Reliability of Bearing Capacity of Footings

by

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Introduction

THE conventional design methods in the field of geotechnical engineering are based on the deterministic approach, as the governing soil properties in the field are considered to be deterministic. However, soils by their inherent nature are variable. It is well known, that the data obtained from soil exploration and laboratory tests show large variations. Amongst the engineering materials, soils possibly have the maximum variability, which leads to uncertainty in the design procedure. It is therefore very appropriate to use probabilistic methods for the soil engineering problems, which take into account the variability and uncertainty of soil behaviour.

The conventional design methods in geotechnical engineering make use of a factor of safety to account for uncertainty and variability. However, it is widely accepted that the choice of a certain value of factor of safety does not convey the safety quantitatively. Shortcomings of the factor of safety have been pointed out by Singh (1971), Kay and Krizek (1971), and Langejan (1965). In this context the reliability based design methods can be of great significance in specifying the safety of a problem quantitatively.

The reliability concepts have been introduced to soil engineering problems by Casagrande (1965), Langejan (1965), Lumb (1966, 1970), Wu and Kraft (1967), Meyerhof (1970), and others. Soil engineering problems in various fields such as settlement, stability and foundation have been studied through probabilistic approach by Kay and Krizek (1971). Singh (1971), Matsuo and Kuroda (1974), Hoeg and Murarka (1974), Vanmarcke (1977), and Folayan et al (1976).

In the present investigation an attempt has been made to predict the bearing capacity of footings using the reliability based design method, which takes into account the variability of the governing soil properties. The Monte Carlo simulation technique and the 'Partial Derivative Method' (PDM) are used for obtaining the design charts.

Formulation based on Probabilistic Approach

In the present investigation the design parameters viz., cohesion c, tangent of angle of shearing resistance tan ϕ. unit weight γ are considered random variables following normal distribution. Many investigators (Lumb 1966, Singh 1971, Matsuo 1976, Madhav and Arumugam 1979) have confirmed the validity of this assumption. The soil properties viz., grain size, water content, void ratio, consistancy indices, compression

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index, coefficient of consolidation, follow either normal or lognormal distribution, (Matsuo 1976). Data on mean, standard deviation and coefficient of variation of soil is scanty in literature. As a guide, one can use the values of coefficient of variation for the variable soil properties as suggested by Meyerhof (1970) and Lumb (1974).

A random variable x, can be expressed using the normality relationship as,

$$x = \bar{x}(1+cv_x\, \xi_x) \qquad \text{...(1)}$$

where $\bar{x}$ = mean value of the random variable·

cv_x = coefficient of variation of x,

ζ_x = standard normal variate.

The bearing capaciiy, q_{ult} from deterministic approach is

$$q_{ult} = CN_c\zeta_c + \tfrac{1}{2}\gamma BN_\gamma\, \zeta_\gamma + \gamma D_f N_q \zeta_q \qquad \text{...(2)}$$

Where B and D_f are width and depth of footing N_c, N_γ and N_q bearing capacity factors, (Terzaghi, 1943) and ζ_c, ζ_γ and ζ_q shape factors (Vesic, 1975). The expression for bearing capacity in the probabilistic approach can be obtained by substituting the normality relationship (Equation 1) for each variable in Equation 2.

Individually these variables are expressed as,

$$\begin{aligned} c &= \bar{c}(1+cv_c\zeta_c) \\ \tan\phi &= \overline{\tan\phi}(1+cv_\phi\, \zeta_\phi) \\ \gamma &= \bar{\gamma}(1+cv_\gamma\, \zeta_\gamma) \end{aligned} \qquad \text{...(3)}$$

The bearing capacity factors become

$$N_c^* = \frac{1}{\tan\phi(1+cv_\phi\, \zeta_\phi)} \exp\left[\pi \tan\phi\, (1+cv_\phi\zeta_\phi)^{1+\tan\phi^2}\right] x Z$$

$$N_\gamma^* = 2\,\overline{\tan\phi}\,(1+CV_\phi\, \xi_\phi)\,[1 + \exp(\pi\, \overline{\tan}\,\phi\,(1 + CV_\phi\, \xi_\phi))]\, x\, Z$$

$$N_q^* = \exp(\pi\, \overline{\tan\phi}\,(1 + CV_\phi\, \xi_\phi))\, x\, Z \qquad \text{... (4)}$$

$$\text{where } Z = \frac{\sqrt{\sqrt{1 + \overline{\tan\phi^2}\,(1 + CV_\phi\, \xi_\phi)^2} + \overline{\tan\phi}\,(1 + CV_\phi\, \xi_\phi)}}{\sqrt{\wedge\, 1 + \overline{\tan\phi^2}\,(1 + CV_\phi\, \xi_\phi)^2 - \overline{\tan\phi}\,(1 + CV_\phi\, \xi_\phi)}}$$

Using Equations 3 and 4, the bearing capacity for strip footing can be expressed in the probabilistic approach as,

$$q_{ult} = \bar{C}(1+CV_C\,\xi_C)\,N_C^* + \tfrac{1}{2}\,\bar{\gamma}\, B\,(1+CV_\gamma\, \xi_\gamma)\,N_\gamma^* + \bar{\gamma}\,(1+CV_\gamma\, \xi_\gamma)\, D_f$$

$$= \bar{C} R_C + \tfrac{1}{2}\,\bar{\gamma}\, B\, R_\gamma + \bar{\gamma}\, D_f\, R_q \qquad \text{...(6)}$$

where R_c, R_γ and R_q are the new bearing capacity factors, in place of conventional bearing capacity factors N_c, N_γ and N_q,

$$R_c = f_1\,(cv_c, \overline{\tan\phi}, cv_\phi),$$

$$R_\lambda = f_2\,(cv_\gamma, \overline{\tan\phi}, cv_\phi\,),$$
$$R_q = f_3\,(cv_\gamma, \overline{\tan\phi}, cv_\phi\,). \qquad \text{... (7)}$$

The bearing capacity factors R_c, R_γ, and R_q in the probabilistic approach are not only dependent on $\overline{\tan\phi}$ and cv_ϕ of the soil, but also depend on the variations in other soils properties viz., cohesion, and unit weight.

