Elastic Settlement of Foundation in Granular Media

by

Banabihari Misra*

Introduction

The calculation of foundation settlements is generally based either on the direct use of elastic displacement theory (Meyerhof and Hanna, 1978 and Parry, 1978) or on the integration of the strains in the soil using elastic stress distribution theory (Dash and Gangopadhyay, 1978, Davis and Poulos, 1968 and Schleicher, 1926). In the context of foundation engineering the values of particular interest are the maximum total and differential settlements at the loaded surface (Baligh and Fuleihan, 1978, Hooper, 1975 and Krizek et al, 1977). The total settlement S_{TF} of a foundation is given by; $S_{TF} = S_{IF} + S_{CF}$, where, S_{IF} is the immediate settlement and S_{CF} is the final consolidation settlement. The settlement caused by the change in shape of the soil medium under load (usually termed immediate, or contact, settlement also called initial, or distortion, settlement) often is over looked ; because, it occurs rapidly and it is usually small compared with consolidation settlement. The small, but sometimes significant (Martin, 1977), settlement caused by elastic deformation of the foundation medium is treated in this paper ; with particular emphasis on establishing the material properties (Misra and Sen, 1975) influencing the immediate settlement.

Use of Elastic Theory

The deflection of an elastic material under load can be analyzed by the same methods used for the computation of vertical stresses. The range of available solutions is much smaller, however. The Boussinesq (1885) analysis for deflection integrated for a loaded area, of width B, with a uniform pressure p, results in the following general expression for S_{IF} (Schleicher, 1926) ;

$$S_{IF} = \frac{pB(1-\mu^2)}{E} I_S \qquad ...(1)$$

where, I_S is a influence coefficient (pure number) which depends on shape and stiffness of the loaded area (foundation) and also on on the position of the given point with relation to the loaded area.

E is the elastic modulus of the semi-infinite body; in the case of soils-deformation modulus for laterally unrestrained compression.

μ is the Poisson's ratio of the medium.

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* Assistant Professor, Department of Civil Engineering, Indian Institute of Technology, Kharagupur, India.

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In practice, Equation 1 is used in the determination of the in-situ deformation modulus E from the results of plate-loading tests, and also it is used in the ccmputation of immediate settlements in the half-space conditions as well as of the individual layers (Parry, 1978). In case of layered soils settlement $\triangle_S$ of individual layers are computed from Eq. 1 by substituting $\triangle I_S = I_{S2} - I_{S1}$ for I_S. The values of I_{S2} and I_{S1}, corresponding to ratios $Z_2 : B$ and $Z_1 : B$ are taken (Vasiljev, 1955); where, Z_1 and Z_2 are depth below base of foundation to top and underside of the given layer respectively.

Applicability of Elastic Theory to Granular Materials

It is seen (Equation 1) that the elastic settlement relies heavily on the use of constant values for the parameters E and μ. In real soil the elastic constants are much stress dependent, although there is indication (Davis and Poulos, 1963) that μ is less stress dependent. Thus the overriding requirement is that the elastic constants for soil should be experimental values determined for a range of stress appropriate to the problem. Anisotropic stress-strain properties of real soil strata may also need to be taken into account if the maximum precision in prediction of initial settlement is to be achieved (Hooper, 1975).

The inability of the classical elastic continunm theory to explain the behaviour of granular masses has been observed in many instances by researchers (Horne, 1965 and 1969, Koerner, 1970 and Rowe, 1954 and 1962). A valued and alternative approach is to regard the discrete and particulate behaviour of the soil mass. Progress in this direction has been made notably in the stress-dilatancy theory of Rowe (1954, 1962) and the extension by Horne (1965, 1969) and Proctor (1974). Rowe (1954), reported that the angle of internal friction in a cohesionless soil which is not failing is mainly a function of the movement of unit length of the planes subjected to maximum shear. The function varied largely with confining pressure, density, strain history, and strain direction, and to a lesser degree with soil grading and rate of strain.

Various researchers in geo-mechanics (Kezdi, 1964 and Winterkorn, 1975) have stated that the variation of the coefficient of earth pressure at rest K_O can represent every condition of the state of the matter. If a solid body displays a very great cohesion ($C \rightarrow \infty$) then K_O tends to zero. With decreasing binding forces between the elementary particles, the K_O value increases and reaches the value of 0.4 to 0.5, which is characteristic for grain assemblies. For viscous liquids, where the internal shearing resistance is much smaller, K_O has greater values and reaches to $K_O = 1$, for ideal liquid with zero internal friction. In the case of gases $K_O > 1$ due to movements of atomic particles.

If a granular mass is loaded, certain parts of the body behave elastically while in other areas the mass may shear off and start to flow more or less like a liquid (Kezdi, 1964). Thus an external force disturbs the balance of positions within the system. This results infinite movements of particles, retarded by mobilised friction. This dialation can be effectively represented through equivalent or apparent elastic properties of the mass as a whole, in state. In a deforming assembly of particles; (i) the deformation occurs as a result of relative motion between groups of particles and (ii) relative movements between groups of particles occur in a certain preferred directions (Horne. 1965).

Need for a new approach to settlement analysis

The development of a theory describing both stresses and displacements in graular materials has encountered and still involves, fundamental difficulties. Many attempts have been made to relate stress and strain in the fallible region (Misra and Sen, 1975-1976), but no complete theory has yet been found, not even in two-dimension. Moreover, each formulation (K_O concept, dilatancy etc.) is valid only for the specific conditions upon which it is based.

