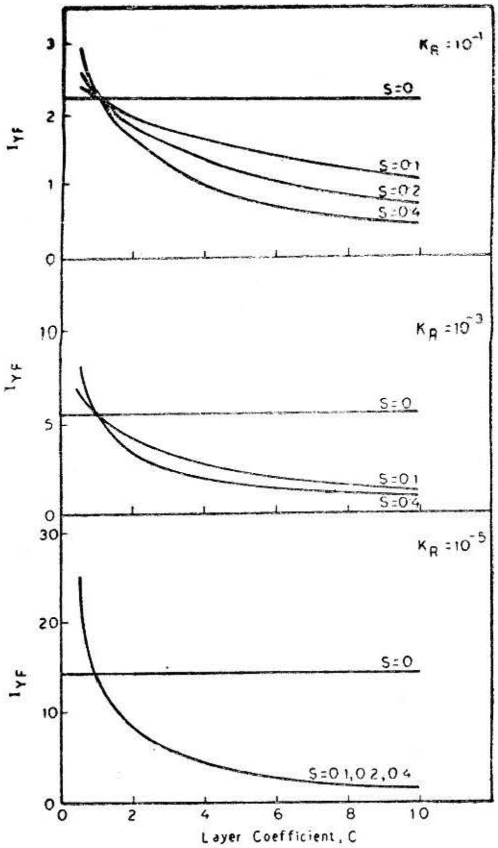

FIGURE 4 I_{YF} **Versus leyer Coefficient** C, $(L/d=100)$

returns with respect to damping of moments are achieved at layer thickness ratios of 0.10 and 0.20 for piles having $K_R=10^{-5}$ and 10^{-3}, and $K_R=10^{-1}$ respectively irrespective of L/d ratio.

Illustrative Example

The hypothetical example solved by Davisson and Gill (1963) is solved here by using author's results.

A 12 inch (0.305 m) steel pipe pile embedded 30 ft (9.15 m) in a clay of medium consistency is considered. The pile head is fixed at the ground surface and subjected to an axial load of 100 kips (45.3 t) and a lateral load of 20 kips (9.06t). The subgrade modulus k_o of the underlayer is taken as 500 psi (35.2 kg/cm^2). The pile has an area A of 14.58 sq. in (94 cm^2) and I_p = 279.3 in^4 (11,600 cm^4) add E_p = 10^7 psi (2.1×10^6 kg/cm^2). The fixed

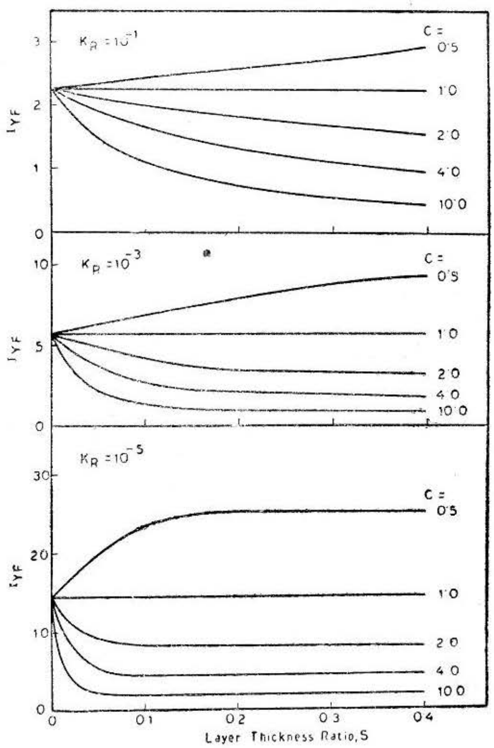

FIGURE 5 I_{YF} Versus S, (L/d=10)

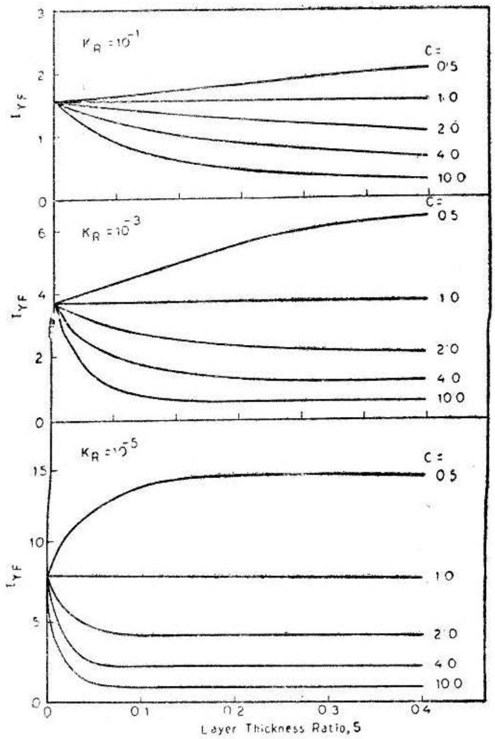

FIGURE 6 I_{YF} Versus S, (L/d=100)

