Investigation of Footings Under Eccentric Load

By

R. Das Purkayastha*

Introduction

The evaluation of bearing capacity forms an important part of the design of foundations. Apart from vertical centric loads, foundations of portal framed buildings are often subjected to eccentric loads caused by wind and earthquake forces. Similarly, foundations of earth and water retaining structures, abutments and similar structures may be subjected to eccentric loads.

Whereas many theories have been developed for the bearing capacity of centrally loaded foundations the analysis of the bearing capacity of eccentrically loaded foundations has not received much attention. The conventional method, where the maximum base pressure at the toe is compared with the ultimate bearing capacity of a centrally loaded footings of the same width, is not adequate. Ramelot and Vandeperre (1950) reported the results of model tests with eccentrically loaded square and circular footings. Meyerhof (1953) developed an empirical concept by which an eccentrically loaded footings may be regarded as a centrally loaded footing of reduced width, the reduction along any direction being equal to twice the eccentricity in that direction. Eastwood (1955) and Dhillon (1961) reported the results of tests on sand with model footings and concluded that Meyerhof's hypothesis is not adequate for predicting the bearing capacity of eccentrically loaded footings. Jumikis (1956) observed that for footings under eccentric load, the failure surface develops on the opposite side of the eccentricity. Lee (1965) found that the peak moment capacity is slightly higher than the value derived from the Meyerhof concept. Zaharescu (1961) photographed the movement of soil beneath foundations and his experimental results agreed well with Meyerhof's hypothesis. Prakash and Saran (1971) attempted a theoretical solution of the problem on the basis of the superposition technique suggested by Terzaghi.

It is apparent that though experimental results on the behaviour of eccentrically loaded foundations have been reported in general, attempts to arrive at a comprehensive solution of the problem on the basis of experimentally determined failure surface have not been made. In this paper, details of experimentation with eccentrically loaded footings for two dimentional and three dimensional cases have been reported. The observed ultimate loads have been compared with the theoretical values obtained from consideration of stability of the sliding soil masses along the experimentally observed failure surfaces. Exponential curves have been fitted for the theoretical reduction factors and suitable expression has been given. A simple method for determining the bearing capacity of eccentrically loaded footings has been suggested.

---

* Department of Civil Engineering, Jadavpur University, Calcutta-700032.

The paper was received in September, 1978 and is open for discussion till the end of November, 1979

Experimental programme

Wooden footings of 4 cms thickness were used in the testing programme. The base of the footing was knurled to simulate the rough base condition of a prototype footing. To allow for the free tilting and lateral movement of the footing, the load was applied through a roller arrangement which was fitted in a groove at the top of the footing. Centric and eccentric grooves were made on the top surface of the footing. Eccentric grooves were made at e/B values of 0.1, 0.2, 0.3 and 0.4, where 'e' is the eccentricity of load and 'B', the width of footing. The footings with the grooves and the roller arrangement are shown in Figure 1.

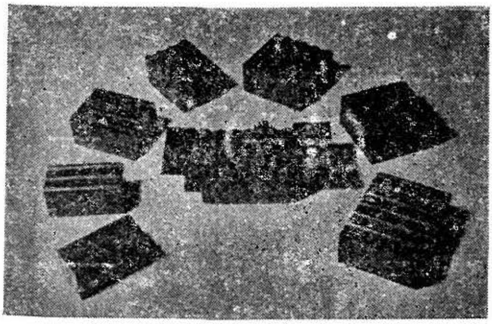

FIGURE 1 Wooden footings showing the grooves and the roller arrangements

Materials Used

White Ennore sand was used for preparing the test bed in the two dimentional and three dimensional tests. This particular sand with sub angular particles of fairly uniform diameter facilitated the reproduction of the test bed properties. The grain size distribution for this sand is shown in Figure 2.

Two Dimentional Tests

A plane strain test box of size $90 \times 45 \times 10$ cm size with perspex wall in the front was used for the two dimensional tests. To maintain the rigidity of the box, removable horizontal stiffeners were used. Four sizes of footings of width 5 cm, 7.5 cm, 10 cm and 12.5 cm with a constant length of 10 cm were used.

The test bed was prepared by pouring sand through a funnel from a constant height of 30 cm. The density achieved for all the tests was 1.62 t/m^3 and the angle of internal friction at this density was 38° as obtained from triaxial tests. The relative density was 49 per cent.

A universal triaxial machine was modified and used as a loading frame for carrying out the tests. An electric motor at the top of the frame actuated a screw which moved downwards. A calibrated proving ring was connected at the lower end of the screw through an adaptor and the tests were conducted at constant rate of strain. The settlement was recorded by dial gauges placed at the top of footings. The machine with the test box is shown in Figure 3.

