Stability of Structure on Scoured Bed

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Introduction

Hydraulic structures founded on permeable foundations have to withstand forces due to seepage flow underneath. The two factors governing the stability of such structures are uplift pressure from below and exit gradient at the downstream side. The solutions for structures founded on homogenous, isotropic and permeable strata of infinite depth, for different boundary conditions have been given by varions authors. In these solutions the upstream and downstream bed of the stream or channel is assumed to be horizontal.

At the time of construction these conditions are similar to those assumed in the solution; but in due course of time, due to scour the conditions change, specially on downstream side. The shape of scoured bed depends upon the extra energy of flow, discharge intensity, design of stilling basin and basin appurtenances. With proper design of stilling basin the scour may not be significant. However, in uncontrolled condition, like that of downstream of a barrage, abnormal scour can occur due to concentration of discharge.

Generally such structures are designed for the worst condition when water is ponded on the upstream and downstream is dry, but assuming the downstream channel bed horizontal. It is obvious that any change in the shape of downstream bed, due to scour, would effect the uplift pressures and exit gradient both.

The shape of scoured bed would change with flow conditions and basin elements. The shape of downstream scoured bed of the channel is either circular or an aerofoil. In view of this, theoretical solution for uplift pressure and exit gradient, for a flat bottom weir, with circular scour profile on downstream boundary, has been worked out.

Theoretical solution

A profile of flat bottom floor is shown in Figure 1. If downstream end

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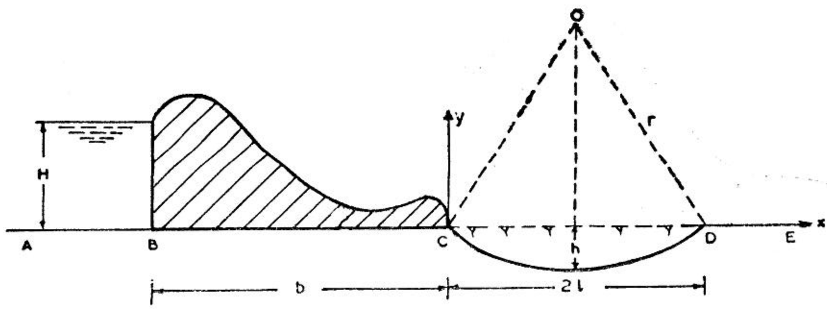


FIGURE 1 **Profile of flat bottom floor**

of the floor, C, is taken as origin, the coordinates of the centre of the circle wold be $\left(l, \frac{l^2-h^2}{2h} \right)$ and equation of circle would be;

$$x^2+y^2-2\,x-\left(\frac{l^2-h^2}{2h}\right).\,2y=0$$

The transformations of the physical flow domain into inverse plane i.e. l/z plane is shown in Figure 2(*a*). In the inverse plane, called Z' plane ($Z' = x' + iy'$), the circular portion $C\,D$ maps into a straight line not passing through the origin and is represented by the equation :

$$y' = \frac{2\,lh}{(l^2-h^2)}\,x' - \frac{h}{(l^2-h^2)}$$

making an angle $\pi\,\theta$ with the horizontal axis. Here

$$\tan \pi\,\theta = \left(\frac{2\,\beta}{1-\beta^2}\right)$$

$$\text{and } \beta = l/h$$

Uplift Pressure

According to Schwarz Christoffel transformation, the mapping of Z'— plane on to upper half to t-plane, is given by

$$\frac{dz'}{dt} = \frac{M}{(1-t)^{(1-\alpha)}} + N$$

where $\alpha = 1-\theta$, and M and N are constants of integration. Integration leads to

$$Z' = -M\,\frac{(1-t)^{\alpha}}{\alpha} + N \qquad \ldots (1)$$

For point D, $Z' = 1/2l$ and $t = 1$;

