Determination of pressure on retaining well due to uniform surcharge

by

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Introduction

When the complete ground surface is occupied by uniformly distributed load, the earth pressure computation is often made by substituting the load by an equivalent height of surcharge (Rao). The height of this surcharge layer is equal to the intensity of load divided by the unit weight of underlying soil as shown in the Figure 1. The pressure on the retaining wall due to distributed load is then the pressure caused by this equivalent height of surcharge which is constant throughout the height of retaining wall and is of value

$$p = K_a \, q$$

Where, K_a is active earth pressure coefficient and q is the intensity of the distributed load.

This problem can be alternatively attacked by computing the pressure on retaining wall by loading the space behind the retaining wall and double integrating Bousinesque function (Bowles 1968, Spangler, Spangler, et al 1956) between proper limits. It is assumed that the sand fill is placed by simply drizzling it after wall construction so that compaction effect on back fill could be neglected. (Aggrow et al 1974).

After integration, the pressures so obtained are compared with conventional value of pressure and suitability of proposed equations based on Boussinesque solution discussed.

Stress Distribution Equations

For the purposes of this presentation, stress functions due to Boussinesque is used.

Boussinesque Equation

The lateral stress generated at any point $(X.Y.Z.)$ due to load p in Figure 2 could be given by the following equation due to Boussinesque.

$$6\,x = \frac{\rho}{2\pi}\left[\frac{3\,X^2\,Z}{R\,5} - (1-2\,\mu)\left\{\frac{X^6 - X^2}{R\,r^2\,(R+Z)} + \frac{Y^2\,Z}{R^3\,r^2}\right\}\right] \quad \ldots(1)$$

Where, μ is Possion's ratio.

If we use this equation, assuming a small element $(dx \;\; dy)$ acted by a load of intensity, q, the lateral stress generated at a point (x, y, z) in

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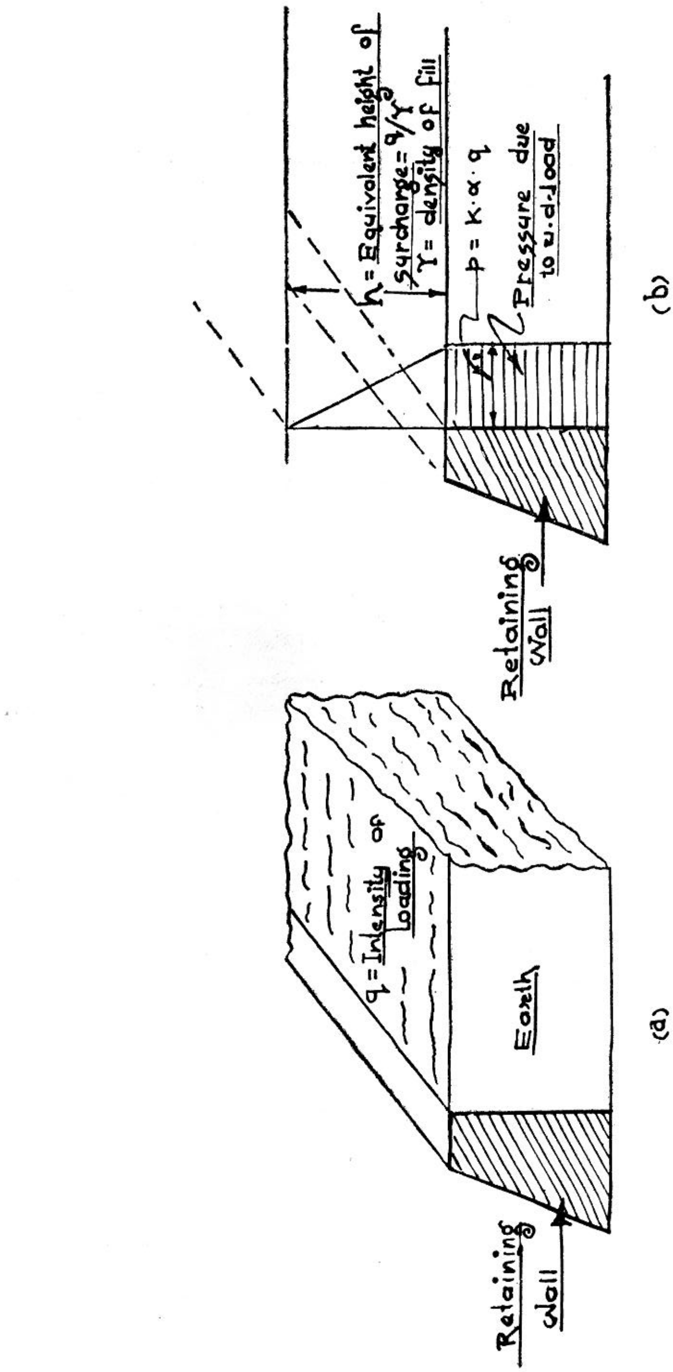
h = Equivalent height of surcharge = q/γ
γ = density of fill
p = K·α·q
Pressure due to u.d.load
Retaining Wall
(b)
q = Intensity of loading
Earth
Retaining Wall
(a)