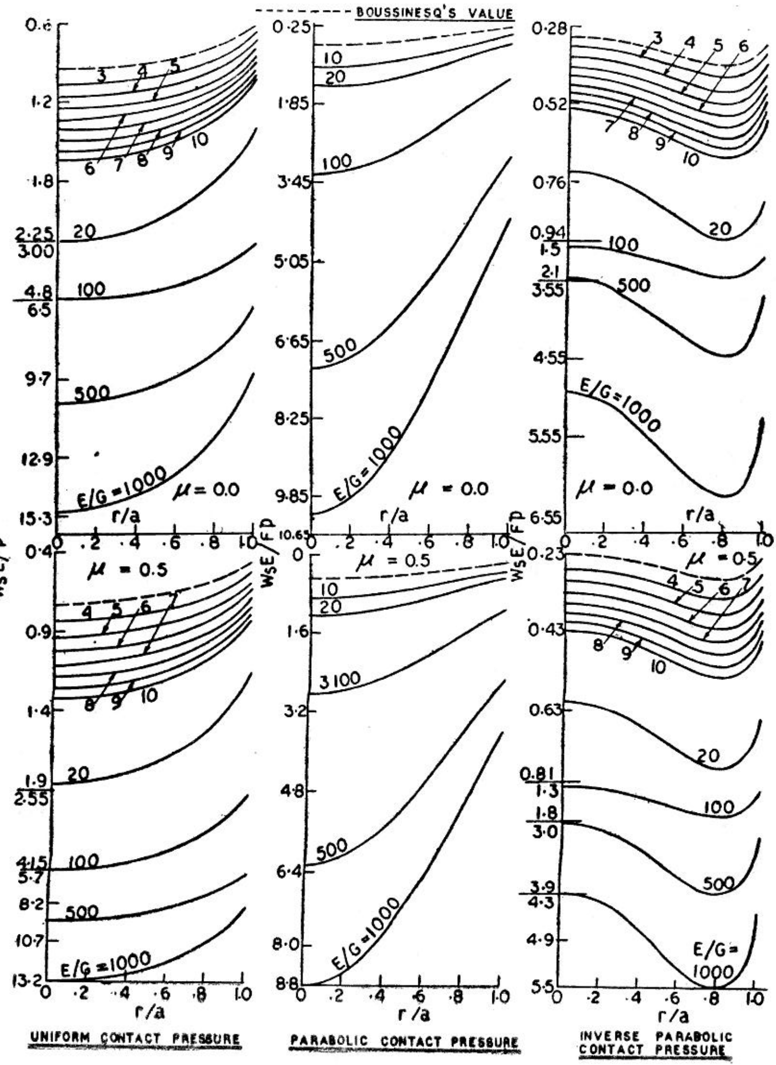

FIGURE 1 Vertical surface displacement versus offset

Even though the K_O concept serves well as a measure of the energy stored in system at different condition of the state of the matter; in the present available form the above philosophy as well as the dilatancy theory are unsuitable for solving the boundary value problems. Because of the great diversity in soil properties, and the many variables involved in the stability problems of soil and structure, the reiationships between these factors are very complex indeed: and in theoretical studies one is forced in many instances to assume idealised conditions and simplfi-cations.

The prediction of the settlement of structures errected on natural deposits may, therefore, prove eroneous if the computations have not taken into account the actual, and practical elastic model (Misra and Sen, 1975) of the respective soil mass. Keeping in view the diversity in the behaviour of granular materials; such as, dialatory (Horne, 1969, Koerner, 1970, and Rowe, 1962), earth pressure in state Ko (Kezdi, 1964 and Winterkorn, 1975), weak resistance to shear (Misra and Sen, 1975 and Weiskopf, 1945), tendency for concentration of stresses (Smoltczyk, 1967) particle sliding in prefereed directions (Horne, 1965); it is felt that for solving boundary value problems the overall behaviour of the masses can best be represented in a model which takes into account the apparent elastic properties (E,G and η) of such medium. Since in this state the granular materials are very weak in resisting shear deformation (resulting in low shear modulus,G), it is more appropriate to represent these apparent elastic properties by a model in which, E:$G > 2(1+\mu)$ as suggested by Weiskopf (1945) and extended by Misra and Sen (1975-1976).

Scope of the present model

The analytical investigation carried out with this concept by Misra and Sen (1975-1976) represents an unified mathematical model applicable not only to granular masses but also to a wider range of materials ranging from classical elastic solids, E:$G = 2(1+\mu)$, at one end to viscous liquid like, E:$G = \propto$, at the other end of the spectrum. It highlighted the influence of the material parameter E:G, by giving a more realistic account of stress and displacement fields in a wider range of materials. Thus, it was thought worthwhile to further investigate the influence of the material parameter E:G on the fonudation settlement.

This material parameter E:G will have different values, between 2 and infinity, for difierent soil structures. It is expected that the softer/looser the medium the higher will be the parameter E:G, because of the poor shearing resistance of such materials. However, it is expected that most of the materials usually encountered in geotechnical applications may have the values of the parameter E:G within 10 (3 for hard stiff clays, around 6 for sand and 10 for very loose sensitive structures). In the absence of established values of this parameter for different types of geotechnical formations, analytical results presented herein give a qualitative representation of their behaviour under load.

Statement of the problem and its solution

The problem deals with a semi-infinite discrete, particulate granular layer of infinite extent in both the horizontal directions and the plane boundary of the upper surface is loaded by an external load, of uniform intensity p, distributed over a circular footing, of radius a. The granular