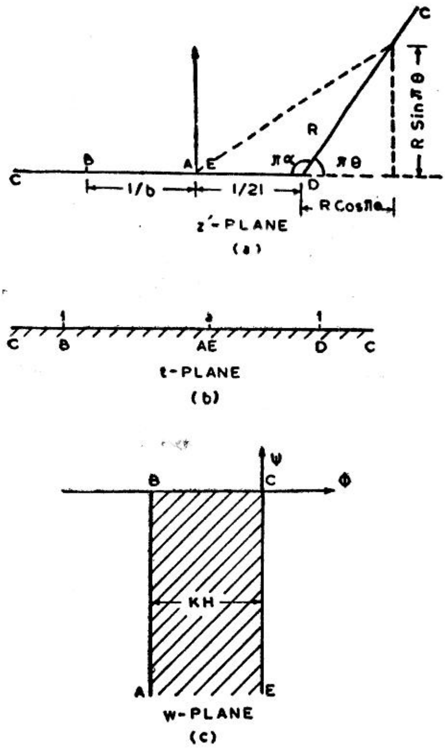


FIGURE 2 Transformations on various planes

Therefore, $N = 1/2l$

For point B, $Z' = -\,1/b$, and $t = -1$,

Hence,
$$M = \left(\frac{1}{2l} + \frac{1}{b}\right)\frac{\alpha}{(2)^{\alpha}}$$

Substituting values of M and N in Equation 1, the relation between Z' and t is found to be

$$Z' = -\left(\frac{1}{2l} + \frac{1}{b}\right)\frac{(1-t)^{\alpha}}{(2)^{\alpha}} + \frac{1}{2l} \qquad \dots (2)$$

Using the relation at $A\,E$ where $Z' = 0$; $t = a$, the unknown 'a' found to be

$$a = 1 - \frac{2}{(1+2\,\xi)^{1/\alpha}}$$

where
$$\xi = \frac{l}{b}$$

Equation 2, can be simplified to

$$t = 1 - 2\left\{\frac{(1-2\,l\,Z')}{(1+2\,\xi)}\right\}^{1/\alpha} \qquad \dots (3)$$

Under the floor $Z' = -\,q/b$ where q varies from 1 to ∞

$$t = 1 - 2\left\{\frac{1+2\,\xi/q}{1+2\,\xi}\right\}^{1/\alpha} \qquad \dots (4)$$

The above expression is valid only for the bottom contour $B\,C$ of the flow domain.

The mapping of w-plane, Figure 2(b), onto upper half of t-plane is given by

$$\frac{dw}{dt} = \frac{M'}{(1+t)^{1/2}\,(t-a)} \qquad \dots (5)$$

Integrating

$$W = \frac{M'}{\sqrt{-1}}\;\frac{2}{\sqrt{1+a}}\;\tan^{-1}\sqrt{\frac{-1-t}{1+a}} + N' \qquad \dots (6)$$

Using the conditions at point B and C the constants M' and N', are found to be

$$M' = \frac{i\,K_H\sqrt{a+1}}{\pi},$$

and

$$N' = -KH$$

Substituting values of M' and N' in Equation 6,

$$W = \frac{2KH}{\pi}\tan^{-1}\sqrt{\frac{(-t-1}{a+1}} - KH \qquad \dots (7)$$

Since along the contour BC, $\psi = 0$, $W = \phi$, along this contour

$$\phi = \frac{2KH}{\pi}\tan^{-1}\sqrt{\frac{(-t-1)}{a+1}} - KH \qquad \dots (8)$$

The pressure distribution under the floor can be obtained by using Equations 4 and 8. For $\xi = 1/8$, $1/12$, $1/16$, $1/32$ and $\beta = 1$ and 10 the pressure distribution has been plotted in Figures 3 to 6. It will be seen that for shallow scours when $\beta = 10$ the change in pressure is nominal. But for deep scours, when $\beta = 1$ the pressure gets reduced. This reduction is maximum near about downstream quarter point.