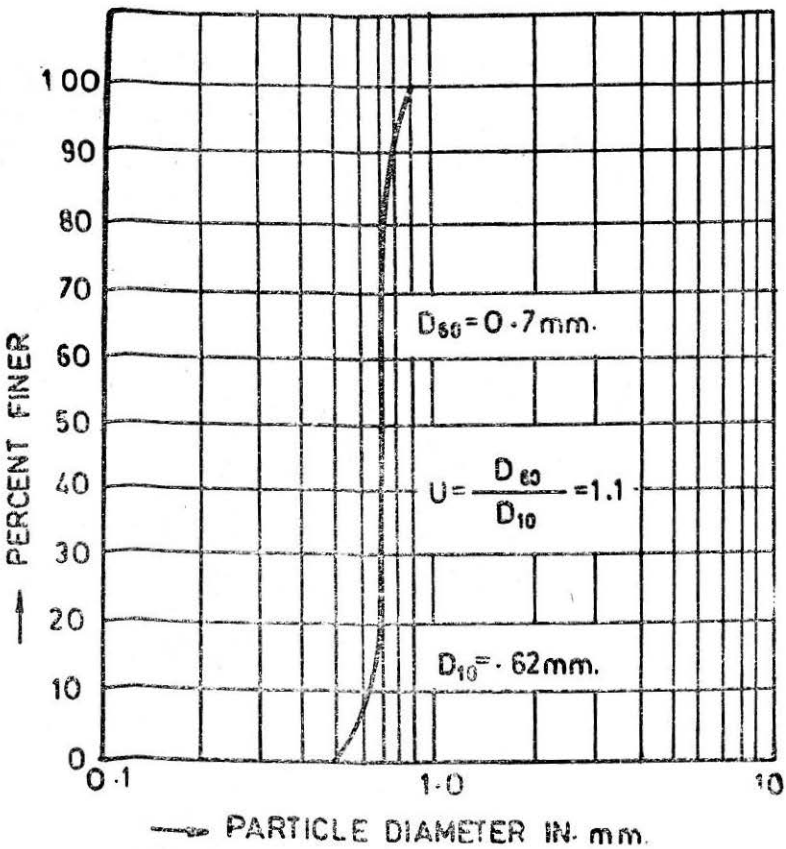

FIGURE 2 Grain size distribution for Ennore sand

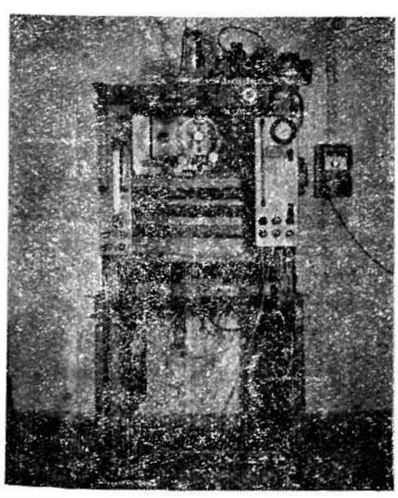

FIGURE 3 Experimental set-up for two dimensional tests

A number of two dimensional tests were conducted to study the failure mechanism of eccentrically loaded foundations by observing the displacements of coloured sand bands in the test box. The one sided failure of footings under eccentric load is shown in Figure 4. The general

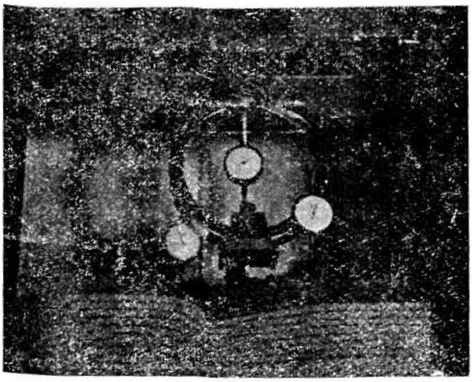

FIGURE 4 Failure of footing under eccentric load

failure pattern as established from the tests is shown in Figure 5.

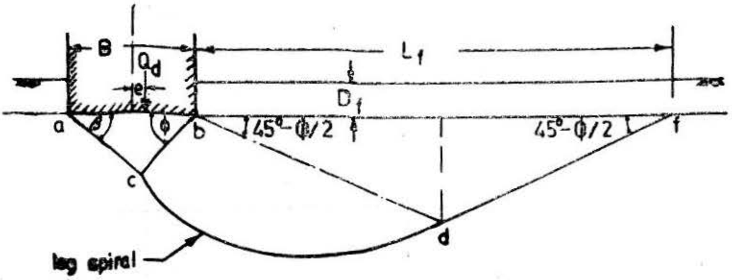

FIGURE 5 General form of failure for loads with eccentricities within the middle third

It has been observed that with increased eccentricity, the failure surface becomes smaller. The angle *abc* is taken as ϕ and the angle *bac* is seen to be a variable β which reduces with the increase in eccentricity. Zaharescu also observed similar phenomenon when he photographed the wedges below the footings and observed that the angle of the wedge on the side of the eccentricity remains almost constant while the other angle reduces with increased eccentricity.

A total of sixty two dimensional tests were conducted with four footing sizes for central loads and eccentric loads with four eccentricity ratios (e/B) as mentioned earlier and three surcharge ratios (D_f/B) of 0.0, 0.25 and 0.5, where D_f is the depth of footing.

Typical load *vs* settlement curves are shown in Figure 6. Details of the tests and results have been reported by Purkayastha (1974).