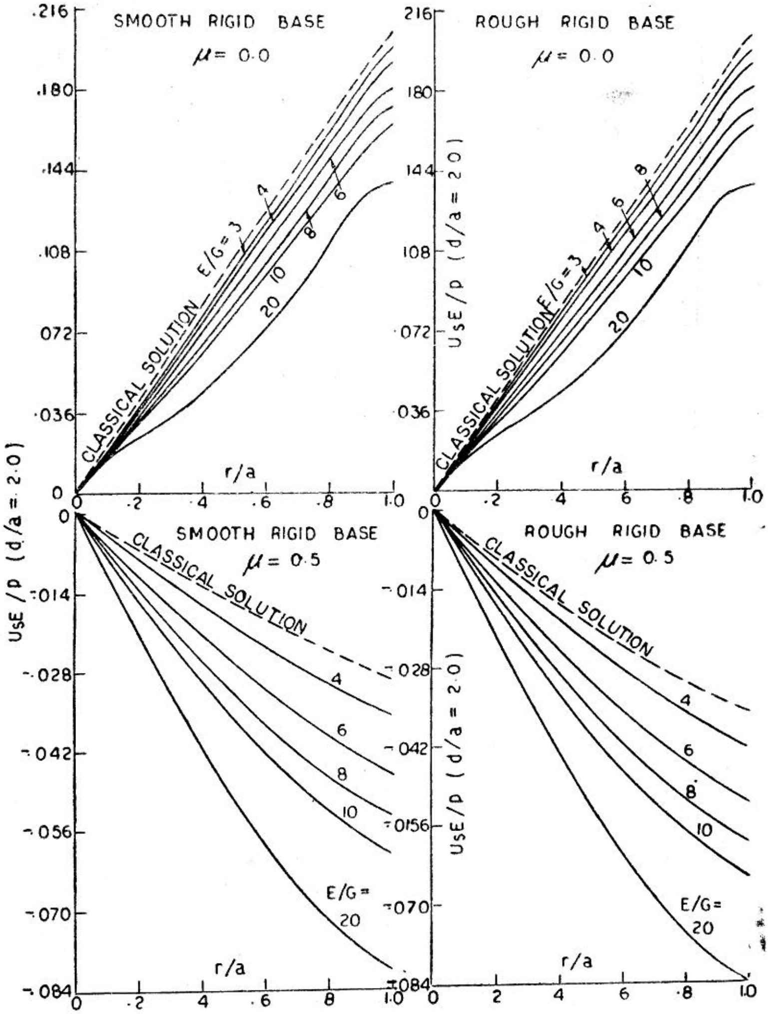

FIGURE 2 Radial displacement of surface under flexible circular footing (uniforom contact pressure)

layer extending up to infinity depth as well as limited by the presence of rigid boundaries, having different frictional characteristics, have been considered for different types of contact pressure (positive and/or negative boundary reactions) transmitted to the granular medium.

In the present problem for the limited thickness d of the granular layer. the plane boundary of the upper surface is denoted by, $z = -d$ plane, whereas for the infinitely thick layer it is denoted by, $z = O$ plane. The Z-axis is drawn with positive direction downward. The surface displacement (U_s—radial and W_s— vertical), beneath the footing, equations resulting from this material parmeter $E{:}G$ concept, with the model $E{:}G > 2(1 + \mu)$, for the various types of contact reactions for different types of boundary conditions (at the underside of the finite granular layer) can be obtained with full details from the works of Misra and Sen (1975-1976). For ready

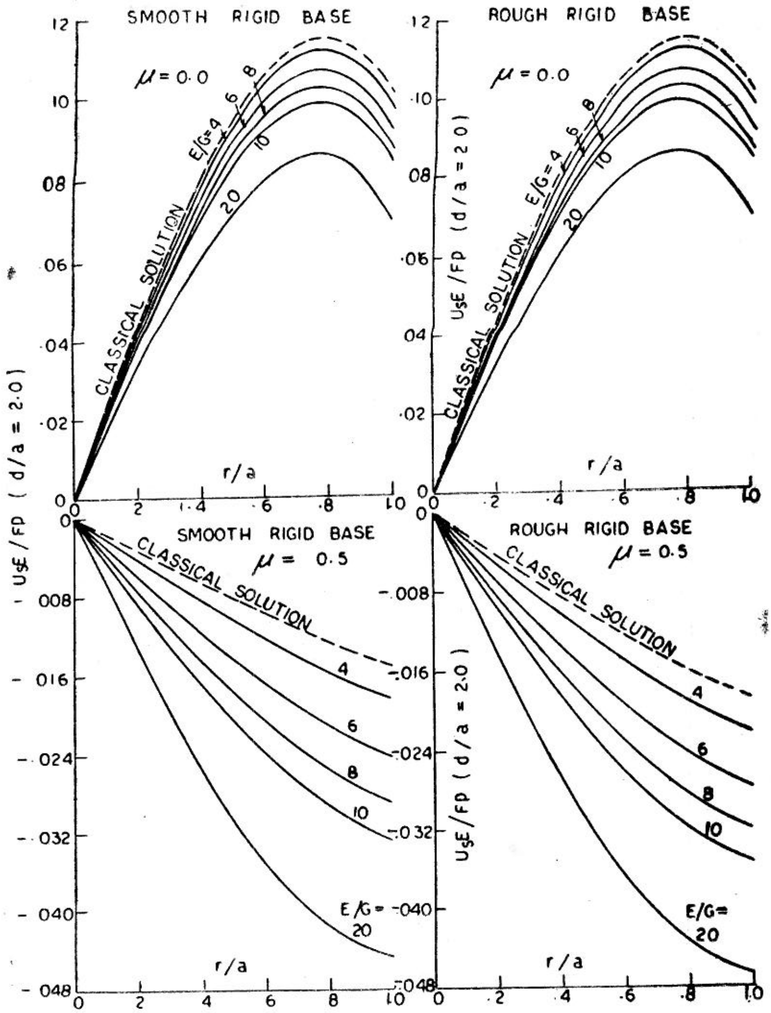

FIGURE 3 Radial displacement of surface under flexible circular footing (parabolic contact pressure)

reference they are reproduced below in the final form, which are used for evaluating the numerical results of the present work.

Case I : Infinitely Thick Granular Layer (Misra and Sen, 1975)

Uniform contact pressure

$$U_S = \frac{a\,p\,(1+\mu)[\beta\,(L+K) - \alpha\,(M+K)]}{E\,(\alpha-\beta)} \int_0^\infty \frac{1}{\lambda}\, J_1\,(\lambda r)\, J_1\,(\lambda a)\, d\lambda \quad \ldots(2)$$

$$W_S = \frac{a\,p\,(1+\mu)\,(M-L)}{E(\alpha-\lambda)} \int_0^\infty \frac{1}{\lambda}\, J_0\,(\lambda r)\, J_1\,(\lambda a)\, d\lambda \quad \ldots(3)$$

Parabolic contact pressure (Negative boundary reactions) having zero intensity at the periphery and maximum at the centre of the footing :

$$U_S = \frac{2Fp\,(1+\mu)[\beta(L+K)-\alpha\,(M+K)]}{E\,(\alpha-\beta)}\int_0^\infty \frac{1}{\lambda^2}\,J_1\,(\lambda r)\,J_2\,(\lambda a)\,d\lambda \qquad \ldots(4)$$

$$W_S = \frac{2\,Fp\,(1+\mu)\,(M-L)}{E(\alpha-\beta)}\int_0^\infty \frac{1}{\lambda^2}\,J_1\,(\lambda r)\,J_2\,(\lambda a)\,d\lambda \qquad \ldots(5)$$