Exit gradient

The exit gradient on downstream side

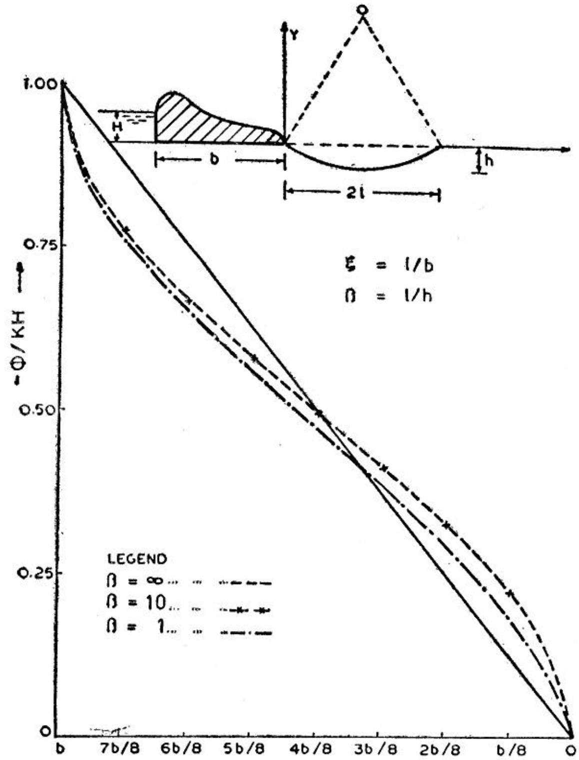
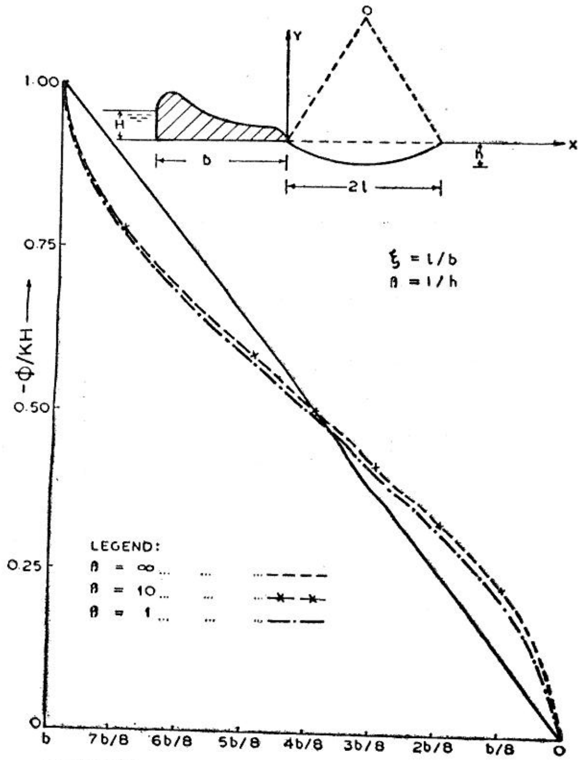