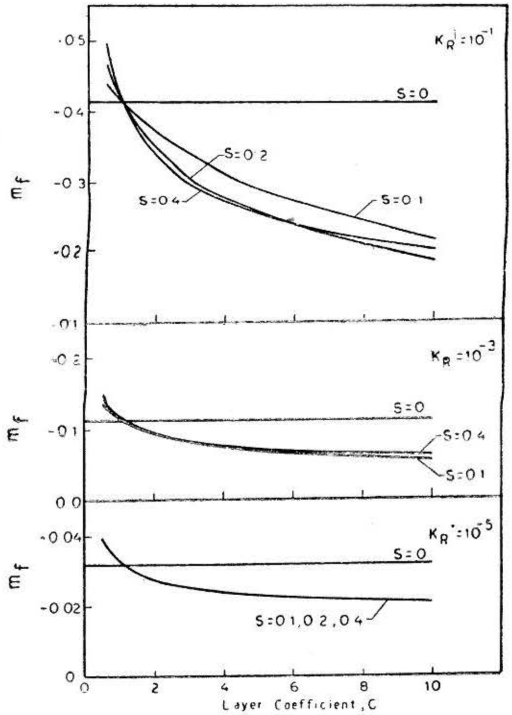

FIGURE 7 Fixed-Head Moment Coefficient m^f Versus C, ($L/d=100$)

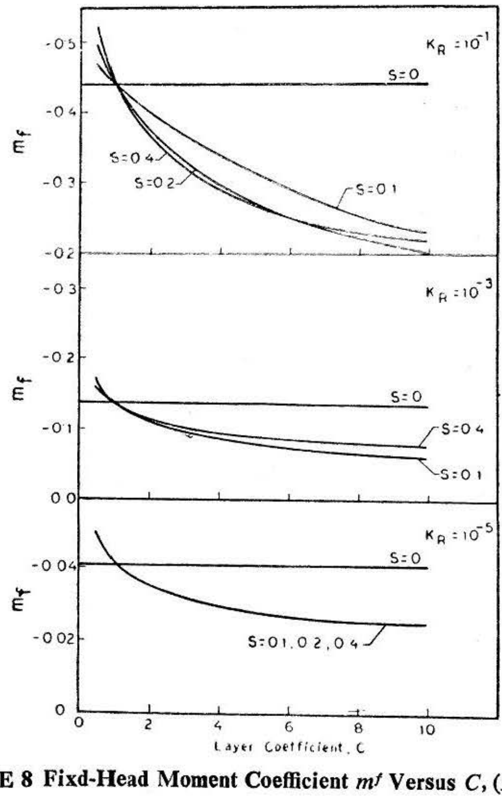

FIGURE 8 Fixd-Head Moment Coefficient m^f Versus C, ($L/d=100$)

head moments and ground deflections are computed for $C = 0.5$, 1 and 6 and surface layer thickness of 2 ft (0.61 m).

To compare the solutions from elastic theory with those given from the theory of subgrade reaction, it is necessary to establish a relationship between Young's moduls of soil and modulus of subgrade reaction k. The most accurate method appears to be to equate the elastic and subgrade reaction solutions for the displacement of a short stiff fixed head pile) rigid pile for uniform soil condition.

The influence of axial load on the lateral behaviour of the pile is neglected.

For piles having $K^r = 1$ and $L/d = 10$ and $C = 1$, L_{yf} was obtained as 1.23.

According to Broms (964) for rigid short fixed head pile embedded in uniform soil,

$$y_g = \frac{H}{k_o L} \qquad \text{... (18)}$$

Equating the displacement as obtained from Equation 15 and 18,

$$E_b = I_y F\, k^o \qquad \text{... (19)}$$

So $E_b = 1.23 \times 500 = 615.0$ psi (43.4 kg/cm²)

From Equation 16; K_R $0.82 \times 10^{-3} \simeq 10^{-3}$

From the given data, $K_R \simeq 10^{-3}$, $L/d = 30$ and $S = 0.067$ for the soil-pile system.

Aproximate interpolations for I_{yF} for $K_R = 10^{-3}$, $L/d = 30$ and $S = 0.047$, at C 0.5, 1.0 and from Figures 5 and 6 give $I_{yF} = 4.83$, 4.30 and 1.95 respectivly; and Figures 9 and 10 give $m_f = 0.135$, 0.12 and 0.070 for the respective layer coefficients C.

From Equations 15 and 17 the ground displacements and the fixed-head moments are respectively calculated. The results are tabulated in Table-1 along with those obtained by Davisson and Gill (1963).

The lateral deflections predicated by elastic theory are found to be less than those predicated by subgrade reaction theory. The predicted fixed-head moments, by both the theories, are, however, in reasonably good agreement.