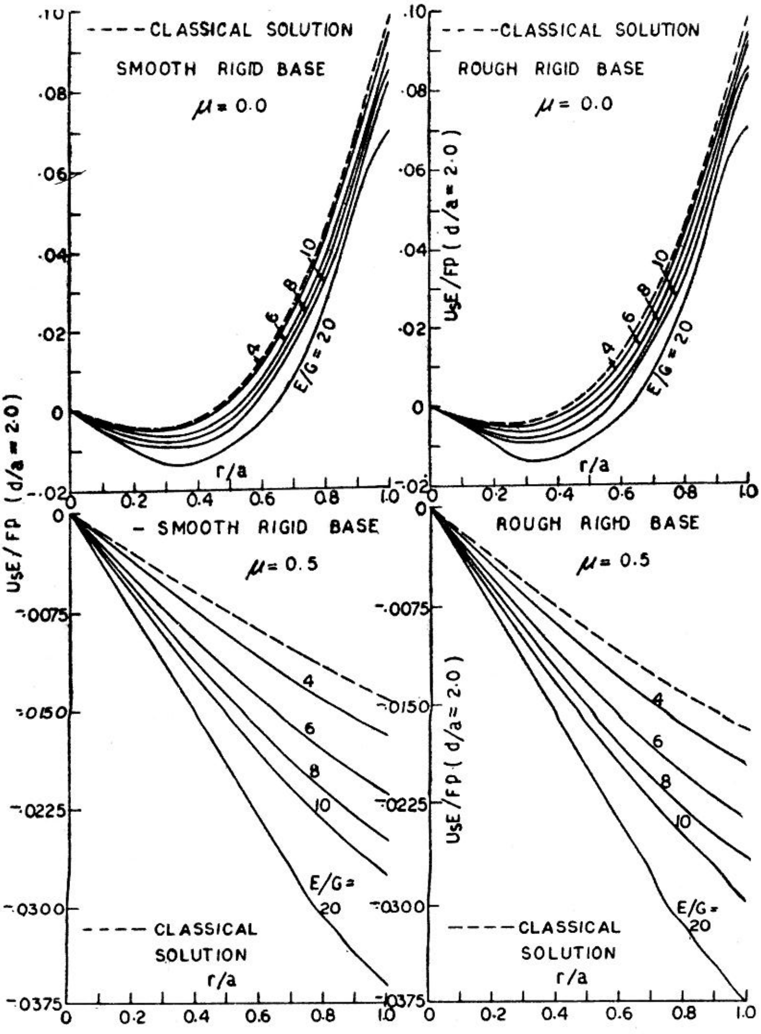

FIGURE 4 **Radial displacement of surface undar flexible circular footing (inverse parabolic contact pressure)**

Inverse parabolic contact pressure (Positive boundary reactions) having zero intensity at the centre and maximum at the periphery of the footing :

$$U_S = \frac{F p\,(1+\mu)\,[\beta\,(L+K) - \alpha\,(M+K)]}{E\,(\alpha-\beta)} \int_0^\infty \frac{1}{\lambda}\, J_1(\lambda r)\, j\,(\lambda) d\lambda \qquad \ldots(6)$$

$$W_S = \frac{Fp\,(1+\mu)\;(M-L)}{E\,(\alpha-\beta)} \int_0^\infty \frac{1}{\lambda}\, J_0\,(\lambda r)\, j\,(\lambda)\, d\lambda \qquad \ldots(7)$$

where, $K = [E/G\,(1+\mu) - 1] > 1$

$L = (K-1)/(\alpha^2-1)$

$M = -\alpha^2 L$

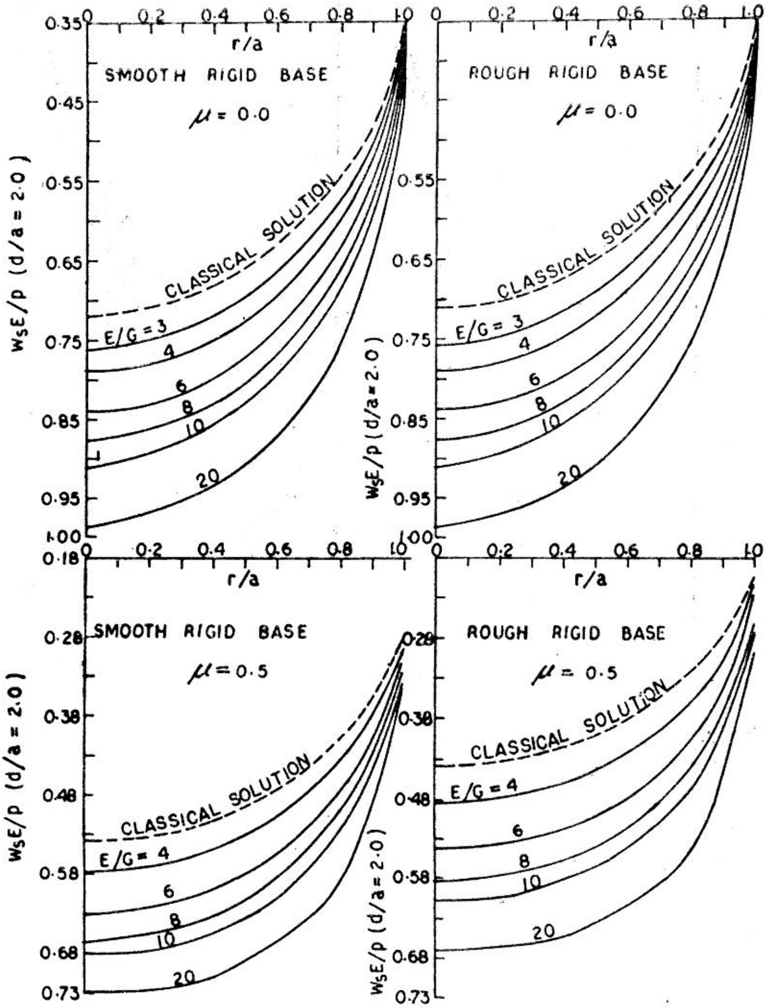

FIGURE 5 Vertical displacement of surface under flexible circular footing (uniform contact pressure)

$$j(\lambda) = \left[a\, J_1(\lambda a) - \frac{2}{\lambda} J_2(\lambda_a) \right]$$

J_0, J_1 and J_2 are Bessel's function of first kind of order zero, one and two respectively. F is a multiplying factor representing the ratio of the maximum magnitude of the boundary reaction to the applied uniform pressure p, transmitted through the footing. r is the radial co-ordinate measured from the centre of the footing. λ is a variable. The effect of the material parameter $E : G$ is reflected through the two coefficients, α and β; where, α^2 and β^2 are the roots of the equation, $x^2 + (K'-2)\,x + 1 = 0$, in which, $K' = (1-K)/(1-\mu)$.

