Draw Down Pore Pressure In Incompressible Homogeneous Dams

by

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Introduction

DRAWDOWN pore pressures form an important but uncertain factor in the stability analysis of upstream slope of an earth dam. Prior to the lowering of the reservoir, the pore pressure distribution is governed by the equilibrium conditions for the flow of water through porous media. At the time of drawdown, seepage forces are reversed and flow sets in from the dam towards its upstream face. The effect of drawdown is two fold. Firstly it establishes new boundary conditions for the flow of water through a dam and an unsteady state of flow is established while the phreatic line adjusts to a new equilibrium position. During the period of drawdown, some water may also drain. Secondly, the stress changes due to drawdown, also cause changes in the pore pressures. These factors complicate the solution of the problem of drawdown pore pressures. A theoretical approach to the problem is discussed herein. For verification of theoretical resusts, observations were taken on three sand models also and the same are also discussed.

Existing Theories

The prediction of drawdown pore pressures in the upstream portion of a dam is the major problem for which no reliable theoretical solution is available. The most common method of estimating these pressures is to assume that the Laplacian equation $\nabla^2\phi = 0$, is applicable and that the reservoir level is lowered rapidly enough to prevent substantial drainage inside the embankment. A flownet can then be drawn for transitory seepage conditions which would exist directly after instantaneous drawdown. For time dependent drawdown Reinius (1948) has said that for incompressible free draining fill, dissipation of pore pressure during draw down is a function of the parameter $\frac{k}{nv}$ where 'k' is permeabillty, 'n' is specific yield and 'v' is rate of drawdown. His study is based on plotting of transient flow net. Cedergren (1948) also refers to the drawing of transient flow net. In practice, it is very difficult to draw transient flow net and also the values of pore pressures so obtained, are many times far out then those actually observed. The analytical approach due to Casagrande and Shanon (1948) is not only applicable to shell portion of a zoned embankment but in fact gives merely the time required for given degree of pore pressure dissipation assuming that drawdown has taken place instantaneously and the transient saturation line is approxi-

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mated to a straight line. In actual practice, some time is bound to take place in drawdown and the transient flow line is never a straight line.

Theoretical Approach

To solve the complex problem of transient pore pressures, the following assumptions have been made.

(*i*) The soil is homogeneous and isotropic.

(*ii*) The problem of flow is two dimensional.

(*iii*) The law of conservation of matter holds good.

(*iv*) Darcy's law of flow through porous media is applicable.

(*v*) Dupuit's assumption viz. (a) the variation of potential with height are negligible and (b) horizontal velocities do not vary with height, are valid in this case.

In homogeneous incompressible earth dam section, the transient pore pressures are given by the equation

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) = \frac{n}{k}\cdot\frac{\partial h}{\partial t} \qquad \ldots(1)$$

which is the general equation for transient flow through porous media and can also be written in the following form

$$\frac{\partial^2 h}{\partial x^2} = -\frac{1}{h}\left(\frac{\partial h}{\partial x}\right)^2 + \frac{n}{k}\cdot\frac{1}{n}\cdot\frac{\partial h}{\partial t} \qquad \ldots(2)$$

where 'h' is pore pressure at any time 't', 'x' is spatial coordinate, 'k' permeability and 'n' is specific yield. The non-dimensionless form of Equation 2 is

$$\frac{\partial^2 H}{\partial X^2} = -\frac{1}{H}\left(\frac{\partial H}{\partial X}\right)^2 + \frac{1}{H}\cdot\frac{\partial H}{\partial T} \qquad \ldots(3)$$

where $H = \frac{h}{L}$, $X = \frac{x}{L}$, $T = \frac{k}{n}\cdot\frac{t}{L}$ and, L is the total impervious length at base.

Non-linear differential Equation 3 has been solved for the known initial and boundary conditions by finite difference method using Douglas and Jone's predictor and corrector Method (Remson et al, 1971). If ΔX and ΔT are space and time intervals respectively, and $w_{i,j}$ is the approximate solution for H at ith space and jth time interval and using implicit finite difference method, the following are the finally rearranged equations for predictor and corrector.