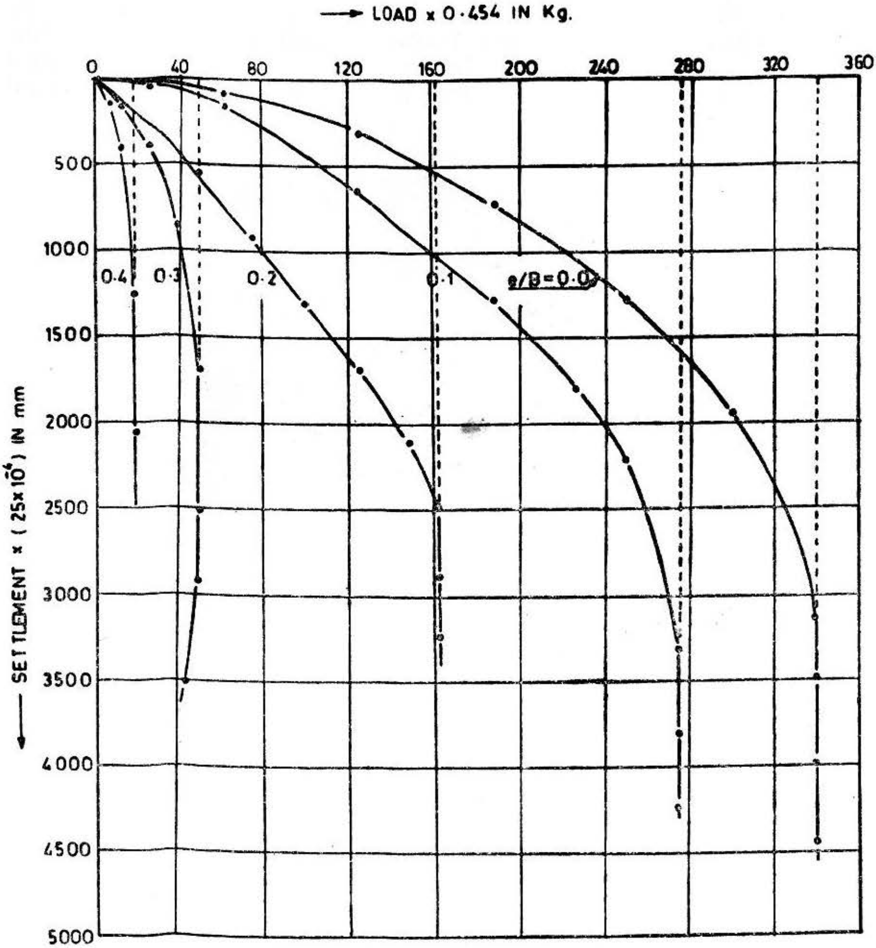

FIGURE 6 Typical load-settlement curves for two dimensional tests footing—10$\times$10 cm; $D_f/B = 0$

Three Dimensional Tests

Three dimensional tests were carried out in a tank of $90\times80\times100$ cm size, made of aluminium alloy sheets stiffened at the base, top and corners by iron angles. Load was applied by a hydraulic jack and recorded through a proving ring. Four sizes of footings 10×10 cm, 12.5×10 cm, 15×10 cm and 20×10 cm were selected. A hopper of size 25×15 cm and 8 cm wide was used for preparing the test bed by pouring Ennore sand from a height of 30 cm. The experimental set up for three dimensional tests is shown in Figure 7. The density of the bed was 1.60 t/m^3 and the angle of internal friction was 37.5°. The relative deosity was 46 per cent.

A total of 20 tests were conducted with centrally and eccentrically loaded surface footings for the four footing sizes, the eccentricity ratios being the same as in two dimensional tests.

The failure was observed to be always one sided and was marked

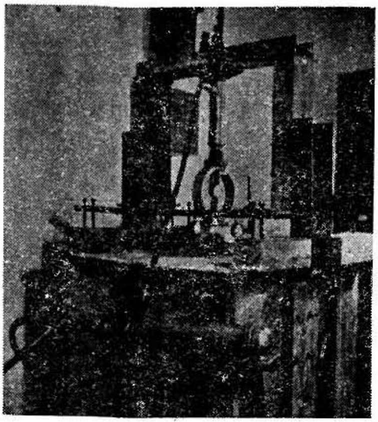

FIGURE 7 Experimental set-up for three dimensional tests

the heaving of sand on the side of eccentricity. Figure 8 shows typical curves of three dimensional tests for one footing size.

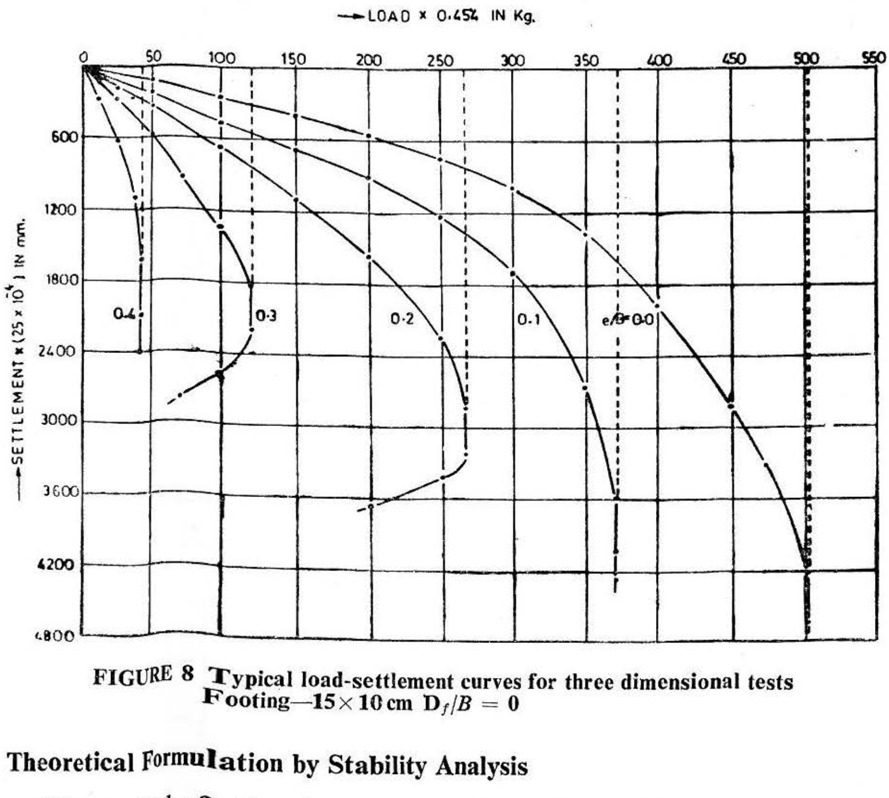

FIGURE 8 Typical load-settlement curves for three dimensional tests
Footing—15 × 10 cm $D_f/B = 0$