The ultimate bearing capacity of square and circular footing can be written in a similar manner.

$$q_{ult} = \bar{c}\,(1 + cv_c\,\xi_c)\,N_c^*\left(1 + \frac{N_c^*}{N_q^*}\right) + \tfrac{1}{2}\,0.6\,\bar{\gamma}\,B\,(1 + cv_\gamma\,\xi_\gamma)\,N_\gamma^*$$
$$+ D_f\,\bar{\gamma}\,(1 + cv_\gamma\;\;\xi_\gamma\,)\,N_q^* \qquad \text{... (8)}$$

Analysis for Probability of Failure

The probability of failure of a foundation is given by Benjamin and Cornell, 1970),

$$P_f = P\,(R < S) = \int_{-\infty}^{S} f(R)\,dR \qquad \text{... (9)}$$

where. P_f = the probability of failure of a foundation, R = the resistance developed, S = the load applied on the foundation, $f(R)$ = the density function of the resistance of the foundation.

For normal density function (Gaussian distribution) Equation 9 reduces to

$$P_f = Y\left(\frac{S - \bar{R}}{\sigma_R}\right) \qquad \text{... (10)}$$

where, Y = the cummulative distribution function of the standardised normal variable, $\bar{R}$ and σ_R—design parameters viz., mean value and standard deviation.

Similarly for lognormal distribution of $Y\,(\bar{m}_Y, \sigma_{log}\,Y)$

$$P_f = Y\left(\frac{\log\,(S/\bar{m}_Y)}{\sigma_{log\,Y}}\right) \qquad \text{... (11)}$$

where, $\bar{m}_Y$ and $\sigma_{log\,Y}$ = parameters of the lognormal distribution.

For a predetermined probability of failure, the values of the standard normal variate are available and given in Table (Benjamin and Cornell, 1970). The design parameters viz., mean and standard deviation for normal or lognormal distribution can be obtained using either Monte Carlo method or PDM (explained in the following section) and the permissible load S determined for a given P_f. A partial factor of safety, F_p, is defined in the following manner (Lumb, 1970),

$$F_p = \frac{\overline{R}}{\overline{S}} \qquad \text{... (12)}$$

Unlike the conventional factor of safety F, the partial factor of safety is based on the probabilitic approach.

Method of Solution

Solution procedure for a reliability based design method involves specifying mean, coefficient of variation of the variables, and the probability of failure. In the present investigation coefficients of variation of cohesion, cv_c, angle of shearing resistance, cv_ϕ, and unit weight cv_γ are chosen on the basis of guidelines suggested by earlier investigators, Lumb (1970). Meyerhof (1970), and Singh (1971). Table 2 shows the ranges of the parameters considered in this study.

Monte Carlo Method

The method consists of the following three steps :

(*i*) Generation of random numbers for variables of the problem.

(*ii*) Finding the value of resistance corresponding to a set of random numbers from the governing equation. This completes one experiment.

(*iii*) Repetition of the experiment to get a large number of experiments.

A number of methods are available to generate random numbers with uniform density function in the range $0 \leqslant x \leqslant 1$. A standard built-in subroutine ('RNDY1' in IBM 7044 computer) is used in the present problem.

The data obtained from the simulation method is used to get an histogram. From the histogram the distribution of the function under consideration is checked and, the design parameters of the distribution (mean and standard deviation) are obtained. Estimation of a failure probability of the order of 10^{-3} requires about 1×10^6 experiments if the answer has to have a confidence level of 90 per cent, and for smaller probabilities a much larger sample size would be required. However, simulation of such a large number of experiments would require large computer time which was not possible with the computer available. Realising this limitation, the mean and standard deviation values are obtained. These values are checked with the Partial derivative method, which is presented in the following section.

Partial Derivative Method

The method is very simple and effective for many engineering problems. The basic principles of the method are as follows, Rao (1978).

$$\text{If } Y = f(x_1, x_2, \ldots, x_n) \qquad \text{...(13)}$$

The approximate mean and variance of Y can be obtained in the following manner. Expanding the function f in Equation (13) by Taylor's series about the mean values $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n$ as,

$$Y = g(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) + \sum_{i=1}^{n} (x_i - \bar{x}_i) \frac{\partial g}{\partial x_i}$$

$$+ \frac{1}{2} \sum_{i=1}^{n} (x_i - {}_i\bar{x}) (x_j - \bar{x}_j) \frac{\partial^2 g}{\partial x_i \, \partial x_j} + \ldots \qquad \ldots(14)$$

Wherein the derivatives are evaluated at $(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$. By truncating the series after the linear terms, the first order approximation to Y can be expressed as,

$$Y = g(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) + \sum_{i=1}^{n} (x_i - \bar{x}_i) \frac{\partial g}{\partial x_i} \mid (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) \qquad \ldots(15)$$

The mean and variance can now be expressed as

$$\bar{Y} = g(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) \qquad \ldots(16)$$

and

$$\sigma_Y^2 = \sum_{i=1}^{n} c_i^2 \ \sigma_{x_i}^2 \qquad \ldots(17)$$

where, c_i is the value of the partial derivative $\frac{\partial g}{\partial x_i}$ evaluated at $(\bar{x}_1, \bar{x}_2 \ldots, \bar{x}_n)$. The standard deviation σ_Y is is expressed as,

$$\sigma_Y = \sqrt{\sigma_Y^2} \qquad \ldots(18)$$

Results and Discussions

The problem of bearing capacity of footings has been studied in the light of probabilistic analysis. A parametric study has been made for the bearing capacity factors by using reliability based design methods.

A typical histogram from Monte Carlo simulation with cummulative distribution function is presented in Figure 1. The distribution of the function (bearing capacity factors in the present study) is established to follow either normal or lognormal by conducting the Chi-square test at 5 per cent significance level. The number of experiments (samples) in the Monte Carlo method has been limited to 3000 only because of lack of a fast and large computer. Few runs with 10,000 experiments were run and the difference between the results of 3000 and 10,000 experiments found to be very small. The limitation of such a small sample size compared to 1×10^6 samples is realised (the distribution in the tail portions is not precisely known). In addition to this method the partial derivative method is also used to get the design parameters. The values of mean and standard deviation obtained by these two methods are compared Table 3 and they are found to be in close agreement.