FIGURE 1

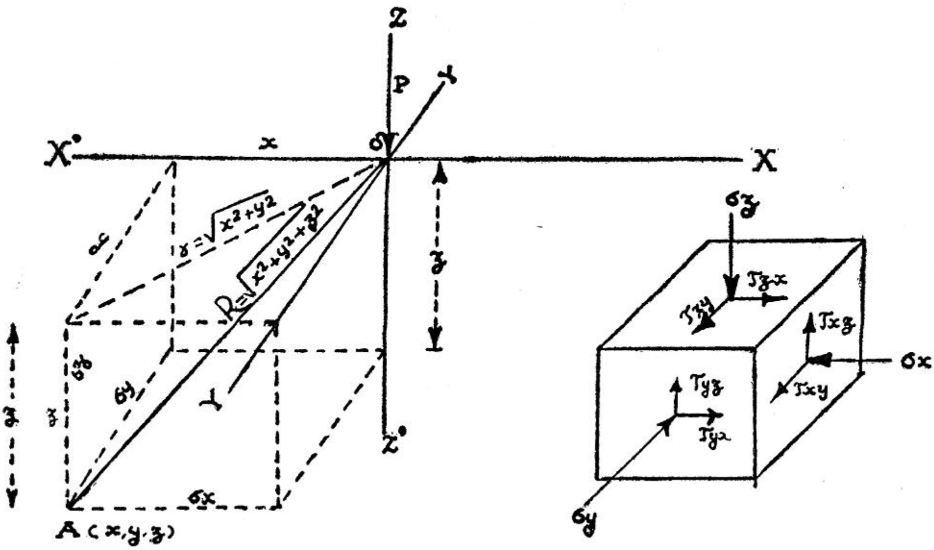

FIGURE 2

Figure 2 due to this loading is

$$\frac{q\,d_x\,d_y}{2\,\pi}\left[\frac{3\,x^2\,z}{R^5}-(1-2\,\mu)\left\{\frac{x^2-y^2}{R\,r^2\,(R+z)}+\frac{y^2\,z}{R^3\,r^2}\right\}\right] \qquad \ldots(2)$$

The lateral stress (p) generated at this point, if the complete backfill is occupied by load of intensity q is

$$p=\int_0^{\infty}\int_{-\infty}^{+\infty}\frac{q\,d_x\,d_y}{2\,\pi}\left[\frac{3\,x^2\,z}{R^5}-(1-2\,\mu)\left\{\frac{x^2-y^2}{R\,r^2\,(R+z)}+\frac{y^2\,z}{R^3\,r^2}\right\}\right] \qquad \ldots(3)$$

