Coulomb's Theory of Earth Pressure for c-ϕ Soils

by

N. Babu Shankar*

Introduction

Coulomb's and Rankine's are the two classical theories of earth-pressure often used in the design of earth-retaining walls. Though both these theories are based on certain simplifying assumptions, *Coulomb's theory* is more general and in fact Rankine's theory can be treated as a particular case of Coulomb's theory.

Most of the text books on Soil Mechanics (Alam Singh, 1975) and Foundation Engineering (Bowles, 1977) give the expressions of active and passive earth pressures for sandy back fills, based on Coulomb's theory. These expressions are cumbersome and as such a number of graphical methods have been suggested. Similar expressions are not available for backfills of c-ϕ soils.

General expression is derived in this paper for the magnitude of active earth pressure, based on Coulomb's theory, for sloping backfills of c-ϕ soils with a uniform surcharge loading at the ground surface taking into the effect of possible craking upto certain depth below ground level. This general expression reduces to that of Rankine as a particular case and similarly all the existing solutions of Coulomb valid for a ϕ-soil may also be obtained as particular cases. Though the expression looks lengthy, it is easily soluable with the help of ordinary pocket calculator as is illustrated by a numerical example. Similar expression for the passive condition can be derived but is not attempted as the Coulomb's theory grossly overpredicts the passive earth pressure (Terzaghi, 1943).

Analysis

The problem under study in show in Figure 1, AB is the rough back of the retaining wall with a better angle of w. AD represents the sloping backfill (with a surcharge angle of i) which has a unit weight of γ, shear parameters of c and ϕ, an adhesion factor of α (i.e. unit adhesion $= \alpha C$) and an angle of wall friction of δ. A uniform vertical surcharge loading of q (per unit of horizontal area) acts along the surface AD. The problem is treated as two-dimensional (plane strain case).

Coulomb analyses the static equilibrium of the forces acting on the trial wedge ABD of the soil adjacent to the wall with an assumed plane rupture surface BD inclined at an angle of θ w.r.t horizontal. The various forces acting on the trial wedge and their directions are indicated in Figure 1 and also in the form of force polygon in Figure 2 W is the weight of the soil wedge. Q is the surcharge force. Both W and Q act vertically downward.

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* Assistant Professor, Civil Engineering Department, Regional Engineering College, Warangal—506 004, INDIA.

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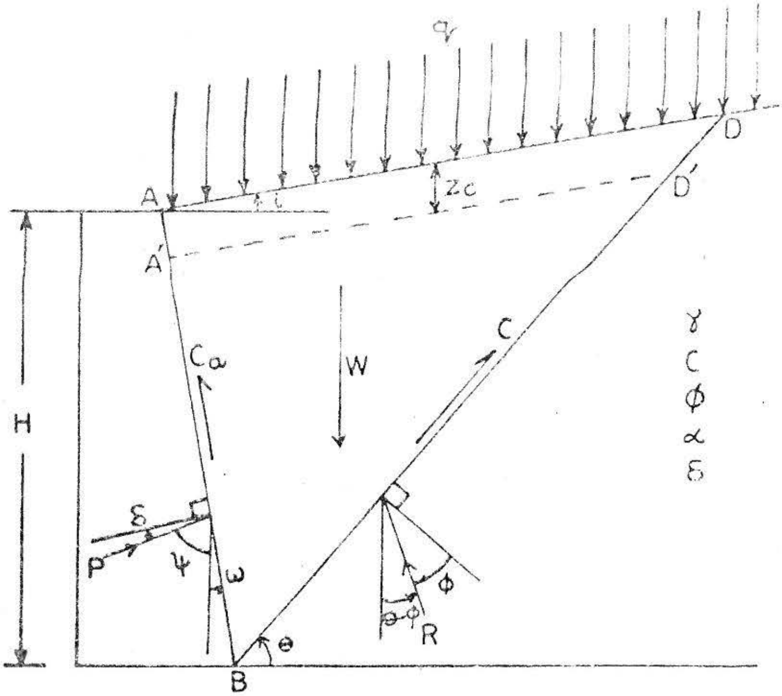

