Shrinkage Stresses in Soil—Cement Base Courses

By

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Introduction

Highway pavements in which soil—cement or similar material is used are susceptible to the detrimental effect of cracking in their early life, before they attain full strength and are opened to traffic. The reflection of these cracks in the upper layers further aggravates the problem (Dunlop *et al*).

The stresses and deformations in pavements caused by superimposed loads and ambient temperature have been studied extensively (Bradbury 1938, Haar and Leonards 1959, Kelley 1939, Teller and Sutherland 1935, Thomlinson 1940, Westergaard 1926, 1927).

The stresses caused by the changes in moisture content or drying shrinkage which is essentially differential in nature are still being investigated. Theoretical expressions for elastic deformations and accompanying shrinkage stresses that occur in concrete beams and slabs during course of drying have been derived by Pickett (1946). Sanan and George (1972) have analysed slabs for shrinkage stresses assuming them as thin plates supported on Winkler foundation giving elaborate expressions.

In the present study, shrinkage stresses have been worked out using the Westergaard's approach because of its simplicity, replacing the temperature strains by the analogus shrinkage strains. Thomlinson's visualization of temperature stresses has also been modified using the same approach. Soil-cement layers dry from one face (top surface) only, consequently the top surface dries more than the bottom surface. The resulting shrinkage is thus not only nonuniform but also nonlinear through the depth of the layer. The expressions have been derived using various assumed shrinkage variations through the depth.

Partial subgrade restraint has been taken into account (using a constant restraining force) and a linear relationship has been concluded between the stresses induced by no base restraint and those induced by complete base restraint.

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Shrinkage Stresses (Elastic Analysis)

Westergaard's approach

Westergaard's approach which has been extended further by Thomlinson, has been adopted here because of its simplicity and as a first approximation.

The shrinkage stresses attain their maximum value at the centre of the pannels (Saman and George 1972), the analysis is thus restricted to the determination of shrinkage stresses in the middle part of the slabs.

Infinite slab (uniform shrinkage variation)

The shrinkage stresses in the central area of a large slabe are given by the expression.

$$\sigma_x = \sigma_y = \frac{sE}{1-\mu} \qquad \ldots(1)$$

where σ_x σ_y are normal tensile stresses at all points through the depth of the slab, E—modulus of elasticity and s—shrinkage strain (assumed uniform).

Infinite slab (linear variation through depth)

The shrinkage stresses are given by

$$\sigma_o = \frac{sE}{2(1-\mu)} \qquad \ldots(2)$$

where σ_o = stress at the top (same in all horizontal directions)

s = shrinkage strain (difference of shrinkage strains between top and bottom)

Long slab of relatively narrow width

For slabs of this nature the deflction is given by the expression

$$Z = -Z_o \frac{2\cos\lambda \cosh\lambda}{\sin 2\lambda + \sinh 2\lambda} \left(-\tan\lambda + \tanh\lambda\right) \cos\frac{Y}{l\sqrt{2}} \cosh\frac{Y}{l\sqrt{2}}$$

$$+ \left(\tan\lambda + \tanh\lambda\right) \sin\frac{Y}{l\sqrt{2}} \sinh\frac{Y}{l\sqrt{2}} \qquad \ldots(3)$$

where $\lambda = \dfrac{b}{l\sqrt{B}}$

Z = deflection (downwards)

$$Z_0 = \frac{(1+\mu)s}{H} l^2 = s\sqrt{\frac{(1+\mu)EH}{12(1-\mu)k}}$$

B = slab width

$l = \sqrt[4]{\dfrac{EH^3}{12(1-\mu^2)k}}$ = radius of relative stiffness of the slab.

k = modulus of subgrade reaction

H = slab depth

Corresponding principal stress (maximum stress) at the top of the slab is given by

$$\sigma_y=\sigma_o\left[1-\frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda+\sinh 2\lambda}\left((\tan\lambda+\tanh\lambda)\cos\frac{y}{l\sqrt{2}}\cosh\frac{y}{l\sqrt{2}}\right.\right.$$

$$\left.\left.+(\tan\lambda-\tanh\lambda)\sin\frac{y}{l\sqrt{2}}\sinh\frac{y}{l\sqrt{2}}\right)\right] \quad \ldots(4)$$

$$\sigma_x=\sigma_o+\mu(\sigma_y-\sigma_o) \quad \ldots(5)$$

$$\text{and} \quad \sigma_o=\frac{Es}{2(1-\mu)}$$

Finite slab

In slabs of finite dimensions, the deflection is obtained by super imposing the deflections in x and y directions *i. e.*

$$Z=Z_1(y)+Z_2(x)$$

where Z_1 — deflection as in Equation 3

Z_2 — deflection obtained by replacing y by x and $\lambda=\frac{B}{l\sqrt{8}}$

B — width of the slab.