Predictor

$$w_{i+1,\,j+\frac{1}{2}} - \left\{2 + 2\cdot\frac{(\Delta X)^2}{\Delta T}\cdot\frac{1}{w_{i,\,j}}\right\}\cdot w_{i,\,j+\frac{1}{2}} + w_{i-1,\,j+\frac{1}{2}}$$

$$= -2 \cdot \frac{(\triangle X)^2}{\triangle T} - \frac{1}{4} \cdot \frac{1}{w_{i,j}} \cdot (w_{i+1,j} - w_{i-1,j})^2 \qquad \ldots(4)$$

Corrector

$$w_{i+1,j+1} - \left\{ 2 + 2 \cdot \frac{(\triangle X)^2}{\triangle T} \cdot \frac{1}{w_{i,j+\frac{1}{2}}} \right\} \cdot w_{i,j+1} + w_{i-1,j+1}$$

$$= -2 \frac{(\triangle X)^2}{\triangle T} \cdot \frac{w_{i,j}}{w_{i,j+\frac{1}{2}}} - (w_{i+1,j} - 2 \cdot w_{i,j} + w_{i-1,j})$$

$$- \tfrac{1}{2} \cdot \frac{1}{w_{,j+\frac{1}{2}}} (w_{i+1,j+\frac{1}{2}} - w_{i-1,j+\frac{1}{2}})^2 \qquad \ldots(5)$$

A computer programme was prepared to solve these equations and the theoretical pore pressures were found out for the known geometry, material properties and rate of drawdown for the three physical sand models, for verification by the observational test data.

Models

For studying drawdown pore pressures in sandy fill material, experiments were carried out in 75 cm high glass flume. The dam section comprises both upstream and downstream slopes of $2H : 1V$, the height being 70 cm. Three models of different sands having different properties, were constructed. The properties of the sands used in the models are given in Table 1.

TABLE 1

Properties of Sand Models

Sand Model No.	D_{10} mm	D_{50} mm	Uniformity coefficient	Permeability K 10^{-2} cm/sec	Specific yield n
1	0.075	0.18	2.3	5	0.18
2	0·10	0.425	6.0	1	0.166
3	0.30	0.85	3.5	15.3	0.34

The sand was hydraulically filled in the flume. Open tube piezometers which are reported to have negligibly small response time for sandy material (Terzaghi and Peck, 1967) were used for observation of pore pressures. These piezometers were fitted on a separate board, the lower ends of which were connected to 20 different tapping points in the model. The tapping points consist of brass pipes welded to a mild steel sheet embedded in the back of the wall of the flume. The ends of the tapping points embedded in the sandy material were covered by means of 75 micron sieve to protect the stand pipe piezometers from clogging. Observations were recorded on a graph sheet attached behind the glass tubes. After achieving the steady state condition, the drawdown in the reservoir level was carried out at the different rates viz. (a) 12 cm/min i.e. almost instantaneous, (b) 3.33 cm/min., (c) 1.667 cm/min and (d) 0.833 cm/min. The pore pressures were observed regularly during and after drawdown at specified time intervals for all rates of drawdown for all the three sand models.

Discussion

The observed pore pressures in terms of dimensionless parameters for three sand models no. 1, 2 and 3 are plotted in Figures 1 to 3 for some