Further mathematical simplification of the factor $[\beta(L+K)-\alpha(M+K)]/(\alpha-\beta)$; appearing in Equations 2, 4 and 6, by replacing the values of L and M, and taking the help of the properties of the roots of a quadratic equation; reduces to $(1-2\alpha)$. Thus, Equations 2, 4 and 6 are completely independent of the influence of the parameter $E : G$ and are identical with those obtained by classical Boussinesqs approach (Timoshenko and Goodier, 1951).

CASE II : Granular Layer Underlain by a Rough Rigid Base (Misra and Sen, 1976)

Uniform contact pressure

$$U_S = ap \frac{1+\mu}{E} \int_0^\infty \frac{1}{\lambda} g_1(\lambda)\, J_1(\lambda r)\, J_1(\lambda a)\, d\lambda \qquad \dots (8)$$

$$U_S = ap\,(M-L)\, \frac{1+\mu}{E} \int_0^\infty \frac{1}{\lambda} g_2(\lambda)\, J_0(\lambda r)\, J_1(\lambda a)\, d\lambda \qquad \dots (9)$$

Parabolic contact pressure

$$U_S = 2F\,p\, \frac{1+\mu}{E} \int_0^\infty \frac{1}{\lambda^2} g_1(\lambda)\, J_1(\lambda r)\, J_2(\lambda a)\, d\lambda \qquad \dots (10)$$

$$W_S = 2F\,p\,(M-L) \frac{1+\mu}{E} \int_0^\infty \frac{1}{\lambda^2} g_2(\lambda)\, J_0(\lambda r)\, J_2(\lambda a)\, d\lambda \qquad \dots (11)$$

Inverse parabolic contact pressure

$$U_S = F\,p\, \frac{1+\mu}{E} \int_0^\infty \frac{1}{\lambda} g_1(\lambda)\, J_1(\lambda r)\, j(\lambda)\, d\lambda \qquad \dots (12)$$

$$W_S = F\,p\,(M-L)\, \frac{1+\mu}{E} \int_0^\infty \frac{1}{\lambda} g_2(\lambda)\, J_0(\lambda r)\, j(\lambda)\, d\lambda \qquad \dots (13)$$

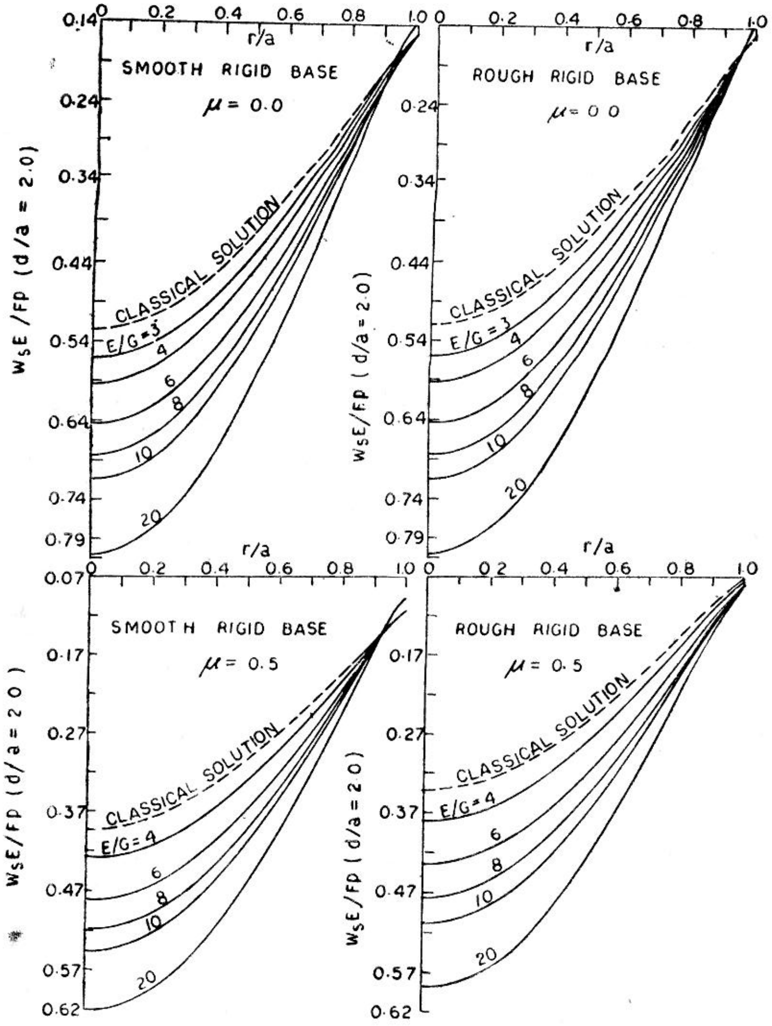

FIGURE 6 Vertical displacement of surface under flexible circular footing (parabolic contact pressure)

where,

$$g_1(\lambda) = \left[\frac{(M+K)(L+M-2)}{(L-1)} \Big(\cosh(\alpha\lambda d)\cosh(\beta\lambda d)) - 1\Big) - \left(\frac{\alpha^2 (M+K)^2}{(L+K)} + \frac{\beta^2 (M-1)(L+K)}{(L-1)} \right) \sinh(\alpha\lambda d) \sinh(\beta\lambda d) \right] \Big/ g(\lambda)$$