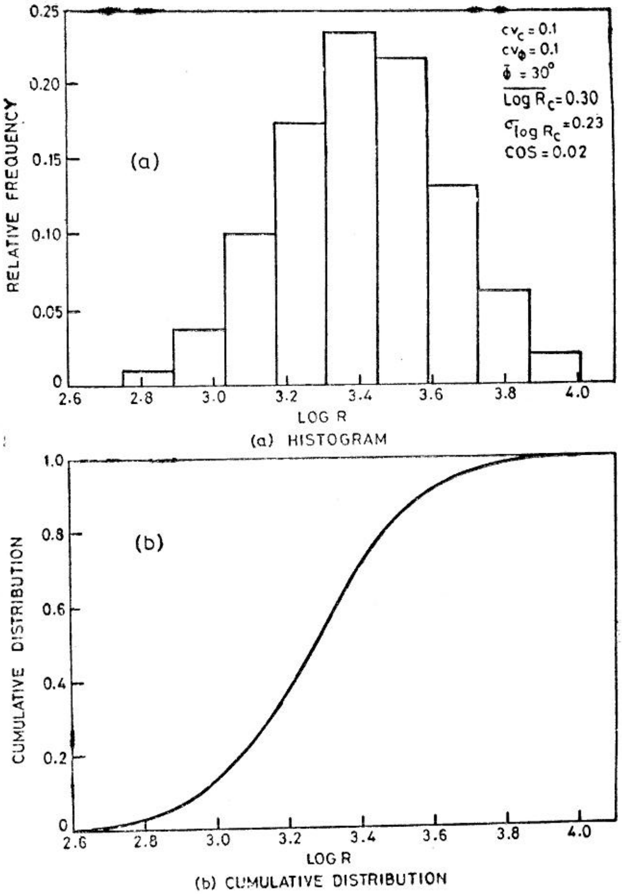


FIGURE 1 Histogram and cumulative distribution of R_c

The probability of failure P_f is considered in the range 10^{-2}—10^{-5} in accordance with Meyerhof (1970). The choice of P_f in a foundation problem is totally governed by the designers judgement based on (*i*) the importance of the structure (*ii*) the confidence with which soil parameters are investigated and (*iii*) the variation of soil properties at the site. For a given P_f the reliability of the foundation R_f is estimated as,

$$R_f = 1 - P_f \qquad \ldots(19)$$

Results for Strip Footing

The bearing capacity factors R_c, R_γ, and R_q follow lognormal distribution. Figures 2 to 4 show the partial safety factors F_p^+ versus P_f^+ relationships for the bearing capacity factors for some typical coefficients of variation of c, tan ϕ, and γ for the mean value of ϕ equal to 30°. Considering the bearing capacity factor R_c, it is seen from Figure 2 that P_f versus F_p relationship is sensitive to coefficient of variation w.r.t. c as well as tan ϕ. For higher values of coefficients of variation the uncertainty becomes

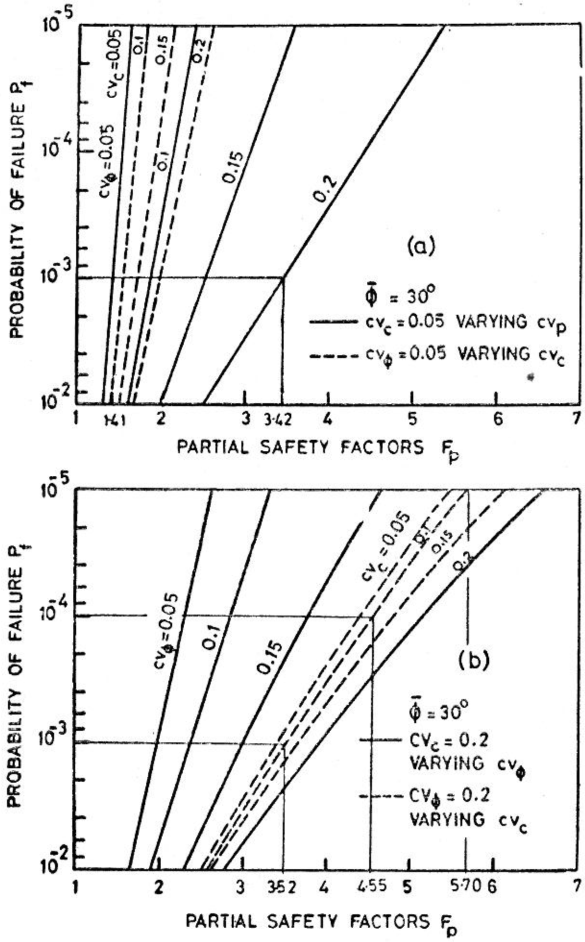


FIGURE 2 Partial safety factors for R_c ($\overline{\phi}$ = 30°)

more as F_p increases. From Figure 2*a* it is seen that for P_f equal to 10^{-3} and $cv_c = 0.05$, F_p is equal to 1.41 for cv_ϕ equal to 0.05; with cv_ϕ equal to 0.2, the F_p value is equal to 3.42 because uncertainty has increased. Figure 2*b* presents F_p values for the bearing capacity factor R_c for cv_c equal to 0.2 and different cv_ϕ, and for $cv_\phi = 0.2$ with varying cv_c. With decreasing P_f, values of F_p increase as expected, because higher safety requirement needs higher values of safety factors. From Figure 2*b* it is seen that for cv_ϕ equal to 0.2 and cv_c equal to 0.1, F_p values are 3.52, 4.55, and 5.70 for P_f values of 10^{-3}, 10^{-4}, 10^{-5}, respectively.

Figures 3 and 4 present the variations of P_f with F_p for the bearing capacity factors R_γ and R_q. These bearing capacity factors depend on the coefficients cv_γ and cv_ϕ. In this case it is observed that for ϕ equal to 30°, the effect of variation of unit weight is insignificant compared to the effect of cv_ϕ. This finding agrees with the earlier results of Singh (1971). For higher ϕ values the safety factors are very sensitive to the value of cv_ϕ.

