Short Communications

Earth Pressures due to Plane Strain Surcharge Loads

by

N. Babu Shankar*

and

Mohammad Ali**

Introduction

Lateral earth pressures on retaining walls due to self weight of backfill soils are estimated from the (classical) earth pressure theories. Such earth pressure theories. Such earth pressures depend on the deformation pattern of retaining structure relative to the backfill. Additional earth pressures are caused on retaining walls due to surcharge loads acting on the surface of the backfill. This paper discusses the earth pressure distribution behind rigid retaining walls due to infinite line loads and strip loads acting parallel to the walls. Solutions are presented for intensities of earth pressure, lateral forces and moments acting on the wall due to above surcharge loads.

Analysis

From Boussinesq's elastic distribution theory (Boussinesq, 1885) the normal stress (σ_x) in x-direction due to a surface point load (Q) acting in an elastic medium with a Poisson's ratio of ν is given by

$$\sigma_x = \frac{Q}{2\pi R^2}\left[\frac{3z\,x^2}{R^3} - (1-2\nu)\left\{\frac{z}{R} - \frac{R}{R+z} + \frac{x^2(2R+z)}{R(R+)z^2}\right\}\right] \quad ...(1)$$

Where x, y, z are the spatial coordinates of the point (at which σ_x is to be evaluated) with reference to Q and $R^2 = x^2+y^2+z^2$. For $\nu = 0.5$, Equation 1 reduces to

$$\sigma_x = \frac{3Q}{2\pi}\,\frac{x^2 z}{R^5} \quad ...(2)$$

If x is treated as the perpendicular distance from the point load to the retaining wall (which is parallel to Y-axis) and z is treated as the depth of soil below ground level, then σ_x may be treated as the intensity of lateral

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*Assistant Professor, **Associate Lecturer — Civil Engineering Department Regional Engg. College, Warangal-506004 AP INDIA

This paper was received in August 1978 and is open for discussion till the end of March 1980.

(horizontal) earth pressure (say p) However Equation 2 is valid for a semi-infinite elastic medium. The presence of a rigid retaining wall restricts the elastic lateral deformations of the backfill. It has, therefore, been found from field instrumented studies that the actual earth pressures behind retaining walls are nearly twice those computed from theory of elasticity (Terzaghi, 1954). Hence Spangler (1960), suggested the modification of Equation 2 as

$$p = \frac{Q\,x^2 z}{R^5} \qquad \ldots(3)$$

Equation 3 which is valid for a point load may suitable be integrated to obtain expressions for earth pressures (p) under different types of surcharge loads.

Line Loads

The intensity of earth pressure (p) due to infinite line load ($\bar{q}$) acting parallel to wall (Figure 1) is obtained from Equation 3 as

$$p = \frac{4\bar{q}}{3}\,\frac{x^2 z}{(x^2+z^2)^2} \qquad \ldots(4)$$

The results of Equation 4 are shown in Figure 2 in non-dimensional form. The curves in Figure 2 show the earth pressure distribution behind the retaininig wall. The areas of the earth pressure distribution diagrams bounded by curves in Figure 2 yield the lateral forces (P) per unit run of the wall

Thus

$$P = \int_0^H p\,dz = \int_0^H \frac{4\bar{q}}{3}\,\frac{x^2 z}{(x^2 z^2)^2}\,dz$$

$$= \frac{2\bar{q}}{3}\,\frac{H^2}{x^2+H^2} \qquad \ldots(5)$$

Where H is the height of the retaining wall.

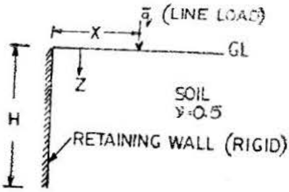

FIGURE 1 **Line load parallel to wall**

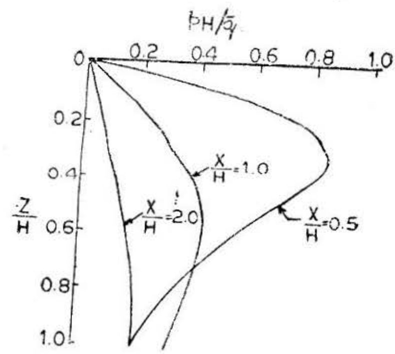

FIGURE 2 **Earth pressure distribution**

The moment (M) per unit run of the wall about the base of the retaining wall is obtained as

$$M = \int_0^H p\ (H-z)dz = \frac{2\bar{q}}{3}\left(H - x\ \tan^{-1}\frac{H}{x}\right) \qquad \ldots(6)$$

The results of Equations (5) and (6) shown in Figure 3. As expected, the lateral force and moment decreases with increasing horizontal distance of the line load away from the wall.

Strip Loads

A row of columns or a series of loading cranes located parallel to a long retaining wall may simulate line loads. However, strip loads due to pavements and cargo are more common types of surcharge loads encountered in retaining walls. Figure 4 shows a typical strip of width 'b' with a uniform load intensity of 'q'.