Theoretical Formulation by Stability Analysis

The method of slices for analyzing the stability of slip surfaces as used by Janbu (1957) for the calculation of bearing capacity under centrally loaded footing, has been extended to the case of eccentrically loaded footings. The following relationship was used to determine the factor of safety F.

$$F = \frac{\Sigma\, \tau_f \cos^{-2}\alpha\, dx}{\Sigma\, (P+t) \tan \alpha\, dx}$$

Where,

$$\tau_f = \text{shearing resistance} = \frac{(p+t) \tan \phi}{1 + \dfrac{\tan \alpha \tan \phi}{F}}$$

p = pressure on the slice

t = unit side friction on the slice

α = angle made by a slice with the horizontal

For an assumed applied load and eccentricity ratio, the maximum and minimum pressures below the eccentrically loaded rigid footing can be determined. Figure 9 shows the pressure distribution and slip surfaces with eccentricities within the middle third. Details of the stability analysis have been reported elsewhere (Purkayastha, 1974; Purkayastha and Char, 1976; Purkayastha and Char, 1977).

By changing the wedge angle β, a set of slip surfaces are generated and each one of them is analyzed for determining the minimum factor of safety corresponding to the critical repture surface. The procedure was programmed and solved by IBM 370 computer. Figure 10 shows the

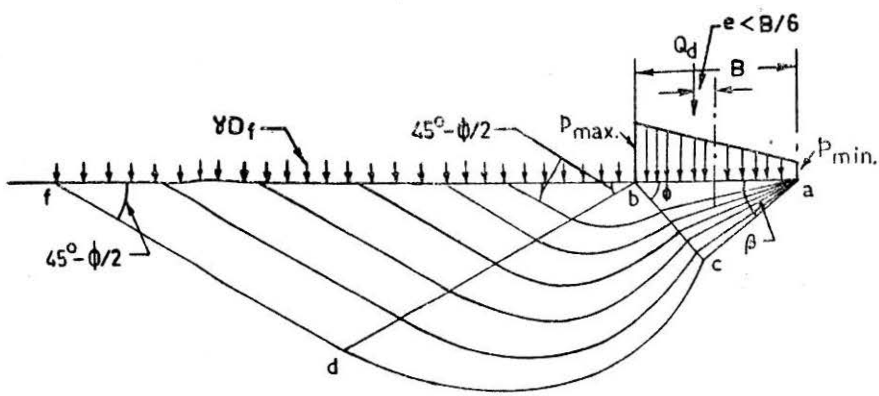

FIGURE 9 Slip surfaces with eccentricity within the middle third

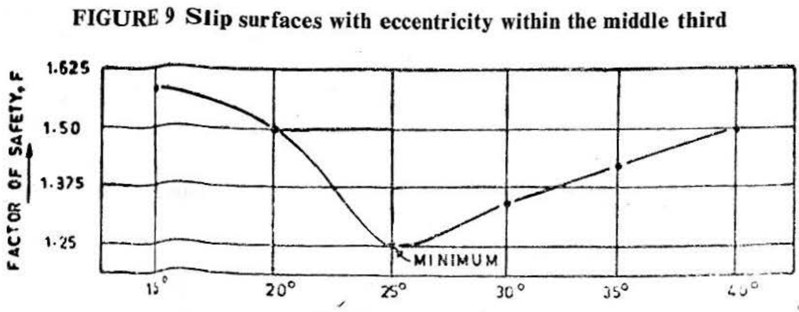

FIGURE 10 Variation of factor of safety with wedge angle Footing—10×10 cm $D_f/B = 0$, $e/B = 0.2$ $\varphi = 38°$

variation of factor of safety with wedge angle β. Analysis was carried out for different sizes of model and prototype footings with varying eccentricity ratios, angles of friction and surcharge ratios.

Results and Discussion

The experimental failure surfaces are found to correspond very closely with the theoretical failure surfaces for the minimum factors of safety as indicated in Figure 11. The theoretical failure surface is drawn for the corresponding plane strain value of the angle of internal friction. It can be seen that the experimental failure surface is slightly smaller in extent than the theoretical surface. This may partly be be due to the friction between the sand and the perspex wall (David and Woodward 1949).

It has been reported in the literature that the plane strain angle of internal friction is higher than the angle as obtained from triaxial tests (Cornforth 1964). From the relationship suggested by Meyerhof (1963), the plane strain value of ϕ is about 1.1 times the triaxial value. With the correction for plane strain, the theoretical ultimate loads agree well with the experimental values.