FIGURE 3 Uplift pressure curves for $\xi_{\mu} = 1/8$



FIGUCE 4 Uplift pressure curves for $\xi_{\mu} = 1/12$

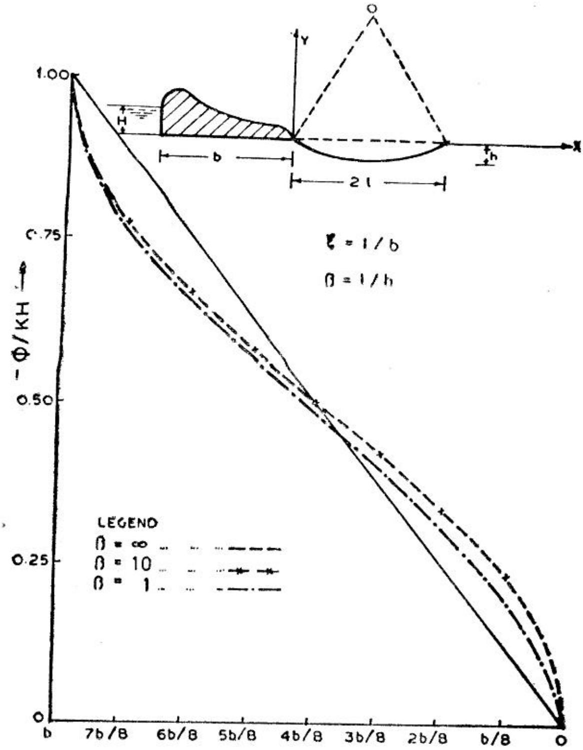


Figure 5 Uplift Pressure Curves for $\zeta_\mu = 1/16$

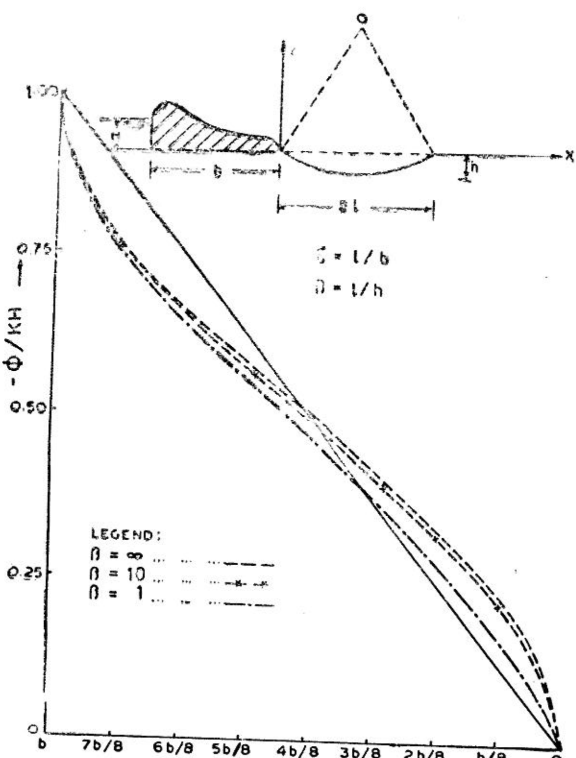


FIGURE 6 Uplift pressure curves for $\xi_\mu = 1/32$

$$I_x = \frac{1}{ki}\left(\frac{dw}{dz}\right) = \frac{1}{ki}\left(\frac{dw}{dt}\cdot\frac{dt}{ds}\right) \qquad \ldots(9)$$

From Equation 7,

$$\frac{dw}{dt} = \frac{ikH\sqrt{a+1}}{\pi(1+t)^{1/2}(t-a)}$$

From Equation 2,

$$\frac{dt}{dz} = -\frac{2^{\alpha}(1-t)^{(1-\alpha)}}{z^{2}\alpha\left(\frac{1}{2l}+\frac{1}{b}\right)}$$

Substituting values of $\frac{dw}{dt}$ and $\frac{dt}{dz}$ in Equation 9.

$$I_x = -\frac{H\sqrt{a+1}}{\pi(1+t)^{1/2}(t-a)}\cdot\frac{2^{\alpha}(1-t)^{(1-\alpha)}}{z^{2}(1/2l+1/b)\alpha}$$

and $$\frac{b}{H}\left|I_x\right| = \frac{|\,l/b\,|\,\sqrt{a+1}\;2^{\alpha-1}}{\pi x\left(\frac{|\,z^2\,|}{b^2}\right)(1+2\xi)} = \frac{|\,(1-t)^{(1-\alpha)}\,|}{(1-t)^{1/2}(t-a)} \qquad \ldots(10)$$

Relation between t and z :

Using Equation 2 for portion between C and D. (Figure 1).

$$(1-t)^{\alpha} = \frac{2^{\alpha}(Z'-1/2l)}{-(1/2l+1/b)}$$

From Figure 2(a)

$$Z' = 1/2l+Re^{i\pi\theta}$$

So $$(t-1)^{\alpha}(-1)^{\alpha} = \frac{2^{\alpha}Re^{i\pi\theta}e^{-i\pi}}{(1/2l+1/b)}$$

Since $\alpha = 1-\theta$ and $\xi = 1/b$ on simplification

$$(t-1) = \frac{2^{(\alpha+1)/\alpha}(lR)^{1/\alpha}}{(1+2\zeta)^{1/\alpha}} \qquad \ldots(11)$$

For portion beyond D (Figure 1)

$$Z' = 1/x$$

Again from Equation 2,

$$(1-t)^{\alpha} = \frac{2^{\alpha}\left(\frac{1}{x}-\frac{1}{2l}\right)}{-\left(\frac{1}{2l}+\frac{1}{b}\right)}$$