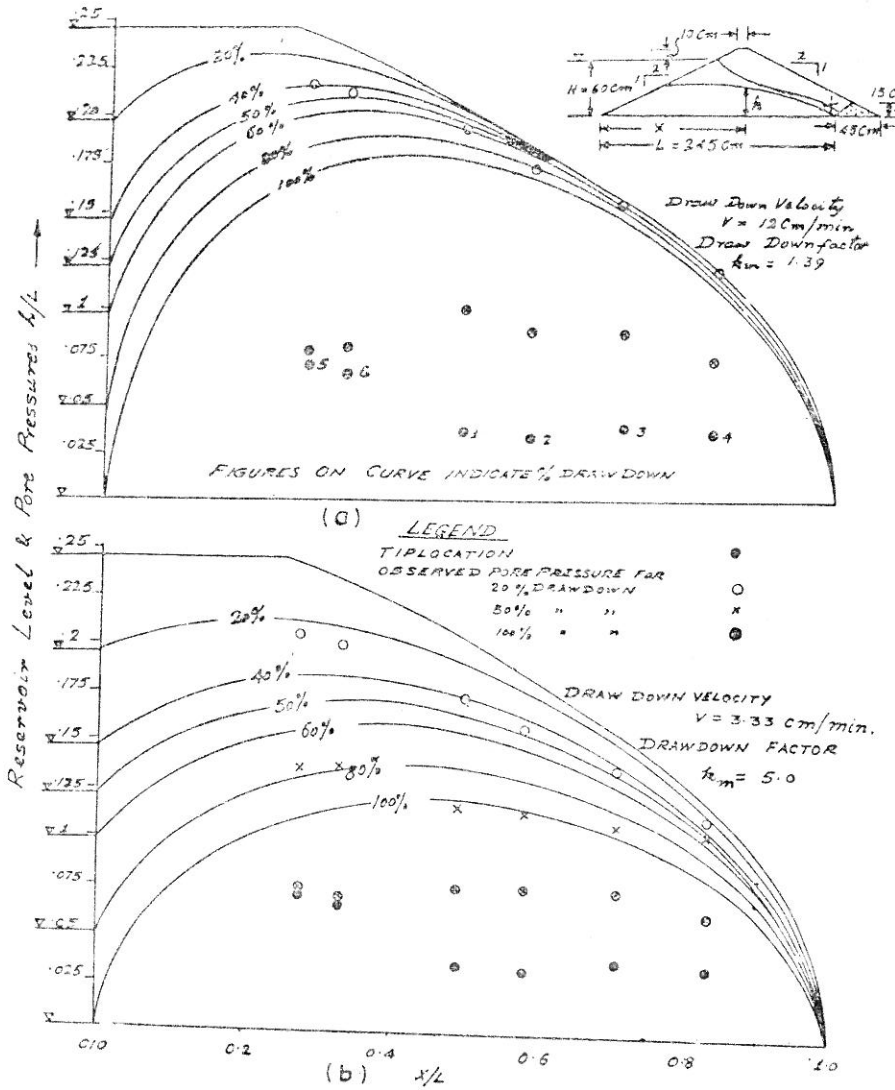

FIGURE 1 (*a*, *b*) Theoretical and observed D.D.P.P. for sand model no. 1, permeability $K = 5 \times 10^{-2}$ cm/sec, specific yield $N_s = 0.18$