Substituting for R and r

$$p=\int_0^{\infty}\int_{-\infty}^{+\infty}\frac{q\,d_x\,d_y}{2\,\pi}\,\frac{3\,x^2\,z}{(x^2+y^2+z^2)^{5/2}}$$

$$-\int_0^{\infty}\int_{-\infty}^{+\infty}\frac{(1-2\,\mu)}{2\,\pi}\,\frac{(x^2-y^2)\,q\,d_x\,d_y}{(x^2+y^2+z^2)y_2\,(x^2+y^2)\,[(x^2+y^2+z^2)y_2+z]}$$

$$-\frac{q\,(1-2\,\mu)}{2\,\pi}\int_0^{\infty}\int_{-\infty}^{+\infty}\frac{y^2\,z\,d_x\,d_y}{(x^2+y^2+z^2)^{3/2}\,(x^2+y^2)} \qquad \ldots(4)$$

Substituting for x and y as $x = \rho \cos\theta$, $y = \rho \sin\theta$, $d_x\, d_y = \rho\, d\rho\, d\theta$

$$p = \frac{3q}{2\pi} \int_0^{+\infty} \int_{-\pi/2}^{+\pi/2} \frac{\rho^3 z \cos^2\theta\, d\rho\, d\theta}{(\rho^2+z^2)^{5/2}} - \frac{q(1-2\mu)}{2\pi}$$

$$\int_0^{\infty} \int_{-\pi/2}^{+\pi/2} \frac{\rho^2 (\cos^2\theta - \sin^2\theta)\, \rho\, d\rho\, d\theta}{(\rho+z^2)^{1/2} \rho^2 [(\rho^2+z^2)^{1/2}+z]} - \frac{q(1-2\mu)}{2\pi}$$

$$\int_0^{\infty} \int_{-\pi/2}^{+\pi/2} \frac{z\, \rho^3 \sin^2\theta\, d\rho\, d\theta}{(\rho^2+z^2)^{3/2}\, \rho^2} \qquad \ldots(5)$$

$$= \frac{q}{2\pi} \left[\int_0^{\infty} \frac{3\rho^3 z\, d\rho}{(\rho^2+z^2)^{5/2}} . 2 . \frac{1}{2} . \frac{\pi}{2} - (1-2\mu) \times 0 - (1-2\mu) \right.$$

$$\left. \int_0^{\infty} \frac{z\, \rho^3\, d\delta}{(\rho^2+z^2)^{3/2} \rho^2} . 2 . \frac{1}{2} . \frac{\pi}{2} \right] \qquad \ldots(6)$$

Substituting $\rho^2+z^2 = t^2$, $\rho\, d\rho = t\, dt$.

$$p = \frac{q}{2\pi} \frac{\pi}{2} \int_z^{\infty} \frac{3(t^2-z^2)\, z \pm dt}{t^5} - \frac{(1-2\mu)}{2\pi} . \frac{q\pi}{2} \int_z^{\infty} \frac{t\, dt}{t^3}$$

$$= \frac{3qz}{4} \left[\frac{1}{z} - \frac{1}{3z} \right] - \frac{z(1-2\mu)}{4} \frac{q}{z}$$

$$= \frac{q}{4} \left[\frac{2}{3z} \right] \times \frac{3z}{1} - \frac{q}{4} [(1-2\mu)] = \frac{q}{4} [1+2\mu] \qquad \ldots(7)$$

If z is depth of point in consideration and γ the density of fill

Vertical stress $= \gamma z$

and *Lateral stress* $= K_a \gamma z$

Where K_a is active earth pressure coefficient. The lateral strain due to this vertical stress $= \frac{\mu \gamma z}{E}$ ...(8)

Where E is the Young's Modulus

But from elasticity considerations

lateral strain $= \frac{K_a \gamma z}{E} - \frac{\mu K_a z}{E}$ ...(9)