Figures 12, 13 and 14 show the comparison of theoretical values with

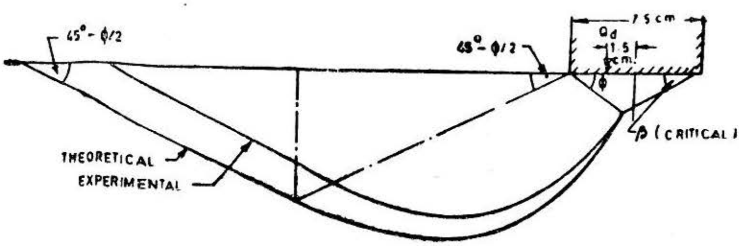

FIGURE 11 Comparison of theoretical and experimental failure surfaces

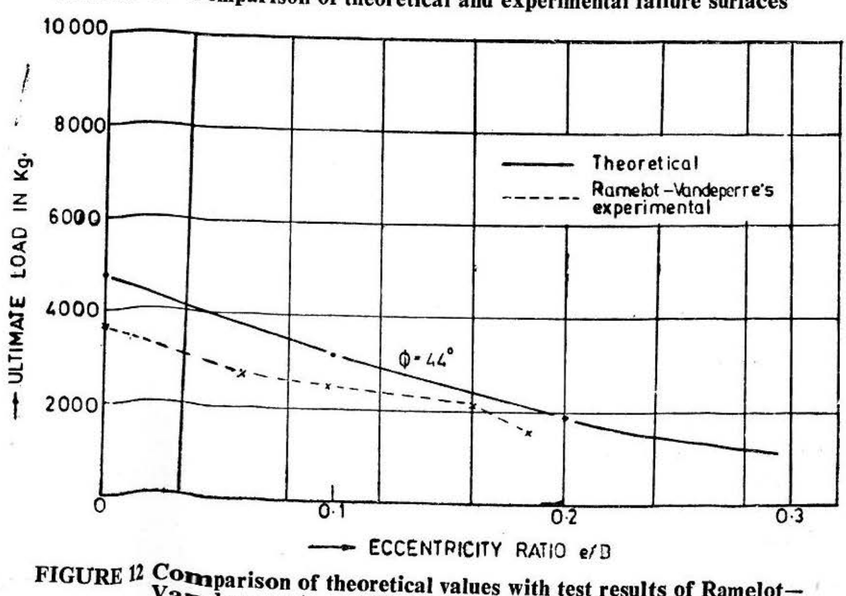

FIGURE 12 Comparison of theoretical values with test results of Ramelot–Van deperre ($\varphi = 44°$)—Footing—30×30 cm

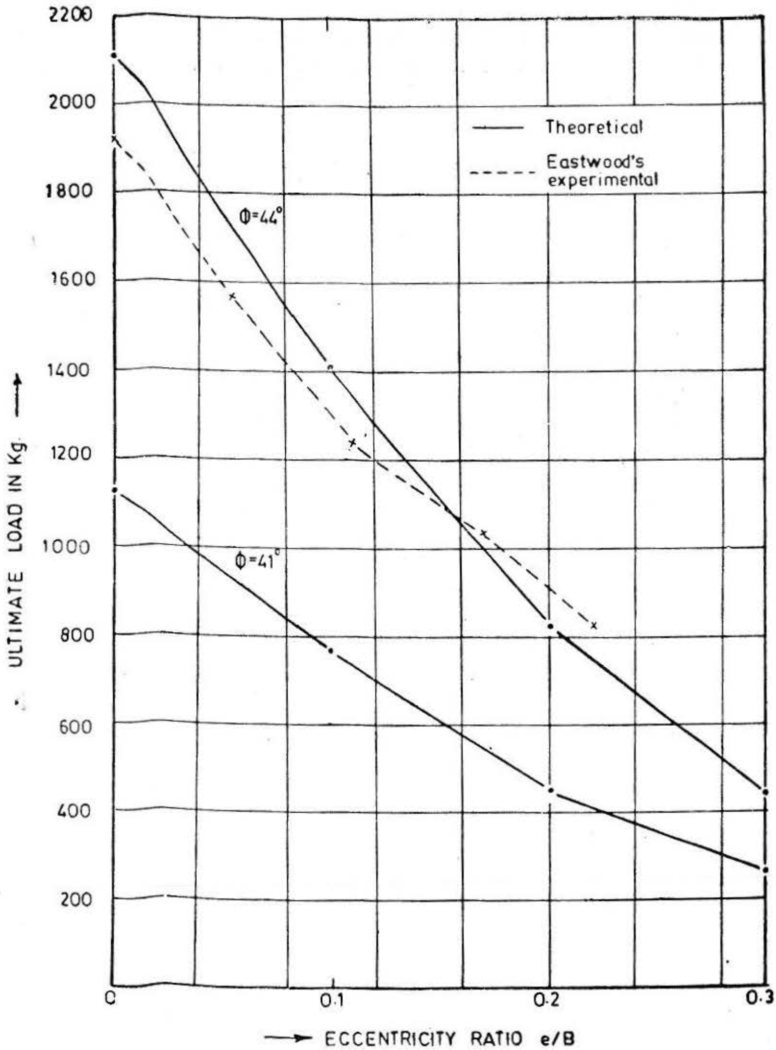

FIGURE 13 Comparison of theoretical values with test results of Eastwood (φ = 43°) Footing—45×15 cm

the test values of Ramelot—Vandeperre, Eastwood and Prakash-Saran. It is seen that the theoretical results agree well with the experimental values of eariier investigators.