FIGURE 1 Earth Retaining Wall Problem Under Study

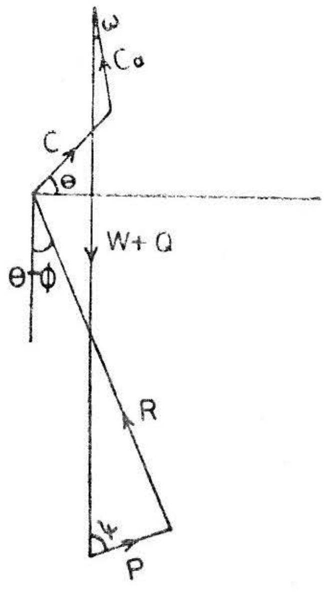

FIGURE 2 Polygon of forces acting on the trial Wedge

P is the (trial) active earth pressure with an angle of obliquity of δ w.r.t. the pressure surface AB. R is the resultant of normal forces and frictional resistance acting on the trial rupture surface BD. C is the cohesive force along the rupture surface. C_a is the adhesive force along the pressure surface BA.

Depending upon the relative values of q and c, there is a possibility of net negative pressure against the wall upto certain depth Z_c below ground surface. As soil cannot take tension, the tension cracks may develop upto that depth vertically below the ground surface. It has been proved by author (Babu Shanker, 1979) that by Rankine's theory.

$$Z_c = \frac{2C}{\gamma}\sqrt{N_\phi} - \frac{q}{\gamma} \qquad \ldots(1)$$

Where $N_\phi = \tan^2\left(45 + \frac{\phi}{2}\right) = \text{flow value}$

If cracking is a possibility, the effective length of pressure and rupture surfaces may respectively be taken as BA' and BD' in assessing the magnitudes of forces C_a and C.

Some of the forces can be quantified as shown below :

$$W = \gamma\,(\triangle\ ABD = \frac{\gamma H^2}{2}\,\frac{\text{Cos}\,(i-w)}{\text{Cos}^2\,w}\,\frac{\text{Cos}\,(\theta-w)}{\text{Sin}\,(\theta-i)}$$

$$= a\,\frac{\text{Cos}\,(\theta-w)}{\text{Sin}\,(\theta-i)} \qquad \ldots(2)$$

$$Q = q\,(AD\cos i) = \frac{q\,H\cos i}{\cos w}\,\frac{\text{Cos}\,(\theta-w)}{\sin\,(\theta-i)}$$

$$= b\,\frac{\text{Cos}\,(\theta-w)}{\text{Sin}\,(\theta-i)} \qquad \ldots(3)$$

$$C_a = c\,(BA') = ac\left[\frac{H}{\cos w} - \frac{Z_c\cos i}{\cos\,(i-w)}\right] \qquad \ldots(4)$$

$$C = c\,(BD') = C\left[\frac{H\cos\,(i-w)}{\cos w} - Z_c\cos i\right]\frac{1}{\sin\,(\theta-i)}$$

$$= g.\,\frac{1}{\sin\,(\theta-i)} \qquad \ldots(5)$$

Thus it is seen that excepting the force C_a, all the other forces are functions of θ, while a, b, g defined in the above equations are all independent of θ.

Assuming that the force polygon is a closed polygon resolving the forces in the direction perpendicular to R-force yields the following equation

$$P = \frac{(W+Q)\sin\,(\theta-\phi) - C\,\text{Cos}\,\phi - C_a\,\text{Sin}\,(\theta-w-\phi)}{\sin\,(\theta+\Psi-\phi)} \qquad \ldots(6)$$

P the trial earth pressure as determined from Equation 6 is a function of θ. The actnal magnitude of active earth pressure (P_a) is obtained from the extremum condition $P_a = (P)_{max}$ or $\frac{\partial P}{\partial \theta} = \theta$.

Let $$\lambda = 2\theta - \omega - \phi \qquad ...(7)$$

Differentiating Equation 6 w.r.t. θ with the notation for λ from Equation 7 and the use of extremum condition results in the following equation.

$$d \sin \lambda + e \cos \lambda - f = 0 \qquad ...(8)$$

where
$$d = m \sin (\delta - i) - C_a \cos i \cos(\phi - i - \psi)$$
$$e = (a+b) \cos (\phi - i - \psi) + C_a \sin i \cos (\phi - i - \psi) + m \cos (\delta + i)$$
$$f = (a+b) \sin (\delta + i) - C_a \cos \delta$$
$$m = 2g \cos \phi + (a+b) \sin (\phi - w) + C_a \cos (\phi + w - i) \qquad ...(9)$$

The solution of Equation 8 is given by

$$\lambda = \pi - \left\{ \sin^{-1} \frac{e}{\sqrt{d^2+e^2}} + \sin^{-1} \frac{f}{\sqrt{d_2+e^2}} \right\} \qquad ...(10)$$

Thus for a given problem knowing the quantities defined in Equation 2 through (5) and Equation 9, one solves for λ which from equation 7 yields the particular value of θ which may be designated as θ_f (the actual inclination of the straight rupture surface). Substitution of θ_f in Equation 6 results in the value P which is the desired value of the magnitude of active earth pressure (P_a).