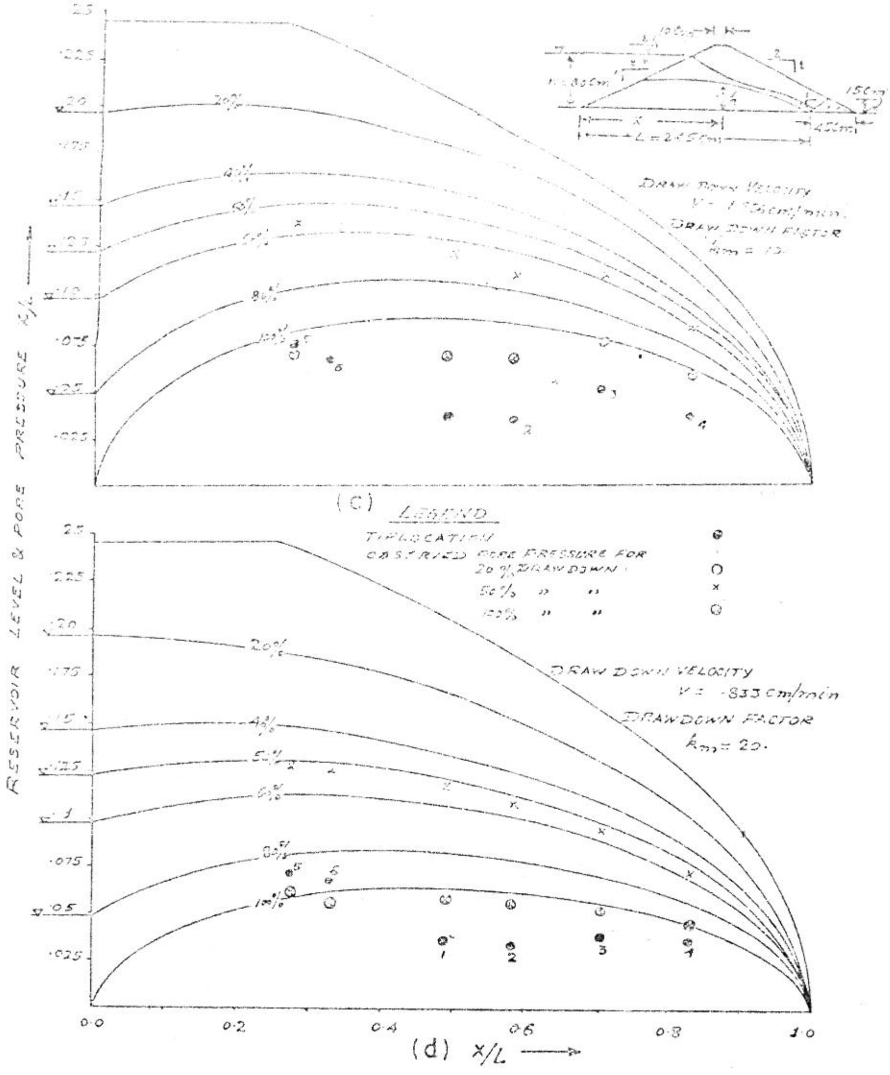

FIGURE 1 (c, d) Theoretical and observed D.D.P.P. for sand model no. 1, permeability $K = 5 \times 10^{-2}$ cm/sec, specific yield $N_s = 0.18$

typical tips for 20 per cent, 50 per cent and 100 per cent drawdown. Nos. (a) to (d) on each figure indicate corresponding four rates of drawdown discussed abovd. The theoretical pore pressures given by Equation 3, for the three models and for four rates of drawdown, have also been plotted in these figures for 20 per cent. 40 per cent, 50 per cent, 60 per cent, 80 per cent and 100 per cent drawdown.

It was noticed that observed steady state pore pressures for models no. 1 and 3 were almost the same as found out theoretically by A. Casagrande method where as for model no. 2, the same were observed to be on lower side.