Apart from the model footings, the theoretical bearing capacities for a wide range of centrally and eccentrically loaded prototype footing sizes are calculated. The valve of B for prototype sizes are varied from 0.5 to 6.0 m at an interval of 0.5 m. The bearing capacities are calculated for different values of ϕ varying from 29° to 44° with an interval of 3° and for different surcharge ratios (D_f/B) varying from 0 to 1.0 with an interval of 0.25. The theoretical reduction factors are calculated for all these

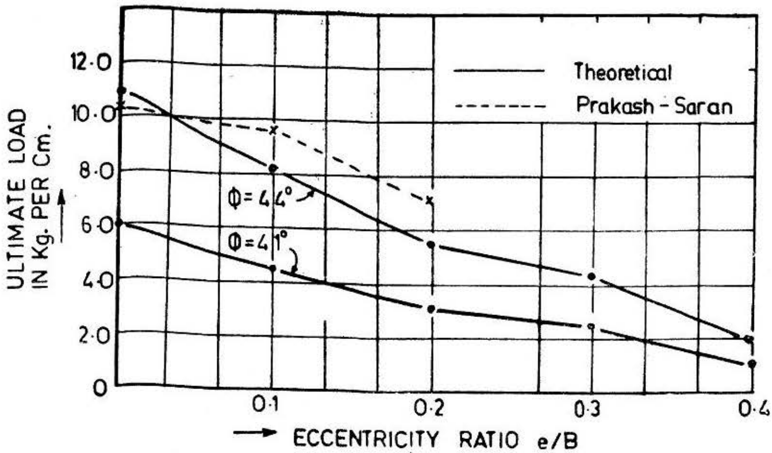

FIGURE 14 Comparison of theoretical values with test results of Prakash—Saran ($\varphi_p = 46.2°$) Strip footing, $B = 5$ cm, $D_f/B = 1$

cases. The reduction factor R_K is defined as

$$R_K = 1 - \frac{\text{eccentric ultimate load}}{\text{centric ultimate load}}$$

The centric ultimate load corresponds to the theoretical value for $e/B = 0$ and the eccentric ultimate load for any e/B ratio is known from the theoretical results. A statistical analysis of all the reduction factors indicates ihat the reduction factor varies with e/B and D_f/B. The width of footing and the angle of internal friction have no significant influence on the reduction factor. Using the reduction factors of all the model and prototype footings, exponential curves for reduction factor versus e/B for different D_f/B values are fitted by the method of least squares according to the following equation

$$R_K = a\,(e/B)^k$$

The fitted curves for different D_f/B values are shown in Figure 15. The values of the constants a and K have been given elsewhere (Purkayastha and Char 1977).

Suggested Method for Prototype Sizes of Footings

From the theoretical results of all the prototype sizes of footings, it is observed that the theoretical centric load agrees very well with the centric load as calculated by Terzaghi's formula. Table 1 shows the comparison of theoretical centric load with Terzaghi's centric load for a typical case.

So, for estimating the bearing capacity of prototype footing, the centric load can be calculated first by Terzaghi's formula and then taking the reduction factor R_K from Figure 15 for the desired eccentricity and surcharge ratios, the required ultimate load can be calculated from the following relation

$$Q_{eccentric} = Q_{Terzaghi}\,(1 - R_K)$$

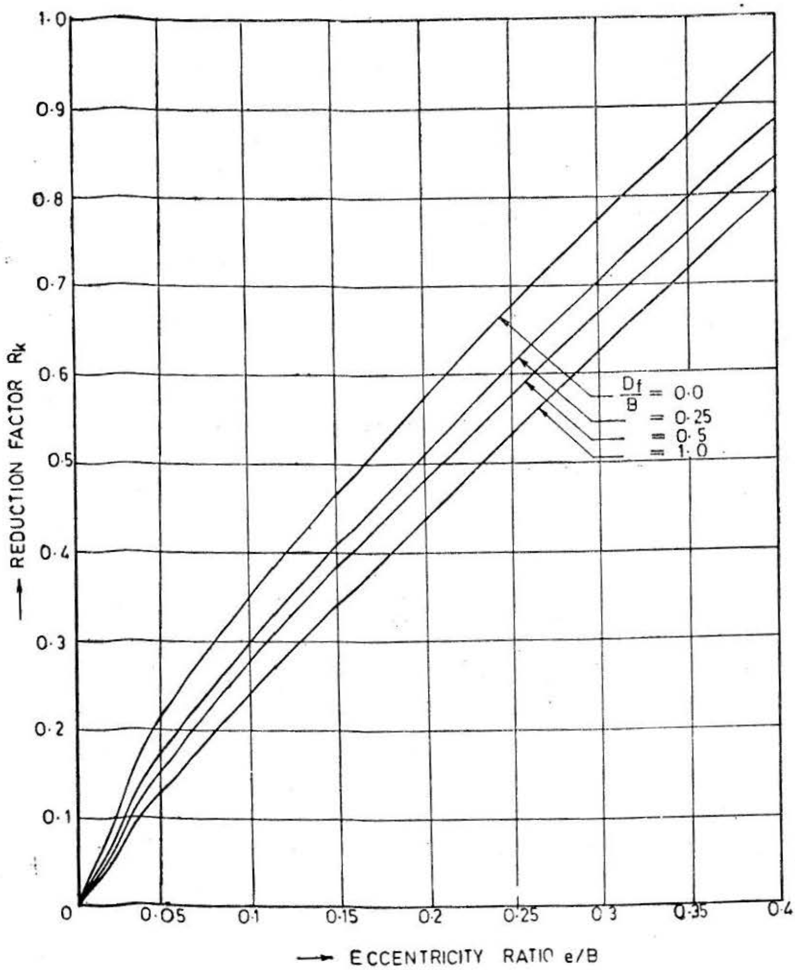

FIGURE 15 Fitted curves for reduction factors

TABLE 1

Comparison of Terzaghi's centric load and theoretical values as obtained by stability analysis

$B = 0.5$ m $D_f/B = 0$

φ (degrees)	Theoretical centric load (Tonne/metre)	Terzaghi's centric load (Tonne/metre)
29	3.42	3.845
32	5.32	5.908
35	9.20	9.54
38	16.38	16.997
41	27.60	28.073
44	52.58	53.88

A comparison of the ultimate load *versus* e/B curves drawn by this suggested method and by the stability analysis is shown in Figure 16. It is evident that the results obtained by the suggested method are very close to those obtained from the stability analysis.