equating Equations (7) and (8) we get

$$\mu = \frac{K_a}{1+K_a}$$

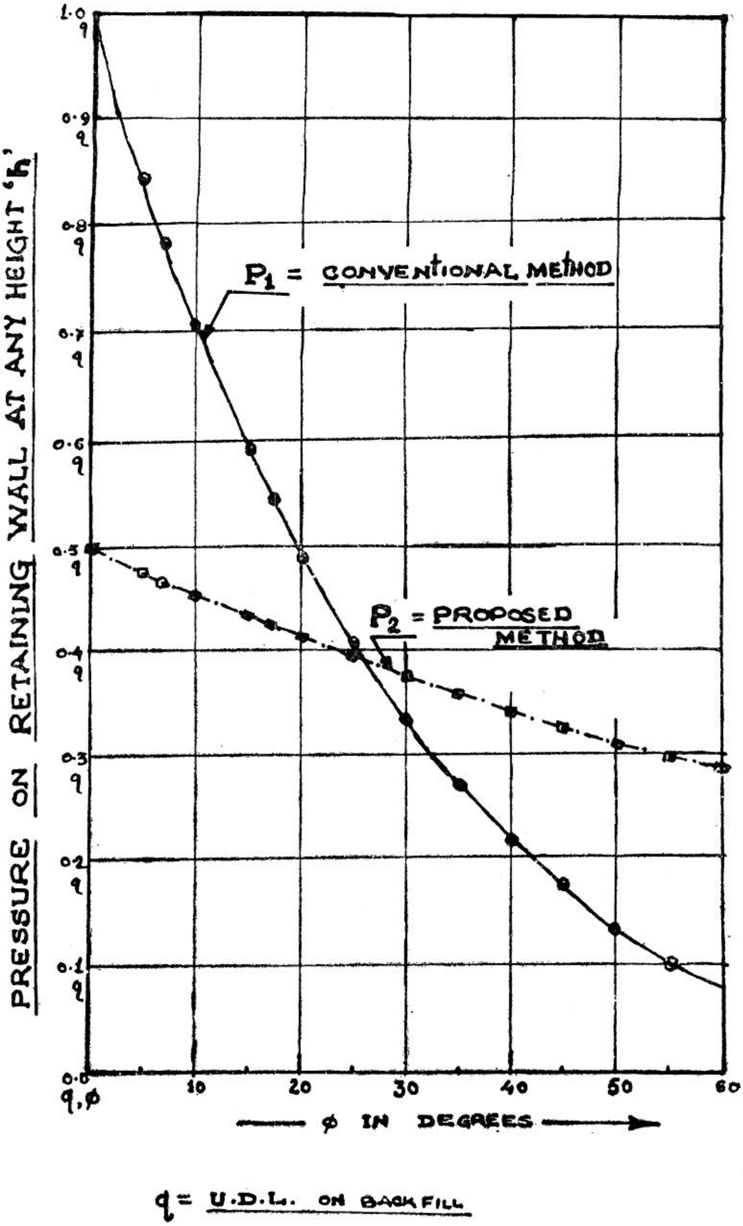

FIGURE 3

and back substituting in Equation (7)

$$p = \frac{q}{4}\,[1+2\,\mu]$$

$$= \frac{q}{4}\left[1+2\,\frac{K_a}{1+K_a}\right]$$

or

$$p_c = \frac{q}{4}\,[2-\sin\phi] \quad \text{if} \quad K_a = \frac{1-\sin\phi}{1+\sin\phi} \qquad \ldots(10)$$

Conventional Method

Conventional method is already indicated in the synopsis of the paper and pressure on retaining wall due to uniformly distributed load at any height of wall is $K_a.\ q$, where K is active earth pressure coefficient and q intensity of loading.

Proposed Method and Suitability

It is suggested that the effect of uniform surcharge on the pressure experted by the backfill should be based on equation $p_c = \frac{q}{4}\,[2-\sin\,\phi]$ where p_c is this effect. Plot is obtained between p_c and angle of internal friction ϕ as shown in Figure 3. The same figure also shows pressure on retaining wall as computed by conventional method.

The conventional method shows that for $p = 0$ the lateral stress is equal to vertical stress which is difficult to conceive.

As the value of ϕ increases, the lateral stress as obtained by conventional and proposed function decreases, however in the case of conventional method, the decline of lateral stress is very steep, which is not so in the proposed method.

In view of this, the proposed method which does not involve imiaginary conversion of actual loads into earth load is recommended.

Conclusion

A function for determination of pressnre distribution of retaining wall due to uniform surcharge is proposed on the basis of Boussinesque equation and is compared with conventional function and points that favour the adoption of proposed method are listed.

The author of the article has not conducted any experiments to demonstrate the validity of method. It may by that where facilities exist, it may be taken.

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