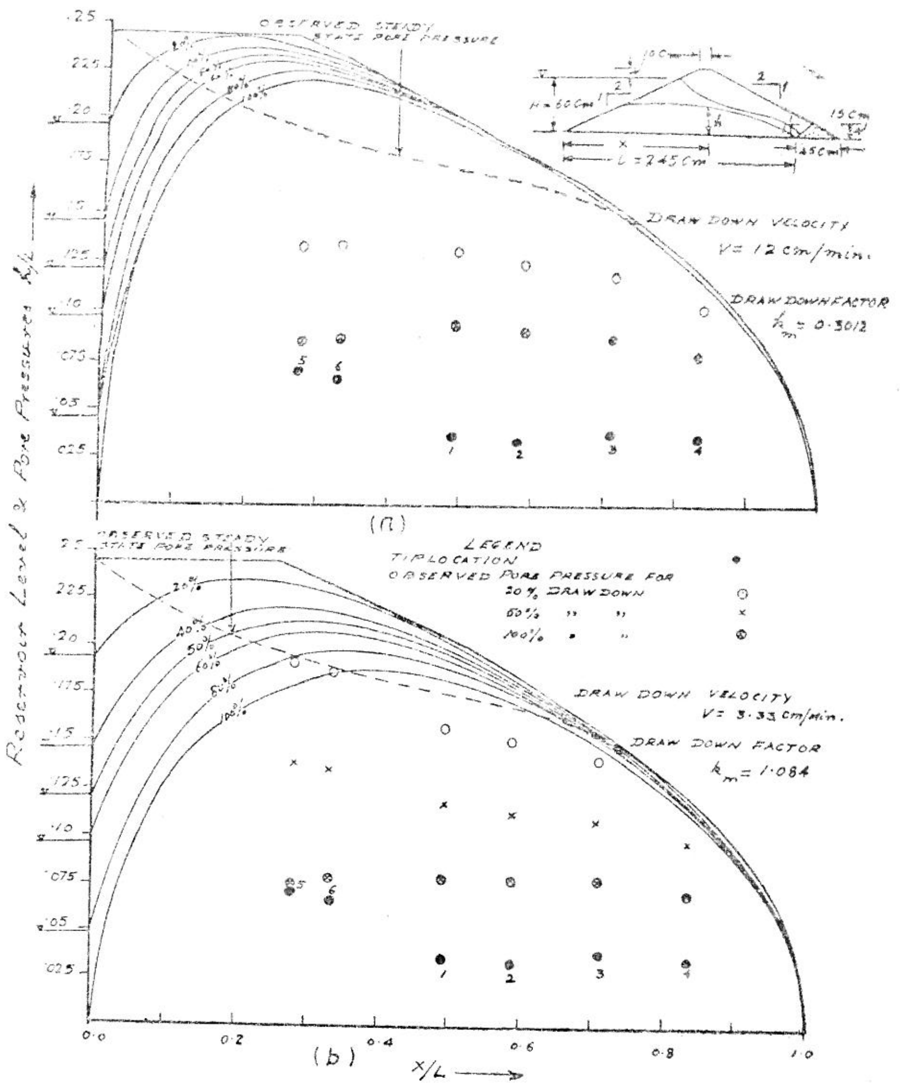

FIGURE 2 (*a*, *b*) **Theoretical and observed D.D.P.P. for sand model no. 2, permeability $K = 1 \times 10^{-2}$ cm/see, specific yield $N_s = 0{\cdot}166$**

A study of results indicates that for sand number 1 with rate of drawdown 12 cm/min, for 20 per cent drawdown, the observed and theoretical values are very close, however for 100 per cent drawdown the observed values are almost half that of theoretical. For rate of drawdown of 3.33 cm/min., for 20 per cent drawdown, the observed values are very close to that of theoretical but for 50 per cent drawdown, the observed values are 25-30 per cent less than theoretical whereas for 100 per cent drawdown, the observed values are near about 40 per cent less than theoretical. For rate of drawdown of 1.667 cm/min, the observed pore Pressure values for 50 per cent and 100 per cent drawdown are quite close to the theoretical values. So is also the case with rate of drawdown of 0.833 cm/min.