TABLE 1

Results of Example Calculation

Layer Coefficient C	Davisson and Gill (1963)				Pise (1979)			
	Fixed-head moment, M_F		Ground displacement y_g		Fixed-head moment, M_F		Ground displacement, y_g	
	In-kips	(t-cm)	in.	(cm)	in-kips	(t-cm)	in.	cm)
0.5	−1015	(−1165)	0.53	(1.35)	−975	(−1120)	0.44	(1.11)
1.0	− 902	(−1038)	0.45	(1.14)	−867	(− 997)	0.39	(0.98)
6.0	− 502	(−557)	0.19	(0.48)	−505	(− 580)	0.18	(0.45)

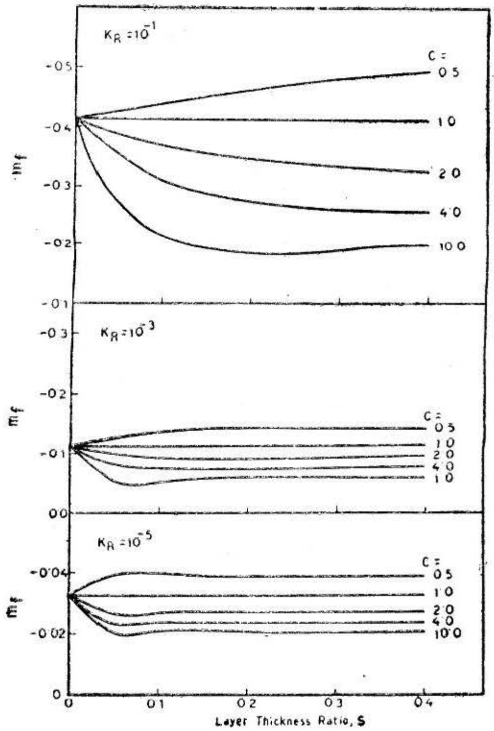

FIGURE 9 Fixed-Head Moment Coefficient m^f Versus S (L/d=10

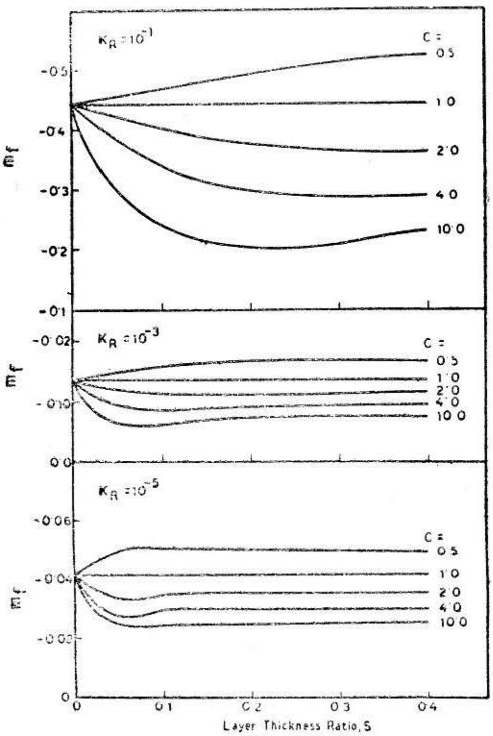

FIGURE 10 Fixed-Head Moment Coefficient m^f Versus S, (L/d=100)

Conclusions

1. The length to diameter ratio, pile flexibility factor, layer thickness ratio, and layer cofficient influence broadly the behaviour of a laterally loaded fixed-head pile.
2. Surface displacements and fixed-head moments are more for piles having large length to diameter ratio and embedded in soils of lower C-values.
3. For piles having $K_R = 10^{-5}$, 10^{-3} and 10^{-1}, points of diminising returns with respect to damping of deflection are reached at layer thickness ratios of 0.05, 0.10 and 0.20 respectively at a layer cofficient of 6 irrespective of L/d ratio.
4. For $K_R = 10^{-5}$ to 10^{-3} and $K_R = 10^{-1}$, points of diminishing returns with respect to damping of moment are achieved at layer thickness ratios of 0.10 and 0.20, at layer cofficients of 4 and 8 respectively.
5. Stiff surface layer upto a depth of about 0.21 has a considerable beneficial effect in reducing the deflection and fixed-head moment in a pile.
6. The illustrative example indicates that the results have got substantive potential to solve the problems of laterally loaded piles embedded in a layered soil system and that they are of acceptable accuracy.

Notations

C = layer cofficient

d = width or diameter of pile

D_j = depth from ground surface to the force P_j

E_b = Young's modulus of bottom soil layer

E_j = Young's modulus of soil at point j

E_p = Young's modulus of pile material

G_j = shear modulus of of soil at point j

H = applied horizontal load

I_p = moment of interia of pile section

I_{yF} = displacement influence factor for fixed-head pile

K_R = pile flexibility factor withe reference to E_b

L = embeded pile length

M_F = fixed-head moment in pile

M_i = bending moment in pile and point i

m_f = fixed-head moment cofficient

n = number of elements dividing pile

P_j = arbitrary horizontal force on pile at point j

S = layer thickness ratio

t = spacing of elements of the pile

y = horizontal displacement of pile

y_g = horiznotal displacement of pile at ground surface

$\bar{y}$ = horizontal displacement of soil

Z_i = depth from ground surface to a point where displacement is desired

V_s = Poisson's ration of soil

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