$$g_2(\lambda) = \left[\frac{(M-1)}{\alpha(L-1)} \text{ cash } (\alpha\lambda d) \text{ sine } (\beta\lambda d) - \frac{\alpha(M+K)}{(L+K)} \sinh(\alpha\lambda d) \cos h(\beta\lambda d) \right] \Big/ g(\lambda)$$

in which,

$$g(\lambda) = \Big[1 + \frac{(M-1)(M+K)}{(L-1)(L+K)} - \left(\frac{(M+K)}{(L+K)} - \frac{(M-1)}{(L-1)} \right) \cosh(\alpha\lambda d)$$

$$\text{cose}\,(\beta\lambda d) + \left(\frac{\alpha^2 (M+K)}{(L+K)} + \frac{\beta (M-1)}{\alpha (L-1)} \right) \sinh(\alpha\lambda d) \sinh(\beta\lambda d) \Big]$$

CASE-III : *Granular Layer Underlain by a Smooth Rigid Base* (*Misra and Sen, 1976*)

The equations for this problem are identical with that of the case II (Equations 8 through 13), except for the values of the trigometric functions $g_1(\lambda)$ and $g_2(\lambda)$ which are identified for this case as $f_1(\lambda)$ and $f_2(\lambda)$

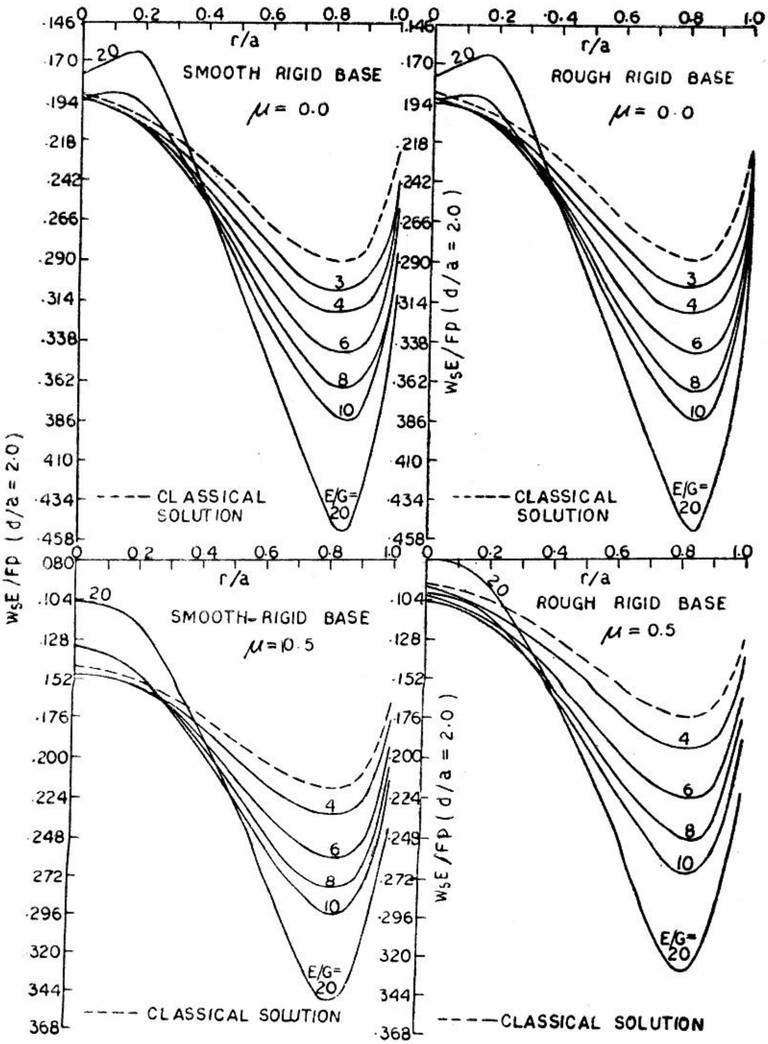

FIGURE 7 Vertical displacement of surface under flexible circular footing (inverse parabolic contact pressure)

respectively. Their values are as follows :

$$f_1(\lambda) = \frac{\beta\,(L+K)\cosh(\alpha\lambda d)\sinh(\beta\lambda d) - \alpha\,(M+K)\sinh(\alpha\lambda d)\cosh(\beta\lambda d)}{\alpha\sinh(\alpha\lambda d)\cosh(\beta\lambda d) - \beta\sinh(\beta\lambda d)\cosh(\alpha\lambda d)}$$

$$f_2(\lambda) = 1/[\alpha\coth(\beta\lambda d) - \beta\coth(\alpha\lambda d)]$$

Analytical Results and Discussions

The radial U_S and vertical W_S displacements of any point on the surface beneath a circular flexible footing can be obtained, by analytical evaluation of the integrals presented in the previous section, with the values of r ranging from 0 to a. The displacements of the surface beneath a circular rigid footing are obtained from the above integrals by replacing r with a,