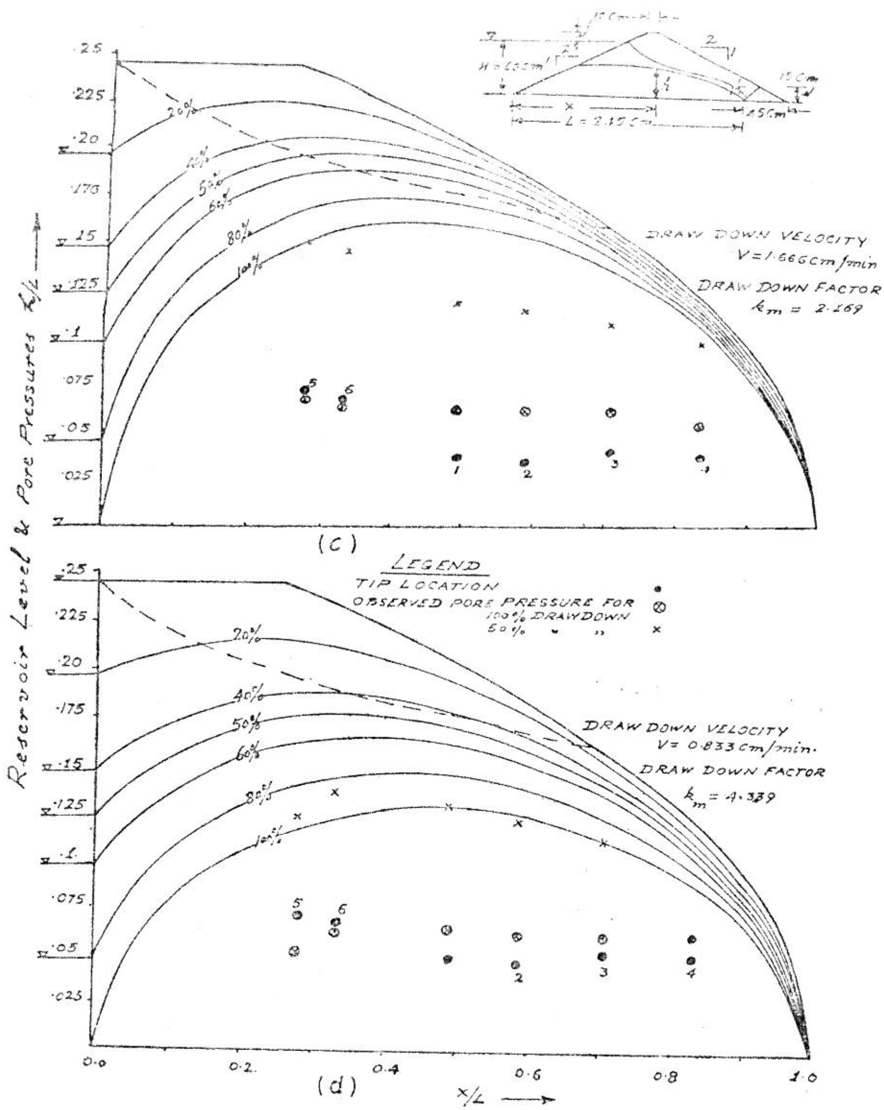

FIGURE 2 (*c*, *d*) **Theoretical and observed D.D.P.P. for sand model no. 2, permeability $K = 1 \times 10^{-2}$ cm/sec, specific yield $N_s = 0.166$**

For sand no. 2, with rate of drawdown 12 cm/min, the observed values both for 20 per cent and 100 per cent drawdown are almost half of theoretical values. For rate of drawdown of 8.33 cm/min, the observed values are near about 20-25 per cent, 40-45 per cent and 50-60 per cent less than that of theoretical values for 20 per cent, 50 per cent and 100 per cent drawdown respectively. The difference in between observed and theoretical pore pressures also pesrists for drawdown rates of 1.667 and 0.833 cm/min.