Though straightaway a general expression for P_a may be tried (without involving $\theta = \theta_f$) such an expression will be too lengthy and cumbersome. Hence the abovs procedure is recommended. A numerical example will illustrate the use of the above method. However it can be shown either numerically or analytically that the general expressions given above reduce to the other simpler expressions valid for some particular conditions. For example, Rankine's classical expression for P_a valid for sandy backfills without surcharge may be letting $C = q = 0 = w = Z_c$; $\delta = i$ for which case one has:

$$d = a \sin \phi \sin 2i$$
$$c = 2a \sin \phi \cos^2 i$$
$$f = a \sin 2i$$

and hence Equation 10 reduce to

$$\sin \lambda = \frac{\sin^2 i + \cos i \sqrt{\cos^2 i - \cos^2 \phi}}{\sin \phi}$$

P_a may be evaluated from Equation 6 using the above condition and value of λ as as

$$P_a = \frac{W \sin(\theta_f - \phi)}{\sin(\theta_f + \psi - \phi)}$$

$$= a \frac{\cos(\phi_f - w)\sin(\theta_f - \phi)}{\sin(\theta_f - i)\sin(\theta_f + \psi - \phi)}$$

$$= a \frac{\sin\lambda - \sin(\phi_f - w)}{\cos(\phi - i - \,_f) - \cos\lambda\sin(\delta + i) + \sin\lambda\cos(\delta + i)}$$

$$= \frac{\gamma H^2}{2}\cos i\ \frac{\cos i - \sqrt{\cos^2 i - \cos^2\phi}}{\cos i + \sqrt{\cos^2 i - \cos^2\phi}}$$

$$= \frac{\gamma H^2}{2N\phi} \text{ for } i = 0 \text{ for which case } \theta_f = 45 + \frac{\phi}{2}$$

which are the familiar expressions.

Numerical Example

Assuming the data, $q = 1$ t/m^2, $c = 0.5$ t/m^2, $\alpha = 0.5$, $\gamma = 2$ t/m^3, $H = 10$ m, $i = 10°$, $w = 5°$ and $\phi = 30°$, we get the following quantities from various equations.

$a =$ 100.382	$d =$ 22.6125	$Z_c =$ 0.37 m
$b =$ 9.8856	$e =$ 122.9269	
$C_a =$ 2.4181	$f =$ 48.9367	
$g =$ 4.815	$m =$ 57.1302	

Thus λ from Equation 15 will be 77.37° $\therefore\ \theta_f = 56.19°$

From Equation 6 are obtains $P_a =$ 35.82 t/m. (which value Agree very nearly with the one obtained by graphical trial wedge method)

The corresponding value of P_a from Rankine's theory which is a special case of Coulomb's theory (with the conditions $\alpha = w = Z_c = 0$, $\delta = i$) is 32.5 t/m. This value is incidentally the same as obtained for a similar problem using Rankine's expressions given in a separate paper by the author (Babu Shankar, '79).

If $c = q = 0$, the value of $P_a =$ 38.72 t/m (which value can also be obtained from the Coulomb's formula given in standard text books for the cohesionless backfill).

Conclusions

Analytical expressions are derived based on Coulomb's theory of earth pressure for the magnitude of resultant active earth pressure for backfills of C-ϕ soil with a sloping ground surface which carries a uniform surcharge load. The effect of cracking due to net tensile forces is also considered. Inclination of the (assumed) straight rupture surface passing through the heel of the wall is also obtained. These general expressions reduce to simpler expressions given in standard text books for particular cases including those of Rankine. Though the expressions are lengthy, computation of earth pressure is facilitated by pocket calculator, without the necessity of resorting to the cumbersome graphical construction, as is demonstrated by numerical examples.

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