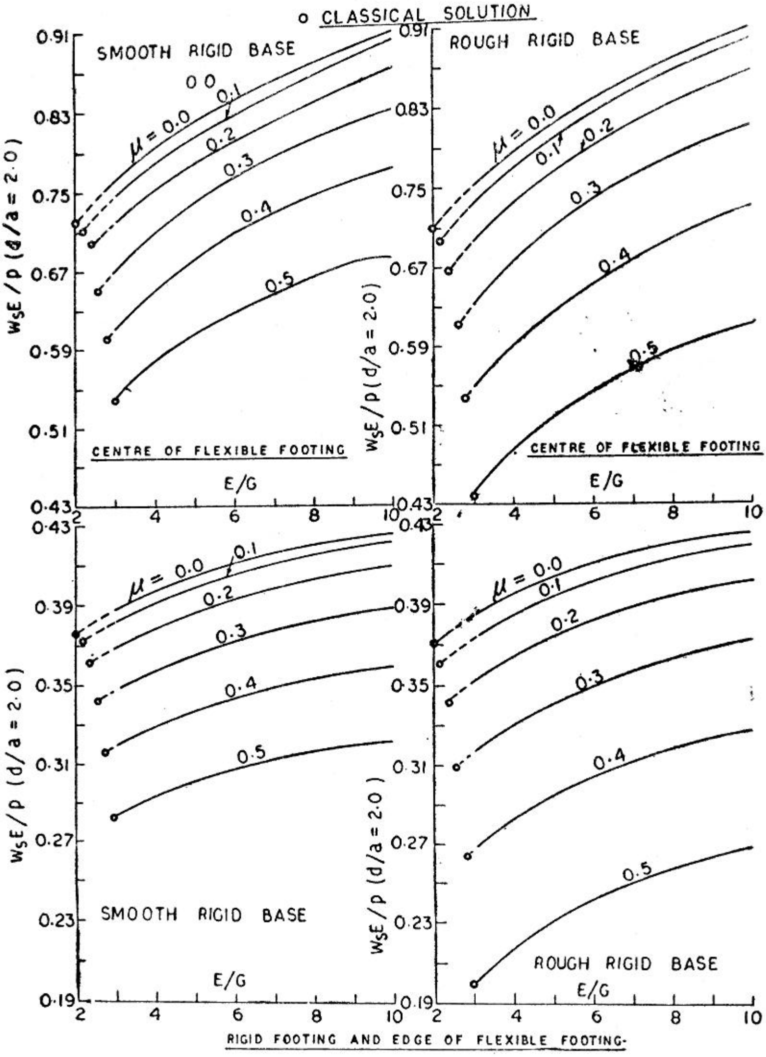

FIGURE 8 **Vertical displacement of surface versus *E/G* (uniform contact pressure)**

thereby, imparting the condition of uniform vertical settlemont across the rigid die. Numerical computations have been made of the present integrals with the help of Simpson's method.

In order to generalise the results as much as possible, and for allowing easy comparison with respect to the influence of the material parameter $E{:}G$ on settlement of structures the results are expressed in terms $\triangle sE/p$, where $\triangle s$ represents the radial/or vertical displacement as the case maybe. Coordinates expressing depth and radial distance and also the thickness of the finite layer are all expressed as multiples of the radius of the footing. Compressive strains and downward displacements are considered positi ve.

Numerical results have been presented for arbitrarily selected values of the parameter $E{:}G$ up to 1000, purely on the consideration of clarity of

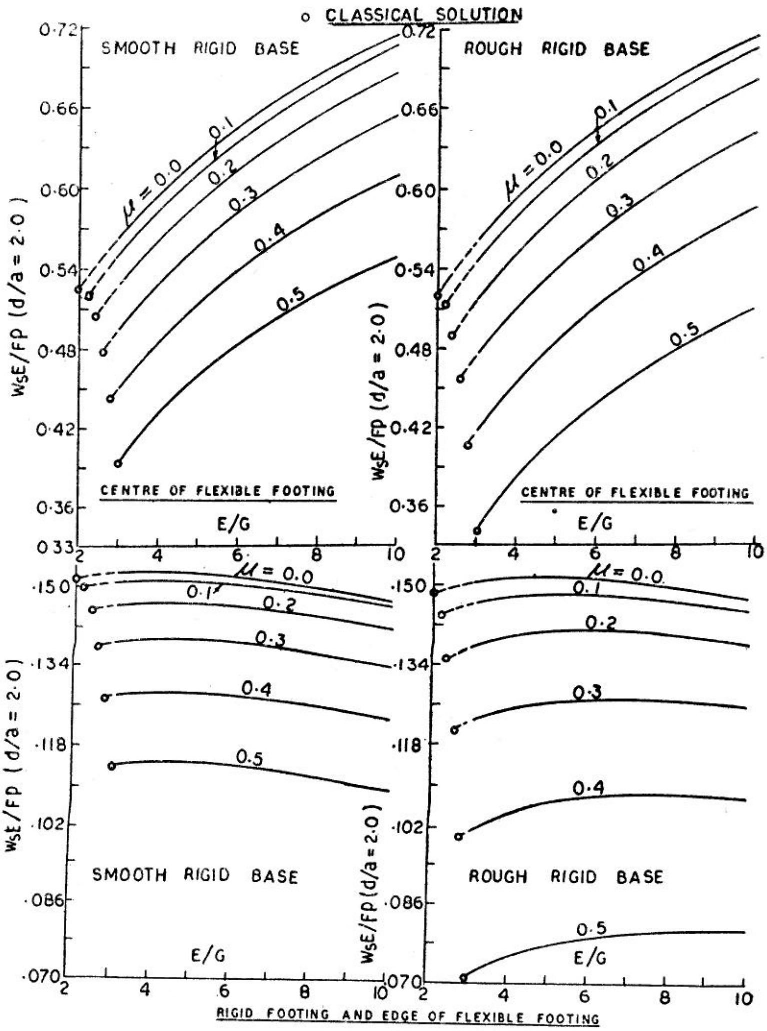

FIGURE 9 Vertical displacement of surface versus E/G (parabolie contact pressure)

representation. Initially close intervals have been selected in view of the fact that most of the materials, with which a practical engineer deals, have E:G values slightly greater than $2(1+\eta)$ and later on higher intervals have been selected in order to demonstrate the nature of surface settlement in a wider range of materials. The deviations of real material behaviour is highlighted by comparing the values of the present analysis with readily available computed solutions of Boussinesq's classical theory.