The bearing capacity of any size of eccentrically loaded prototype footing for a specified value of ϕ and D_f/B can easily be obtained by this suggested method. The ultimate load versus e/B plot according to the suggested method for a prototype footing size with varying ϕ and D_f/B are shown in Figure 17 which shows the effect of surcharge on the ultimate bearing capacity.

Till now, only two theories by Meyerhof and Prakash-Saran are available for calculating the bearing capacity of eccentrically loaded footings. Table 2 shows a comparison of the author's suggested method with the two available theories. Whereas a better agreement of the suggested method is observed with the theory of Prakash and Saran, Meyerhof's values appear to be slightly on the conservative side.

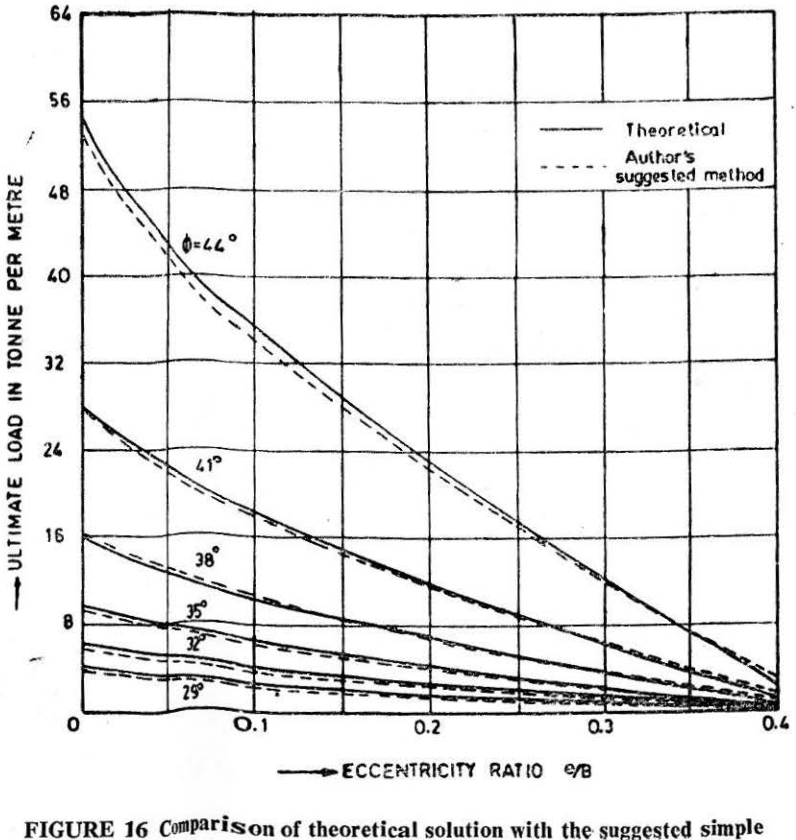

FIGURE 16 Comparison of theoretical solution with the suggested simple method $B = 0.5$ m, $D_f/B = 0$

TABLE 2

Comparison of the author's suggested method with the theories of Meyerhof and Prakash-Saran

	$B = 0.5$ M	$D_f/B = 0.0$	$\phi = 38°$	$\gamma = 1.8$ t/m^3
e/B	Meyerhof		Prakash—Saran	Author
0.1	9.36		10.8	10.4
0.2	5.26		6.3	6.88
0.3	2.34		3.38	3.68
0.4	0.585		0.9	0.64