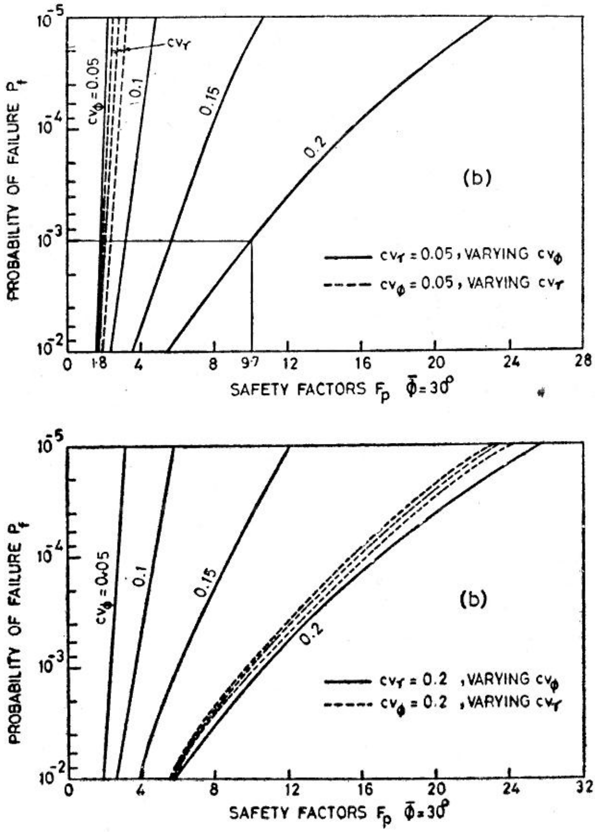


FIGURE 3 Partial safety factors for R_γ ($\phi = 30°$)

Figure 3*a* shows P_f versus F_p for the R_γ term for $cv_\gamma = 0.05$ and varying cv_ϕ. It is seen that for P_f equal to 10^{-3}, F_p equal to 1.80 for cv_ϕ equal to 0.05, and F_p increases to 9.70 for $cv_\phi = 0.2$. Figure 4 shows results for the bearing capacity factor R_q, similar to the factor R_γ.

Results for smaller ϕ values of 10° are presented in Figures 5 to 7. For smaller ϕ values, effects of variation of cv_c in the case R_c and effects of variation of cv_γ in the case of R_γ and R_q are significant as seen from the figures.

The effect of coefficient of variation of ϕ values is shown in Figures 8 through 10. P_f versus F_p relationships for the minimum and maximum range of the variables are plotted. For lower values of ϕ determination of cv_c and cv_γ should be more precise, whereas for higher values of ϕ, cv_ϕ should be given more weightage compared to cv_c and cv_γ.

The bearing capacity factors for circular or square footings formulated in Equation 8 are checked for their distribution. These factors also are found to follow lognormal distribution. The shape factoıs ζ_c and ζ_q are taken to be functions of soil property ϕ, whereas ζ_γ is constant. In case

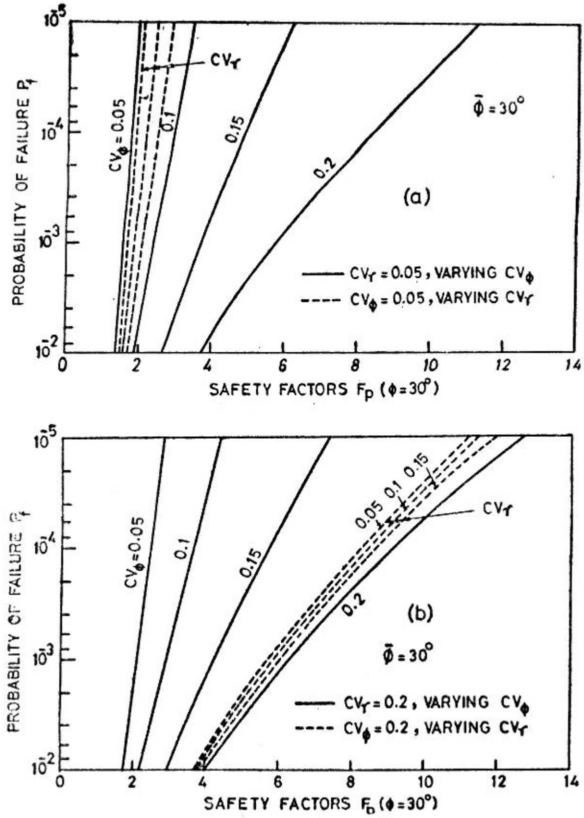


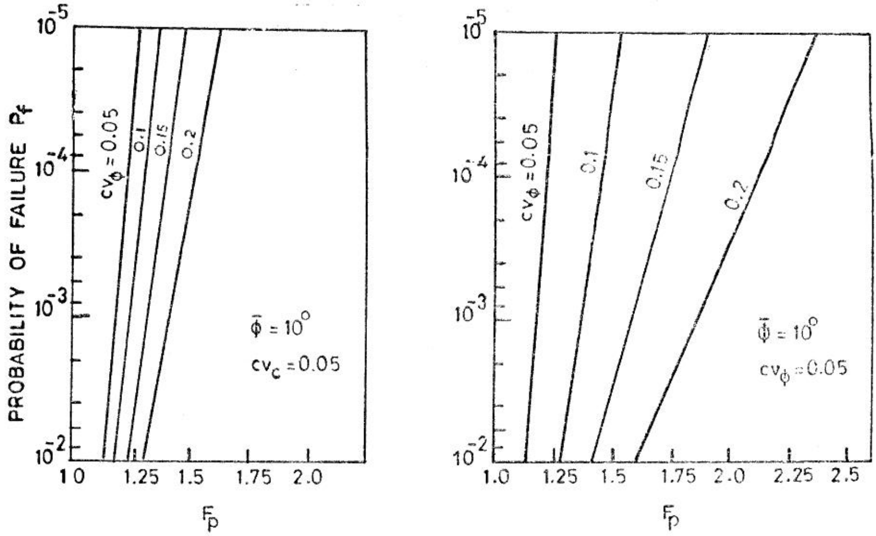
FIGURE 4 Partial safety factors R_q ($\bar{\phi}$ = 30°)

of square or circular footings the findings are similar to those observed in the case of strip footing.