The vertical surface displacements across a flexible circular footing when laid different types of infinitely thick materials (represented by the ratio E to G are given in Figure 1, for the three conventional types of boundary reactions. Although the results show the same general pattern, the vertical surface displacement in general is found to be higher for materials having low Poisson's ratio. *For the inverse parabolic type of* contact pressure the greatest displacements is found to occur at $r/a=0.8$,

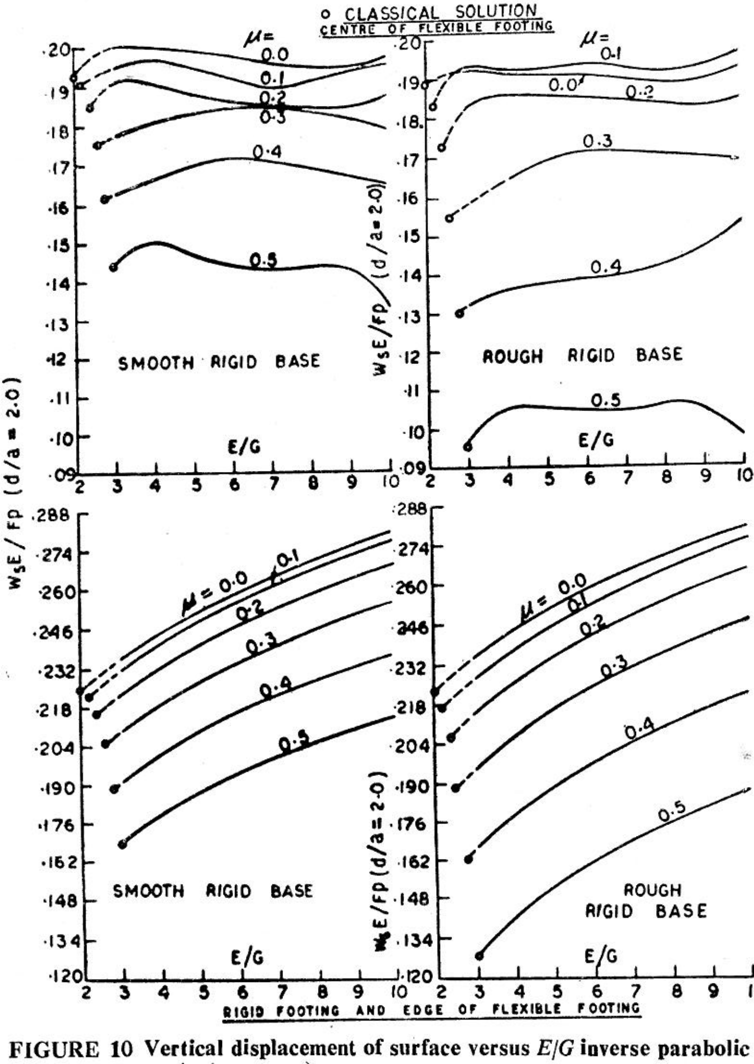

FIGURE 10 Vertical displacement of surface versus E/G inverse parabolic contact pressure)

independent of the values of $E{:}G$ and μ, whereas, as expected for the other two types of contact pressures the greatest displacement occurs at the centre of the footing $r/a = 0$. The model very effectively explains the natural fact that; the looser/softer the foundation medium (represented in this model by higher $E{:}G$ values) the greater is the settlement of the structure.

In geotechnical problems one frequently comes across a layered system in which an elastic layer of limited thickness rests on a rigid unyielding base, the latter being idealised as either perfectly rough or perfectly smooth. The results of the two cases, namely a finite layer underlain by (*i*) a rough, and (*ii*) a smooth rigid base, have been grouped together herein for the purpose of convenience; and for the fact that. mathema-cically it is a question of changing a single boundary condition, while the equations, identical and, indeed, even the numerical results are similar. The influence of the material parameter $E{:}G$ being well-established in the case of infinitely thick layers, the displacement values (U_S and W_S) for these cases have been presented for selected values of $E{:}G$ up to 20.

The surface displacements, both radial and vertical, across a flexible circular footing on the surface of a finite layer of thickness equal to twice the radius of the footing ($d/a = 2$) are presented in Figures 2 through 4 and Figures 5 through 7 respectively. It is observed that for $\mu = 0$, the radial surface displacements decreases in magnitude with the increase in the values of $E{:}G$ and is directed towards the axis; whereas, for $\mu = 0.5$, it increases with the increase in the values of $E{:}G$ and is directed away from the axis. On the other hand, the vertical surface displacement increasses with the increase in the values of $E{:}G$, independent of Poisson's ratio. It is also observed that for $\mu = 0$, the nature of interface has little effect on the magnitude of either the radial or the vertical surface displacement; whereas, for $\mu = 0.5$, rough interface gives rise to higher radial displacements and lower vertical displacements compared with the smooth one.

The influence of the material parameter $E{:}G$ on the vertical surface displacement of both the centre of the flexible footing and the edge of the flexible footing (which also equals with that for the rigid footing) is shown in Figures 8 through 10. It is observed that the displacement value increases with the increase in the values of $E{:}G$, independent of the Poisson's ratio. By extrapolation of the results of the present model, $E{:}G>2(1+\mu)$, the values far a finite thick layer composed of classical elastic solid material, $E{:}G = 2(1+\mu)$, are obtained, which are exactly to those given by the classical approach.

Summary and Conclusions

This study has indicated that the analysis, based on the material parameter $E{:}G$ concept, has successfully covered a fairly wide field, and has furnished useful and a more realistic information on the behaviour, of a wider range materials, under the action of surface loads. The hypothesis, $E{:}G>2(1+\mu)$, very ably explains the observed fact in the actual soils; namely, the looser/softer the foundation medium the greater is the immediate settlement of the structure. This can be effectively improved upon by adopting a more realistic mathematical model; a variable material parameter $E{:}G$ with depth in some form, for geotechnical formations.

The most startling information obtained from this study lies in the fact that, the radial displacement of the surface beneath the footing is independent of the nature of the material comprising the infinitely thick medium. On the other hand, the properties of the material (represented by the ratio E to G), comprising a finite thick foundation layer, influences the development of the radial surface displacement beneath the footing.

The results of the present analysis, as well as the works of Misra and Sen (1975-1976), has indicated that by adopting the model, $E:G>2(1+\mu)$, no loss of generality has occurred with respect to the elastic behaviour of the foundation material: because of the fact that one can always get the results of classical elastic approach from this model too. The model can be very effectively used for solving boundary value problems concerning dialatory materials like soils.

At a later date, by extensive investigation it may become possible to evolve suitable experimental procedures to determine the elastic properties E, G (or $E:G$) and μ, independently from the first principles. Once such methods are devised, it is hoped that the concept of material parameter $E:G$ will go a long way in bridging the wide gap existing in the understanding of the material characteristics responsible for the stress distribution in a variety of foundation materials occurring under various natural conditions of state.

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