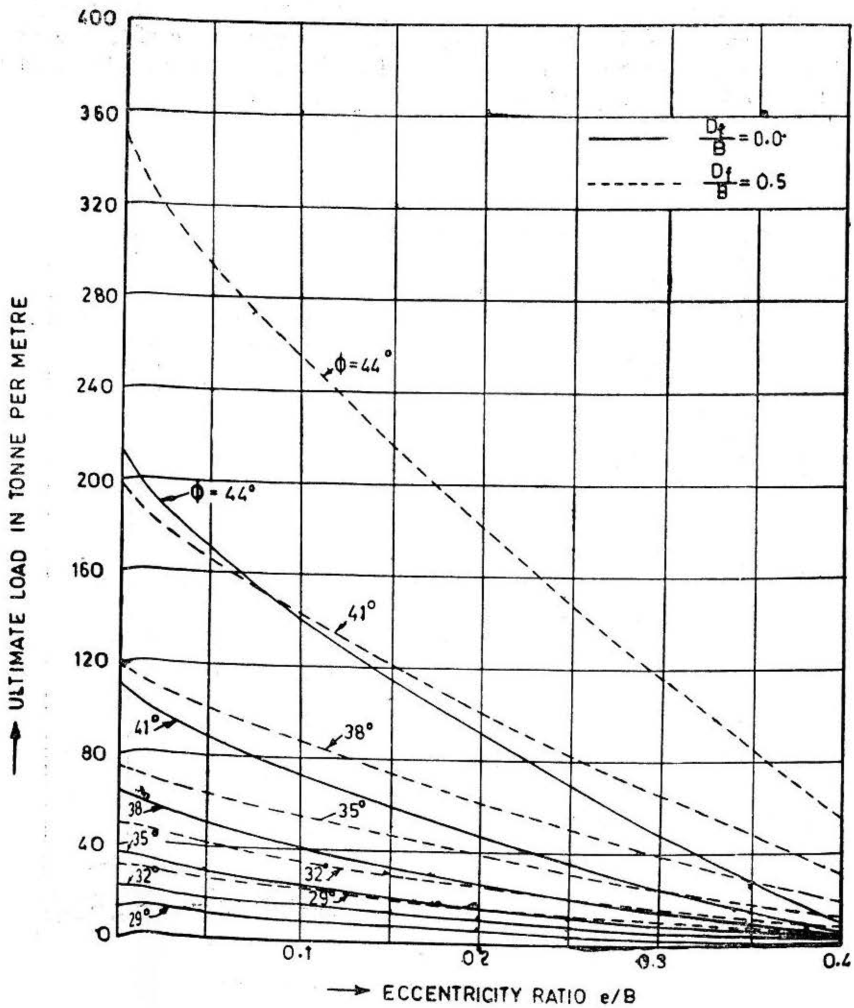

FIGURE 17 **Ultimate load *versus* e/B plot for a prototype footing size by the suggested method—$B = 1.0$ m**

Conclusions

The failure of eccentrically loaded footings resting on sand is a distinct phenomenon and for unrestrained footings, the failure surface always develops on the side of the eccentricity of load. The extents of the experimental failure surfaces as observed at the perspex walled side of the test box are always slightly smaller than those of the theoretical surfaces due to the friction between the sand and the perspex walls. The method of calculating the bearing capacity of foundations by analyzing the stability of slip surfaces which has so far been in use for centrally loaded footings can be extended to the case of eccentrically loaded footings too. A good agreement between the theoretical bearing capacities of eccentrically loaded footings as obtained by stability analysis and the experimental two dimensional and three dimensional ultimate loads is observed. The theoretical bearing capacities of eccentrically loaded footings agree well with the experimental values of earlier investigators. The theoretical bearing capacities for centrally loaded footings on sand agree well with the corresponding values of Terzaghi. A simple method based on stability analysis has been suggested for calculating the bearing capacity of eccentrically loaded prototype footings.

Acknowledgement

The work described in the paper was carried out while the author held a research fellowship at I.I.T. Kharagpur. The author is grateful to Dr. A.N.R. Char, Asst. Professor, Civil Engineering Department, I.I.T., Kharagpur for his guidance during the work. The author also likes to express his gratitude to the authorities of I.I.T. Kharagpur for the facilities provided.

References

CORNFORTH, D.H., (1964) "Some Experiments on the Influence of Strain Conditions on the Strength of Sand", *Geotechnique*, Vol. 14, No. 2, p.-143.

DAVIS, H.E. and WOODWARD, R.J., (1949) "Some Laboratory Studies of Factors Pertaining to the Bearing Capacity of Soils," *Proceedings, Highway Research Board*, Vol. 29, p. 467.

DHILLON, G.S., (1961) "The Settlement, Tilt and Bearing Capacity of Footings on Sand Under Central and Eccentric Loads." *Journal of National Building Organisation*, New Delhi, Vol. 6, No. 1 p. 66.

EASTWOOD. W., (1955) "The Bearing Capacity of Eccentrically Loaded Foundations on Sandy Soils," *The Structural Engineer*, Vol. 33, No. 6, June, p. 181.

JANBU, N., (1957) "Earth Pressures and Bearing Capacity Calculations by Generalised Procedure of Slices," *Proceeding, Fourth International Conference on Soil Mechanics and Foundation Engineering*, Vol. 2, p. 207.

JUMIKIS, A.R., (1956) "Rupture Surfaces in Sand Under Oblique Loads." *Journal of the Soil Mechanics and Foundations Division, A.S.C.E.*, Vol. 82, No. SMI, p. 1.

LEE, I.K. (1965) "Foundations Subject to Moment," *Proceedings, Sixth International Conference on Soil Mechanics and Foundation Engineering*, Vol. 2, p. 181.

MEYERHOF, G.G., (1953) "The Bearing Capacity of Footings Under Eccentric and Inclined Loads, "*Proceedings, Third International Conference on Soil, Mechanics and Foundation Engineering*, Vol. 1, p. 440.

MEYERHOF, G.G., (1963) "Some Recent Rresearch on the Bearing Capacity of Foundations," *Canadian Geotechnical Journal*, Vol. 1, No. 1, p. 16.

PRAKASH, S. and SARAN, S., (1971) "Bearing Capacity of Eccentrically Loaded Footings," *Journal of the Soil Mechanics and Foundations Division, A.S.C.E.*, Vol. 97, No. SMI, p. 95.

RAMELOT, C. and VANDEPERRE, L., (1950) "Travaux de la commission d'Etude des fondations de pylones," *Compt. Rend Rech.*, I.R.S.I.A , Brussels, No. 2.

PURKAYASTHA, R.D., (1974) "A Stability Analysis for Eccentrically Loaded Footings Resting on Sand," *Ph.D. Thesis submitted at I I.T. Karagpur.*

PURKAYASTHA, R.D. and CHAR, A.N.R., (1977) "Stability Analysis for Eccentrically Loaded Footings," *Journal of the Geotechnical Enginnering Division, A.S.C.E.*, Vol. 103, No. GT6 p.647.

PURKAYASTHA, R.D. and CHAR, A.N.R., (1976) "Stability of Eccentrically Loaded Footings," *Proceedings of the Symposium on Foundations and Excavations in Weak Soil*, Calcutta, Vol. 1, No. C-2, p.11.

ZAHARESCU, E., (1961) "The Eccentricity Sense Influence of Inclined Loads on the Bearing Capacity of Rigid Foundations," *Journal of National Buildings Organisation*, New, Delhi, Vol. 6, No. 2, p. 283.