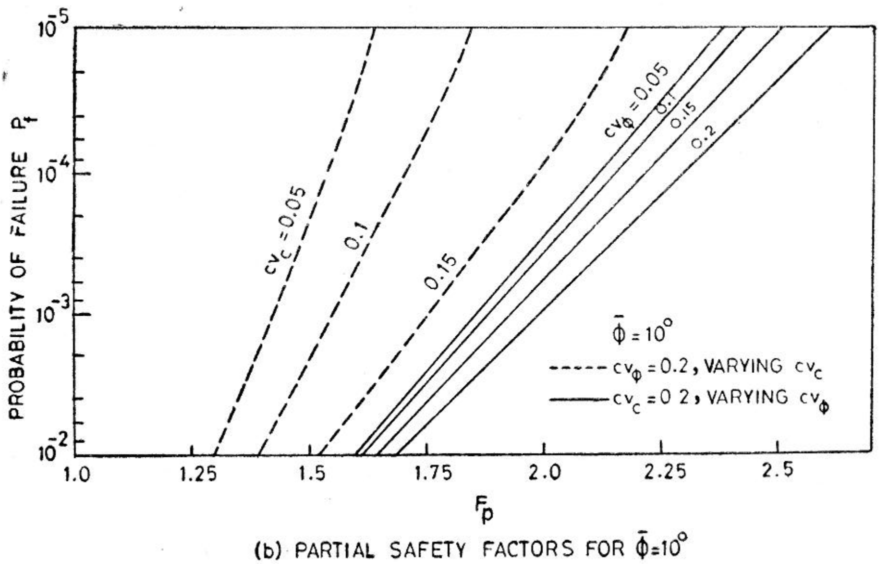
Design Tables

For ready use, it is preferred to present Tables 4-7 giving the partial safety factors for a wide range of cv and P_f values for the bearing eapacity factors for strip and circular or square footings. For a given mean value of ϕ Table 4 gives the mean values of the bearing capacity factors. These factors are similar to those used in the deterministic analysis for average value of ϕ. For a given P_f and coefficients of variation of c, tan ϕ, and γ the partial safety factors either of strip or circular and square footings are directly obtained from Table 5-7 and using Equation (12) the allowable load on the footing can be readily estimated.

The use of factor of safety (usually 2-3) in the conventional method gives equal weightages for uncertainty in N_c, N_γ, and N_q terms. Such a method has certain limitations because as seen from the preceding discussions the effects of variations in the terms c, tan ϕ, and γ are different on the terms R_c, R_q, and R_γ. The present method based on probabilistic concept is believed to the more rational as it gives proper weightages depending upon the coefficients of variations of each design variable.



(a) PARTIAL SAFETY FACTORS FOR $\bar{\Phi}=10°$



(b) PARTIAL SAFETY FACTORS FOR $\bar{\Phi}=10°$

FIGURE 5 Partial safety factors for R_c ($\overline{\phi} = 10°$)

Example Problem

A square isolated footing 2m × 2m rests in a soil at a depth of 1m. The average soil properties are $c = 1$ t/m², unit weight $= 1.8$ t/m³ and angle of shearing resistance $\phi = 30°$.

(*i*) *Conventional Procedure*

As per the conventional procedure, ultimate bearing capacity of the footing is estimated using Equation (5.3) and equals,

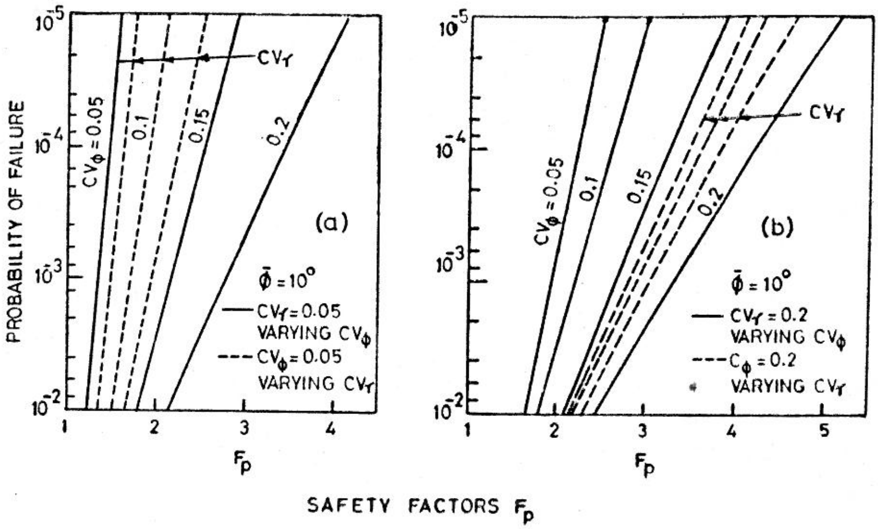


FIGURE 6 Partial safety factors for R_γ ($\overline{\phi} = 10°$)

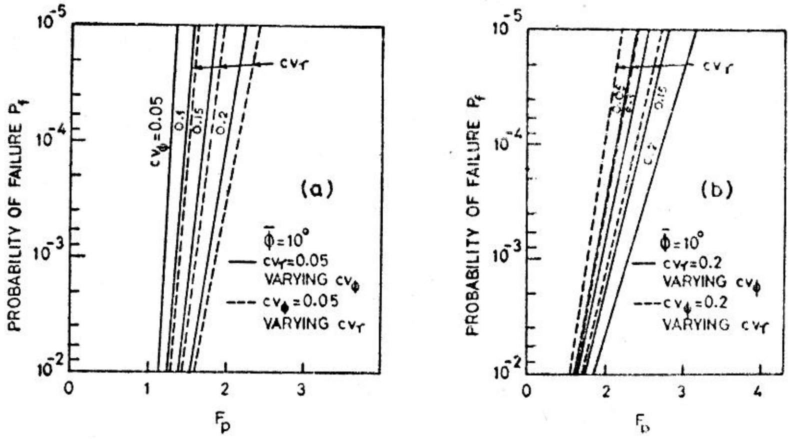


FIGURE 7 Partial safety factors R_q ($\overline{\phi} = 10°$)

$$q_{ult} = 125.39 \text{ t/m}^2$$

Using a factor of safety of 3,

$$q_{all} = 41.80 \text{ t/m}^2.$$

The q_{all} determined does not convey any reliability or safety in quantitative terms.