The earth pressure (p), lateral force (P) and lateral movement (M) per unit for this case can be obtained in a similar manner as for line loads from the integration of basic Equation 3 as

$$p = \frac{2q}{3}\left[\left\{\tan^{-1}\frac{x+b}{z} - \tan^{-1}\frac{x}{b}\right\} - \left\{\frac{(x+b)z}{(x+b)^2+z^2} - \frac{xz}{x^2+z^2}\right\}\right] \qquad \ldots(7)$$

$$P = \frac{2qH}{3}\left[\tan^{-1}\frac{x+b}{H} - \tan^{-1}\frac{x}{H}\right] \qquad \ldots(8)$$

$$M = \frac{q}{3}\left[Hb + (x^2+H^2)\tan^{-1}\frac{H}{x} - \left\{(x+b)^2+H^2\right\}\tan^{-1}\frac{H}{x+b}\right] \qquad \ldots(9)$$

The results from these equations are shown in non-dimensional form in Figures 5, 6, and 7. It is seen that closer the proximity (that is lesser the

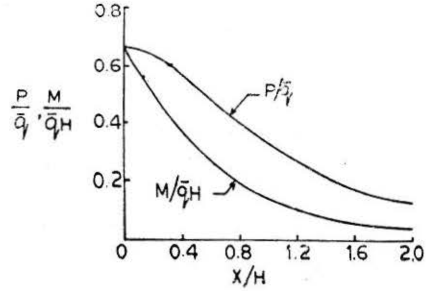

FIGURE 3 Lateral forces and moments

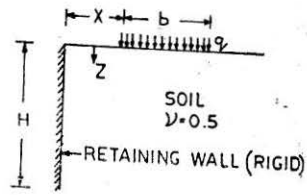

FIGURE 4 Strip load parallel to wall

edge distance (x) and larger the width (b) of loading, more will be the earth pressures, forces and moments on the wall as expected.

Example

Use of graphs shown in Figure 6 will be illustrated by an example of problem shown in Figure 8 wherein it is required to find the magnitude of active earth pressure due to (strip load) surcharge only.

For $\frac{x}{H} = 0.4$ and $\frac{b}{H} = 0.5$ from Figure 6 $\frac{P}{qH} = 0.23$

$\therefore P = (0.23)\,(10)\,(2) = 4.6$ t/m

However, this value of P is correct provided the wall is rigid indicating almost k_o conditions. If earth pressure for active condition is desired,

$$P_a = \left(\frac{k_a}{k_o}\right)(P) \simeq \left(\frac{1}{1+\sin\phi}\right)(P) = (0.67)\,(4.6) = 3.1 \text{ t/m}$$

Using Jaley's expression for k_o and Rankine's expression for k_a.

The active earth pressure calculated based on the procedure suggested

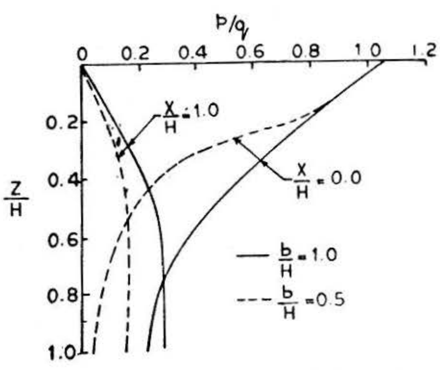

FIGURE 5 Earth pressure distribution

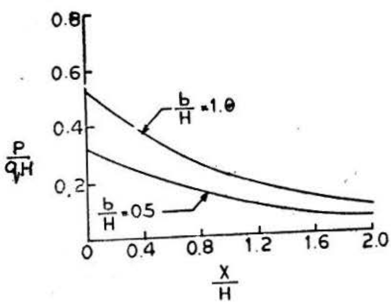

FIGURE 6 Lateral forces

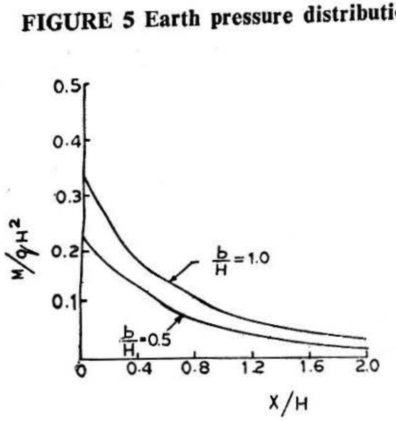

FIGURE 7 Lateral moments

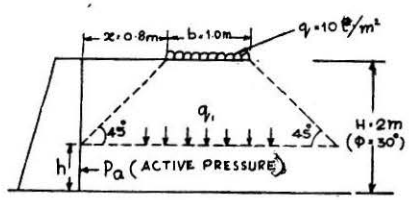

FIGURE 8 Sketch for the example illustrated

in India Rly Code (1963) is as follows:

$$P_a = q_1 \, h \, k_a = \frac{qb}{(b+2X)} \, (H-X) \left(\frac{1-\text{Sin}\,\phi}{1+\text{Sin}\,\phi} \right)$$

$$= \frac{(10)\,(1)}{(1+1.6)} \, (1.2) \, (\tfrac{1}{3})$$

$$= 1.5 \text{ t/m}$$

Thus Spangler's solutions presented in the paper yields almost twice the values calculated based on IRS procedure. However, the coincidence is remarkable if Boussinesq's Equation 2 was used instead of Equation 3.

Conclusions

Solutions are obtained for the earth pressures, lateral forces and moments acting on rigid retaining walls due to plane strain surcharges like line loads and strip loads. Graphical results presented in non-dimensional form indicate that as the loads move away from retaining wall the lateral earth pressures, forces and moments on the wall reduce. Increasing width of strip loading will increase the earth pressures.

Acknowledgements

Encouragement given by Principal, Dr. K. Koteswara Rao of the College is gratefully acknowledged. Authors thank Mr. Ramakrishna Reddy for the help rendered in calculations and drawing graphs.

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