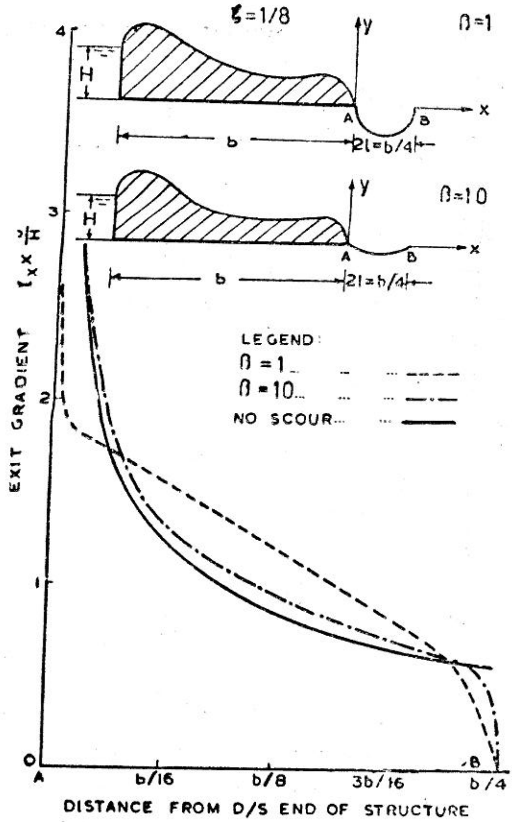


FIGURE 7 Exit gradient in scoured portion

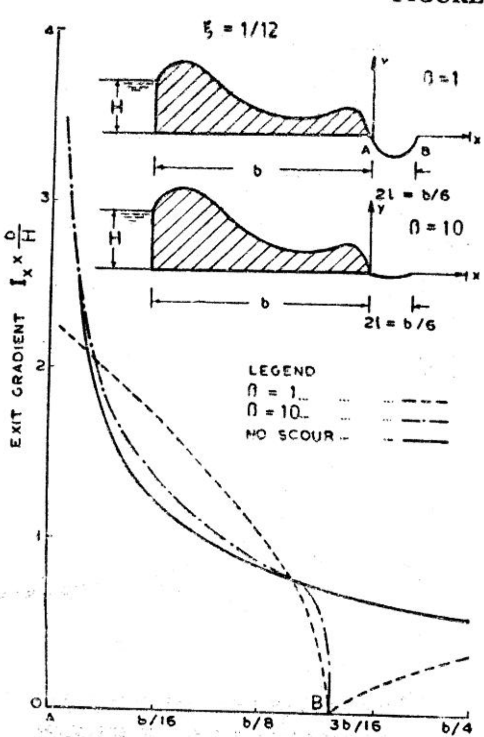


FIGURE 8 Exit gradient in scoured portion

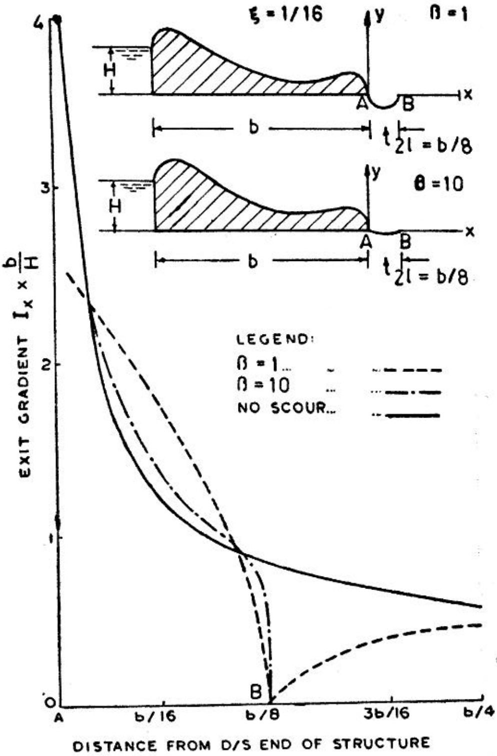


FIGURE 9 Exit gradient in scoured portion

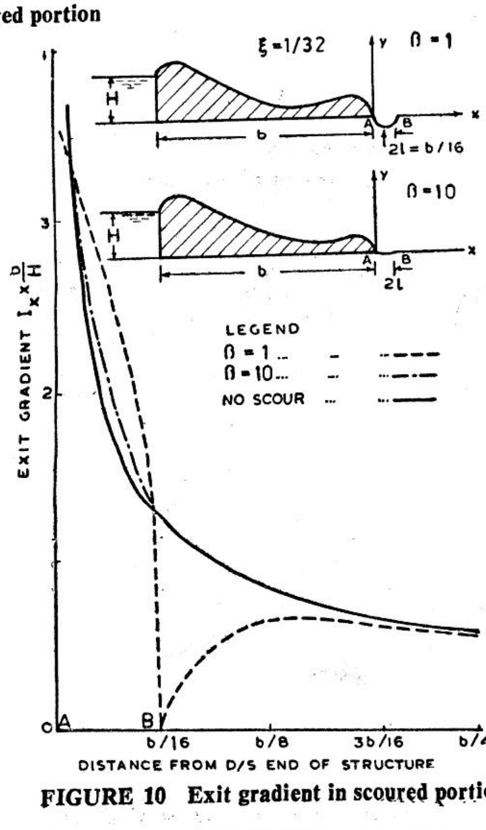


FIGURE 10 Exit gradient in scoured portion

where $x > 2l$

On simplification, the above expression gives

$$t = 1-2\left[\frac{b}{x}\left(\frac{x-2l}{b+2l}\right)\right]^{1/\alpha} \quad \ldots(12)$$

This distribution of exit gradient at various points in the scoured portion can be obtained by using Equations 10 and 11 and beyond the scoured zone the exit gradient can be obtained from Equations 10 and 12. The value of exit gradient for ζ equal to 1/32, 1/16, 1/12 and 1/8 and β equal to 1 and 10 have been plotted in Figure 7 to 10. $\beta = 1$ corresponds to semi circular scour profile. *It will be seen that for deep semi circular* scour the exit gradient becomes steeper in almost the entire scoured portion. In small portion near downstream end of scour the exit gradient becomes safer. At the end of scour, which is a singular point, exit gradient becomes zero. It being junction of two equipotential surfaces at an angle less than 180 degree, velocity of flow there has to be zero. For shallow scours exit gradient does not increase so much. It is to be noted here that the exit gradient acts normal to the circular profile.