(ii) Reliability Based Design Method

The coefficient of variation of soil properties cv_c, cv_ϕ, and cv_γ are taken equal to 0.1 (Here the values are selected arbitrarily; but in an actual field situation the values are determined in a rational way). Referring to Table 4 and 8 the partial safety factors are obtained and the

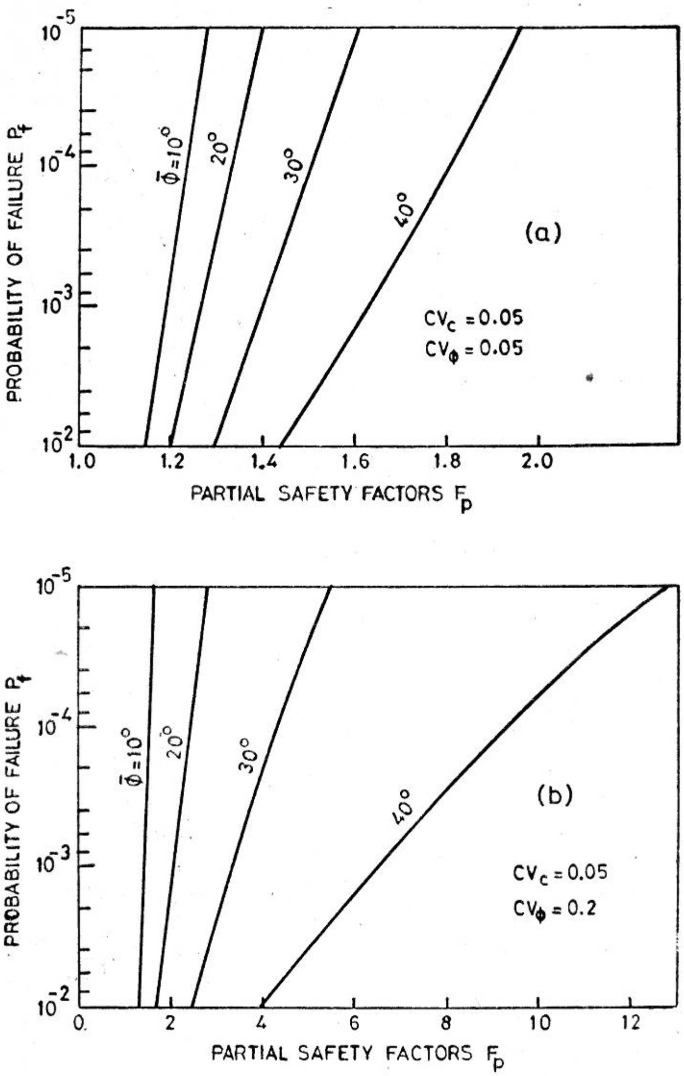


FIGURE 8 Effect of variation of CV_{ϕ} on R_c

allowable bearing capacity estimated for different levels of probability of failure as,

$$q_{all} = 44.78 \text{ t/m}^2 \text{ for } P_f \text{ of } 10^{-3} \text{ i.e., } R_f = 99.9 \text{ per cent}$$

$$= 37.38 \text{ t/m}^2 \text{ for } P_f \text{ of } 10^{-4} \text{ i.e., } R_f = 99.99 \text{ per cent}$$

$$= 31.64 \text{ t/m}^2 \text{ for } P_f \text{ of } 10^{-5} \text{ i.e., } R_f = 99.999 \text{ per cent}$$

The reliability based design method gives the allowable load for different levels of P_f quantifying the safety. The example, thus shows the advantage of the reliability based design method in predicting allowable bearing capacity of footings.

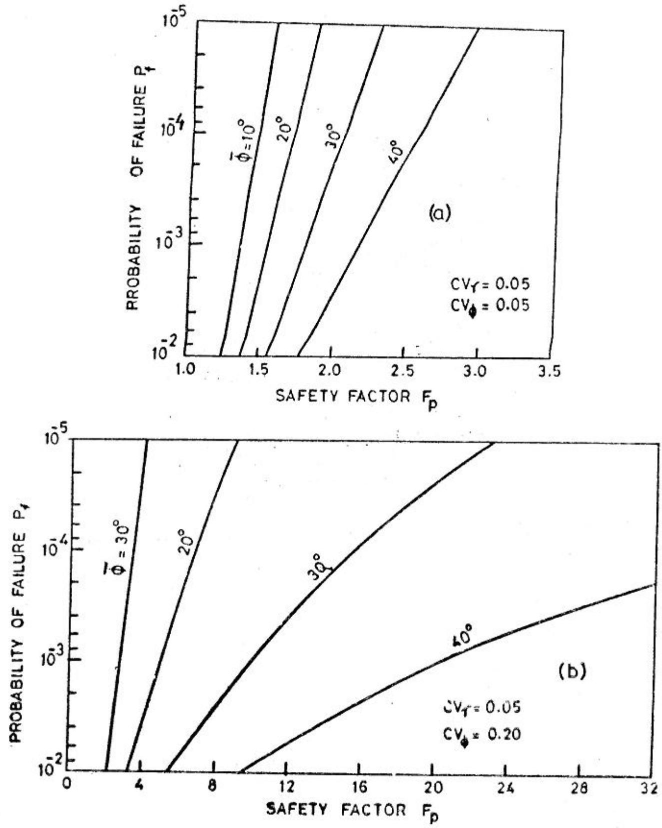


FIGURE 9 Effect of CV_ϕ on $R\gamma$

TABLE 1

Standard Normal Variables Versus P_f

(Ref. Benjamin and Cornell, 1970)

P_f in per tent	Reliability in per cent	Standard Normal Variate
50	50	0
20	80	-0.85
15	85	-1.04
10	90	-1.28
1	99	-2.32
0.1	99.9	-3.09
0.01	99.99	-3.72
0.001	99.999	-4.27

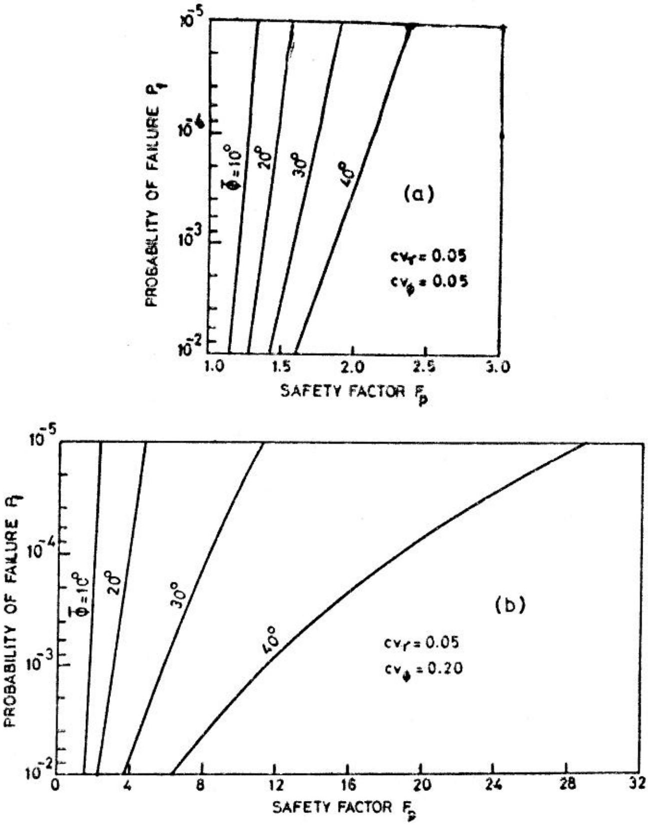


FIGURE 10 Effect of CV_ϕ on R_q

TABLE 2

Parameters And Their Ranges

Parameter	Range
(i) For footings on homogeneous soil	
ϕ	10°—40°
cv_c and cv	0.05—0.2
cv_ϕ	0.05—0.2