Observational results for 100 per cent drawdown plotted for sand no. 3

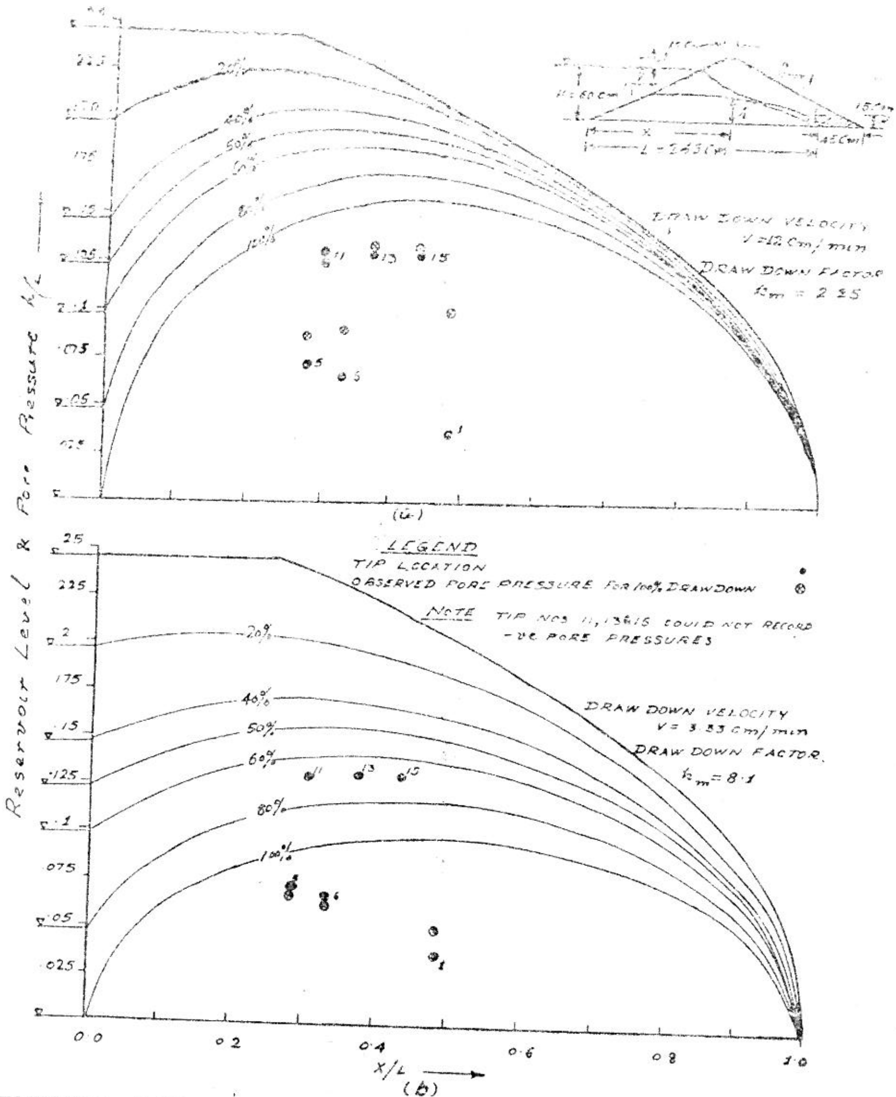

FIGURE (a, b) **Theoretical & Observed D.D.P.P. for sand model No.3, permeability $K = 15. \times 10^2$ cm/sec, specific yield $N_s = 034$**

indicate that negative pore pressures were there for tips no. 11, 13 and 15 but the same could not be recorded by tips. The other three tips indicate that the observed pore pressures for 12, 3.33 and 1.67 cm/min rate of drawdown are less than the theoretical values but as the rate of drawdown decreases, the difference in between observed and theoretical values also reduces and for 0.833 cm/min rate, the two values almost coincide.

The above discussions indicate the following

(*i*) For slow rates of drawndown, the observed pore pressure tend to coincide with the theoretical values.