The variation in exit gradient at three points A, B and C at $b/64$, $b/32$ and $3b/64$ from downstream end of floor, for same depth of scour, but different lengths, have been plotted in Figure 11. The points A and B lie in each case with in the upper half portion of scoured length. As these

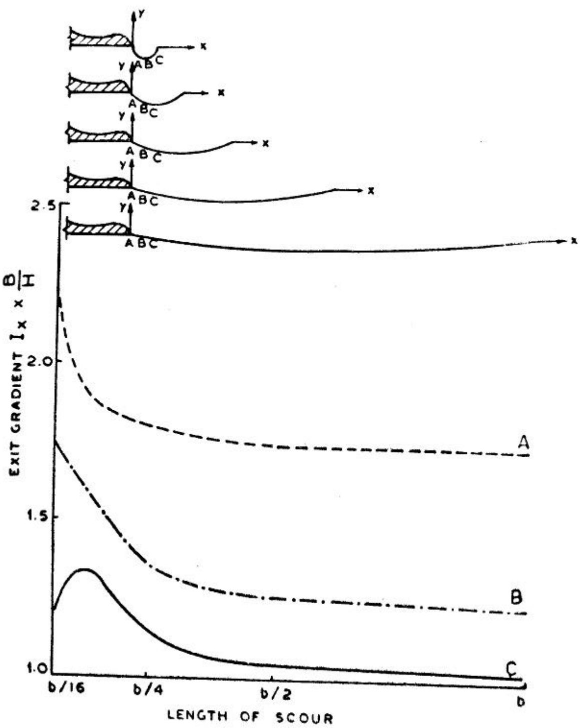


FIGURE 11 Change in evit gradient with scour length

points the exit gradient decreases with increase in scour length for same maximum depth of scour. The point C in the first case lies in the lower half of scoured portion. In all subsequent cases it lies in the upper half. Therefore for this point the exit gradient first increases with scour length and thereafter it decreases.

Conclusion

The effects of circular scour downstream of a flat bottom impervious floor are as follows :

(1) It reduces uplift pressure under the floor. But the reduction is marginal and mainly depends upon depth of scour.

(2) The exit gradient in the scoured portion becomes steeper. For long and shallow scours the change in exit gradient is marginal. But for deep scours it tends to increase and should be guarded against.