TABLE 3

Comparison of Mean and Standard Devilations by Monte Carlo and Partial Derivative Methods

$cv_c = 0.1,\quad cv_\gamma = 0.1,\quad$ and $cv_\phi = 0.1$

	R_c				R_γ				R_q			
ϕ	Monte Carlo method		PDM		Monte Carlo method		PDM		Monte Carlo method		PDM	
	$\bar{R}_c$	σR_c	$\bar{R}_c$	σR_c	$\bar{R}_\gamma$	σR_γ	$\bar{R}_\gamma$	σR_γ	$\bar{R}_q$	σR_q	$\bar{R}_q$	σR_q
10°	8.30	0.96	8.34	0.94	1.21	0.24	1.22	0.23	2.46	0.34	2.47	0.33
20°	14.76	2.33	14.83	2.28	5.34	1.52	5.38	1.49	6.37	1.36	6.40	1.33
30°	30.02	6.81	30.13	6.67	22.22	8.62	22.40	8.52	18.26	5.75	18.40	5.49
40°	75.03	24.08	75.29	23.69	108.30	54.80	109.37	54.37	63.75	26.32	64.17	25.97

TABLE 4

Mean Values of Bearing Capacity Factors for Strip, Circular, and Square Footings

	$\overline{R}_c$	$\overline{R}^*_c$	$\overline{R}_\gamma$	$\overline{R}^*_\gamma$	$\overline{R}_q$	$\overline{R}^*_q$
ϕ	Strip	Circle or square	Strip	Circle or square	Strip	Circle or square
10°	8.34	10.81	1.22	0.73	2.47	2.90
20°	14.83	21.23	5.38	3.17	6.40	8.73
30°	30.13	48.33	22.40	13.44	18.40	29.02
40°	75.29	139.46	109.37	65.62	64.17	118.02

Conclusions

A reliability based design method has been presented for predicting bearing capacity of footings resting on homogeneous soil profile conditions. The necessity of the probabilistic analysis for predicting the bearing capacity arises because of the variation in soil properties. The conventional factor of safety does not convey reliability of the foundation quantitatively and due weightages should be given to variation of soil properties.

The bearing capacity factors are expressed as functions of mean value of angle of shearing resistance and variations of cohesion, tangent of angle of shearing resistance, and unit weight of the soil. Bearing capacity of strip, circular and square footings are evaluted.

The Monte Carlo method is used to generate a large number of samples, which are tested for the mode of distribution. It is osberved that the bearing capacity factors follow lognormal distribution. The PDM is used to get the mean and standard deviation values of the functions for different sets of coefficients of variation. A close agreement is found between the results of the Monte Carlo simulation and PDM.

The partial safety factors suggested by Lumb (1970) are obtained to arrive at the values of allowable design loads for a given probability of failure. Higher values of coefficient of variation of the soil properties and lower values of probability of failure give rise to larger values of partial safety factors.

Design tables are presented for estimating the bearing capacity of footings resting on homogeneous soil. These tables given for a wide range of coefficients of variation and failure probability of 10^{-2} to 10^{-5} can readily be used. To highlight the investigation an example problem is solved which compares the deterministic and probabilistic approaches.

TABLE 5

Partial Safety Factors

(Angle of Shearing Resistance $\phi = 10°$)

cv_c or cv_γ	cv_ϕ	Probability of Failure P_f 10^{-2}		10^{-3}		10^{-4}		10^{-5}	
		Strip	Circle or square	Strip	Circle or square	Strip	Circle or square	Strip	Circle or square
0.05	0.05	1.14*	1.14	1.19	1.20	1.23	1.24	1.27	1.28
		1.25**	1.25	1.35	1.35	1.43	1.43	1.51	1.51
		1.17+	1.18	1.23	1.25	1.28	1.31	1.33	1.36
	0.10	1.18	1.20	1.25	1.27	1.31	1.34	1.36	1.40
		1.49	2.16	2.79	2.79	3.44	3.44	4.13	4.13
		1.27	1.31	1.37	1.43	1.47	1.54	1.55	1.64
	0.20	1.30	1.35	1.42	1.50	1.53	1.62	1.63	1.74
		2.16	2.16	2.79	2.79	3.44	3.44	4.13	4.13
		1.54	1.65	1.78	1.95	2.00	2.23	2.22	2.52
0.1	0.05	1.27	1.27	1.38	1.38	1.47	1,47	1.55	1.56
		1.35	1.35	1.49	1.49	1.62	1.62	1.74	1.74
		1.29	1.30	1.40	1.42	1.50	1.52	1.60	1.62
	0.10	1.30	1.31	1.41	1.43	1.52	1.54	1.62	1.65
		1.56	1.56	1.81	1.81	2.04	2.04	2.27	2.27
		1.37	1.40	1.52	1.57	1.65	1.71	1.78	1.86
	0.20	1.39	1.44	1.56	1.65	1.70	1.79	1.84	1.95
		2.22	2.22	2.89	2.89	3.58	3.58	4.33	4.33
		1.61	1.72	1.89	2.05	2.15	2.38	2.41	2.70
0.2	0.05	1.60	1.60	1.86	1.87	2.12	2.12	2.37	2.37
		1.65	1.95	1.95	2.23	2.23	2.52	2.52	2.52
		1.61	1.62	1.88	1.89	2.14	2.16	2.40	2.42
	0,10	1.61	1.62	1.89	1.91	2.16	2.17	2.41	2.44
		1.82	1.82	2.22	2.22	2.62	2.62	3.02	3.02
		1.66	1.69	1.97	2.01	2.26	2.32	2.55	2.62
	0.20	1.68	1.72	2.00	2.06	2.31	2.38	2.61	2.71
		2.44	2.44	3.28	3.28	4.18	4.18	5.16	5.16
		1.87	1.96	2.30	3.45	2.72	2.94	3.16	3.45

*Safety factor to be used with R_c and R^*_c

**Safety factor to be used with R_γ and R^*_γ

+Safety factor to be used with R_q and R^*_q.