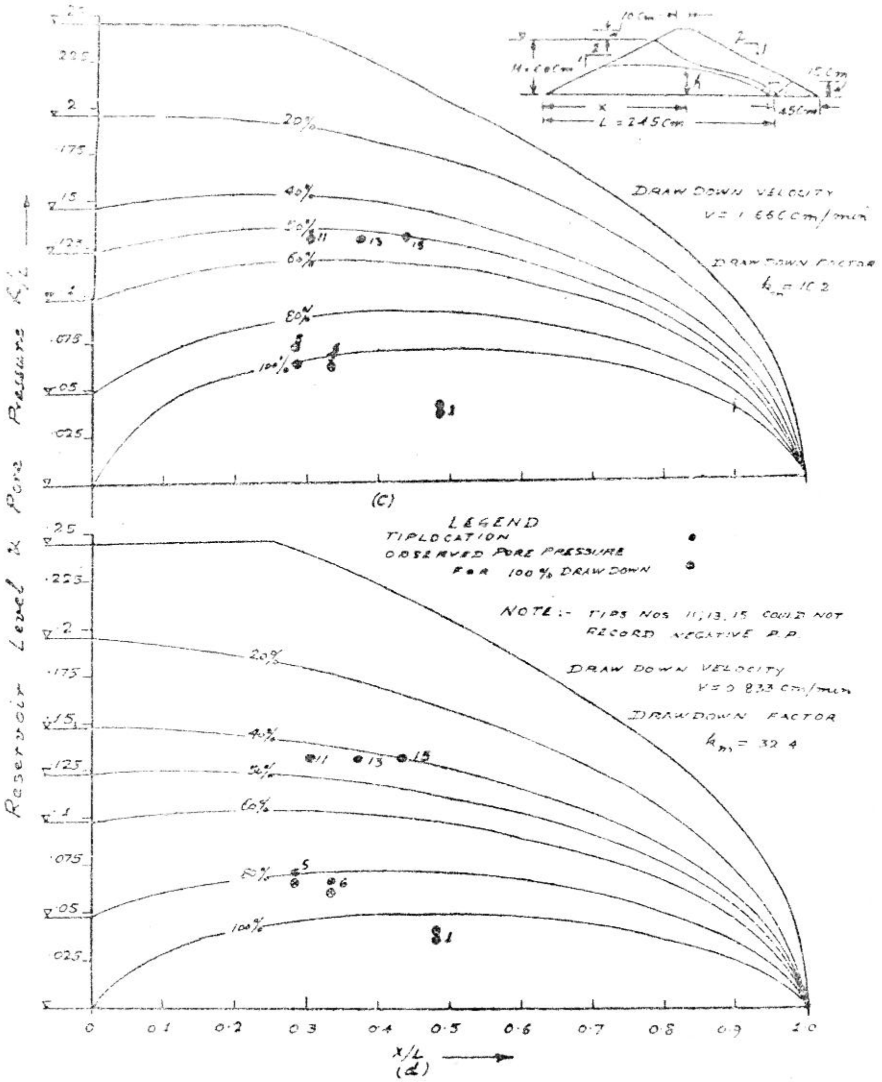

FIGURE 3 (c,d) Theoretical & Observed D.D P.P. for sand model No. 3, permeability $K = 15.3 \times 10^2$ cm/sec, specific yield $N_s = 0.34$

(*ii*) At complete drawndown, difference in observed and theoretical pore pressure values tend to increase with increasing rate of drawdown but at slow rates of drawdown, the observed and theoretical values almost coincide.

(*iii*) Sand No. 2 has shown exceptionally low observed values. Incidentally the observed steady state pore pressures before drawdown were also lower than the theoretical values.

First and second indications tend to believe that the difference in between observed and theoretical pore pressures is mainly because of negative pore pressures due to capillarity effect. As during early perioc of drawdown, sufficient area above free surface line is not available to generate complete capillary zone, the quantum of negative pore pressures is also less and hence less difference in observed and theoretical values. However as the drawdown increases, more area is available above free surface line and hence observed pressures were much less. But as the rate of drawdown slows down, the capillary zone tends to become dry and hence negative pressures dissipate, thus bringing observed pore pressures nearer to theoretical values. The chances of higher negative pore pressures are more for fine sands and as such, noticeable difference in between observed and theoretical values for sand no. 2 stands explained in the light of development of negative pore pressures. Negative pore pressures in capillary zone are effecting the observational test results considerably for model because of scale effect. The dam height in the model was only 70 cm where as reservoir level was 60 cm from bottom. On actual prototype, the relative effect of negative pore pressure because of capillarity will be much less than to that observed on the model.

The study has indicated that drawdown pore pressures for incompressible fills are primarily dependent upon (*i*) permeability of fill 'k', (*ii*) specified yield 'n' and (*iii*) rate of drawdown 'v'. The three parameters when grouped together are represented by one factor named 'draw-down factor' k_m given by the relationship $k_m = \frac{k}{n.v}$. This is incidentally the same factor as envisaged by Reinius (1948) but with different approach.

Conclusion

The comparison of observed pore pressure values with those of theoretical values after keeping in view the capillarity effect, generates confidence that the theoretical model, can be adopted for dealing drawdown problems in earth dams. The drawdown pore pressures in incompressible fill have been found to be dependent upon geometry of dam section and the drawdown factor k_m.

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Notations

h = Pore pressure at any time 't'

$H = h/L$

i = Space interval number

j = Time interval number

k = *Permeability*

L = Total length at base from toe to the point on d/s side where seapage pressure is either zero or equal to d/s tail water level

n = Specified yield

ϕ = Potential

t = Any time

$$T = \frac{k}{n} \cdot \frac{t}{L}$$

W = Approximate solution for H

x = Distance of any point from u/s of dam towards downstream toe

$X = x/L$

Δ = Small change

∂ = Partial differential operator

∇^2 = Laplacian operator