TABLE 6

Partial Safety Factors

(Angle of Shearing Resistance ϕ = 20°)

cv_c or cv_γ	cv_ϕ	Probability of Failure P_f 10^{-2}		10^{-3}		10^{-4}		10^{-5}	
		Strip	Circle or square	Strip	Circle or square	Srip	Circle or square	Strip	Circle or square
0.05	0.05	1.19*	1.22	1.27	1.30	1.33	1.37	1.39	1.43
		1.38**	1.38	1.53	1.53	1.67	1.67	1.80	1.80
		1.27+	1.31	1.38	1.43	1.47	1.54	1.56	1.64
	0.10	1.34	1.40	1.48	1.57	1.60	1.72	1.70	1.86
		1.84	1.84	2.25	2.25	2.66	2.66	3.07	3.07
		1.55	1.65	1.80	1.95	2.02	2.23	2.25	2.51
	0.20	1.74	1.90	2.09	2.36	2.43	2.81	2.77	3.27
		3.30	3.33	4.96	4.96	6.88	6.88	9.15	9.15
		2.35	2.66	3.13	3.68	3.94	4.80	4.83	6.06
0.10	0.05	1.30	1.32	1.43	1.45	1.54	1.57	1.64	1.68
		1.46	1.46	1.66	1.66	1.83	1.83	2.01	2.01
		1.37	1.40	1.52	1.56	1.65	1.71	1.78	1.86
	0.10	1.43	1.48	1.61	1.68	1.77	1.88	1.93	2.06
		1.90	1.90	2.35	2.35	2.80	2.80	3.26	3.26
		1.62	1.71	1.90	2.05	2.17	2.37	2.43	2.69
	0.20	1.80	1.96	2.19	2.46	2.57	2.92	2.95	3.46
		3.39	3.39	5.07	5.07	7.07	7.07	9.44	9.44
		2.41	2.72	3.22	3.78	4.10	4.96	5.04	6.29
0.20	0.05	1.62	1.63	1.90	1.92	2.17	2.19	2.43	2.46
		1.74	1.74	2.09	2.09	2.42	2.42	2.76	2.76
		1.66	1.67	1.97	2.01	2.27	2.32	2.56	2.62
	0.10	1.71	1.75	2.04	2.11	2.37	2.46	2.69	2.81
		2.13	2.13	2.74	2.74	3.37	3.37	4.03	4.03
		1.87	1.96	2.31	2.45	2.74	2.94	3.18	3.44
	0.20	2.04	2.19	2.58	2.85	3.14	3.52	3.71	4.24
		3.61	3.61	5.53	5.53	7.84	7.84	10.63	10.63
		2.63	2.93	3.62	4.20	4.71	5.62	5.92	7.26

*Safety factor to be used with R_c and R^*_c

**Safety factor to be used with R_γ and R^*_γ

+Safety factor to be used with R_q and R^*_q

TABLE 7

Partial Safety Factors

(Angle of Shearing Resistance $\phi = 30°$)

cv_c or cv_γ	cv_ϕ	Probability of Failure 10^{-2}		10^{-3}		10^{-4}		10^{-5}	
		Strip	Circle or square	Strip	Circle or square	Strip	Circle or square	Strip	Circle or square
0.05	0.05	*1.29** 1.55** 1.41+	*1.34* 1.55 1.47	*1.41* 1.80 1.59	*1.47* 1.80 1.67	*1.51* 2.03 1.74	*1·59* 2.03 1.86	*1.60* 2.25 1.89	*1.71* 2.25 2.04
	0.10	1.60 2.36 1.94	1.72 2.36 2.11	1.88 3.14 2.42	2.06 3.14 2.70	2.13 3.96 2.89	2.39 3.96 2.31	2.39 4.86 3·39	2.72 4.86 3.95
	0.20	2.52 5.51 3.71	2.91 5.51 4.39	3.42 9.70 5.73	4.16 9.70 7.18	4.40 15.42 8.18	5.16 15.42 10.73	5.48 23.11 11·16	7.17 23.11 15.24
0.10	0.05	1.39 1.62 1.49	1.42 1.62 1.55	1.54 1.91 1.70	1.60 1.91 1.79	1.69 2.17 1.90	1.76 2.17 2.01	1.82 2.44 2.09	1.91 2.44 2.23
	0.10	1.67 2.42 2.00	1.79 2.42 2.17	1.98 3.24 2.52	2.17 3.24 2.80	2.28 4.11 3.04	2.54 4.11 3.45	2.57 5.07 3.58	2.91 5.07 4.15
	0.20	2.57 5.57 3.77	2.97 5.57 4.45	3.52 9.86 5.85	4.26 9.86 7.31	4.55 15.72 8.38	5.73 15.72 10.96	5.70 23.62 11.48	7.42 23.62 15.62
0.20	0.05	1.68 1.88 1.76	1.71 1.88 1.81	1.99 2.31 2.13	2.04 2.31 2.26	2.29 2.74 2.48	2.36 2.74 2.59	2.59 3.19 2.84	2.68 3.19 2.98
	0.10	1.92 2.64 2.23	2.03 2.64 2.39	2.39 3.64 2.91	2,56 3.64 3.19	2.84 4.73 3.61	3.10 4.73 4.04	3.33 5.95 4.37	3.67 5.95 4.97
	0.20	2.79 5.84 4.00	3.19 5.84 4.70	3.93 10.48 6.33	4.49 10.48 7.84	5.16 16.93 9.22	6.43 16.93 11.94	6.62 25.72 12.84	8.47 25.72 17.25

*Safety factor to be used with R_c and R^*_c

**Safety factor to be used with R_γ and R^*_γ

+Safety factor to be used with R_q and R^*_q.

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Notation

Bar signifies mean; suffix of a variable corresponds to the parameter defined.

B = width of footing

C = cohesion

CV = coefficient of variation

D_f = depth of footing

F_p = partial safety factors
N_c, N_q, N_γ = bearing capacity factors
P_f = probability of failure
q_{ult} = ultimate bearing capacity
R = resistance developed
R_f = reliability of foundation
S = load applied on the footing
x = random variable
Y = cumulative distribution function
γ = unit weight
ζ_c, ζ_q, ζ = shape factors
σ = standard deviation
ϕ = angle of shearing resistance
ζ = standard normal variate