Principal stress at the top are given by

$$\sigma_x=\sigma_o(1+\mu-F_x-\mu F_y) \quad \ldots(6)$$

$$\sigma_y=\sigma_o(1+\mu-\mu F_x-F_y) \quad \ldots(7)$$

$$\text{where} \quad F_x=\frac{2\cos\varphi\cosh\varphi}{\sin 2\varphi+\sinh 2\varphi}\left((\tan\varphi+\tanh\varphi)\cos\frac{X}{l\sqrt{2}}\cosh\frac{X}{l\sqrt{2}}\right.$$

$$\left.+(\tan\varphi-\tanh\varphi)\sin\frac{X}{l\sqrt{2}}\sinh\frac{X}{l\sqrt{2}}\right)$$

$$\text{and} \quad F_y=\frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda+\sinh 2\lambda}\left\{(\tan\lambda+\tanh\lambda)\cos\frac{Y}{l\sqrt{2}}\cosh\frac{Y}{l\sqrt{2}}\right.$$

$$\left.+(\tan\lambda-\tanh\lambda)\sin\frac{Y}{l\sqrt{2}}\sinh\frac{Y}{l\sqrt{2}}\right\}$$

$$\varphi=\frac{L}{l\sqrt{8}},\ \lambda=\frac{B}{l\sqrt{8}} \text{ and } \sigma_0=\frac{Es}{2(1-\mu)}$$

Generalized shrinkage stress and strains (Elastic)

Thomlinson's modification

Thomlinson (1940) taking a temperature gradient through the depth of the slabs, has arrived at what he calls internal stresses

$$f_{\theta, x} = f_{\theta, y} = -\frac{E\,\alpha\,\theta}{1-\mu} + \frac{}{H(1-\mu)}\int_0^H E\,\alpha\,\theta\,dz + \frac{12\left(\frac{H}{2}-Z\right)}{H^3(1-\mu)}$$

$$\times \int_0^H E\,\alpha\,\theta\left(\frac{H}{2} - Z\right)dz \qquad \ldots(8)$$

The first term on the right hand side gives the stress due to the distributed end force, whilst the second and third terms give the stress due to the concentrated end force, and α, θ are thermal expansion coefficient and temperature variation respectively. Shrinkage stress are evaluated by replacing the temperature strains by the analogous shrinkage strains.

Shrinkage stresses and strains (no base restraint)

Equation 8 for infinite slab can be modified for shrinkage stresses as or directly from theory of elasticiiy, Figure 1*a*. Strains are taken as positive in compression.

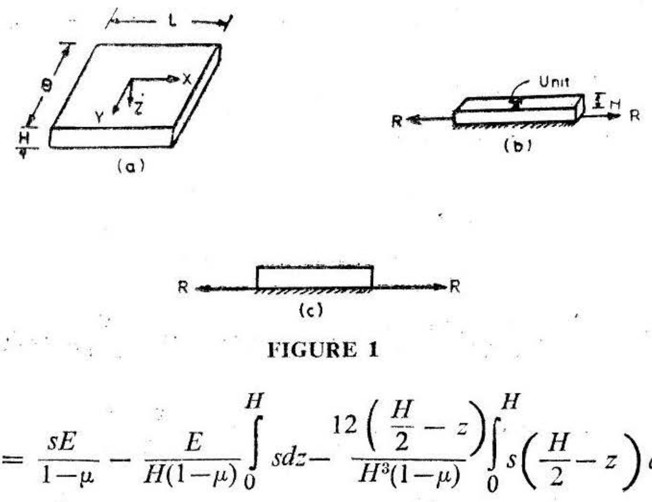

FIGURE 1

$$\sigma x = \sigma y = \frac{sE}{1-\mu} - \frac{E}{H(1-\mu)}\int_0^H s\,dz - \frac{12\left(\frac{H}{2}-z\right)}{H^3(1-\mu)}\int_0^H s\left(\frac{H}{2}-z\right)dz$$

$$= \frac{E}{1-\mu}\left\{ s - \frac{1}{H}\int_0^H s\,dz - \frac{12}{H^3}\left(\frac{H}{2} - z\right)\frac{H}{2}\int_0^H s\,dz + \right.$$

$$\left.\frac{12}{H^3}\left(\frac{H}{2} - z\right)\int_0^H s\,z\,dz\right\}$$

$$= \frac{E}{1-\mu}\left\{s - \frac{1}{H}F - \frac{12}{H^3}\,\frac{H}{2}\,\frac{H}{2}F + \frac{12}{H^3}Z\frac{H}{2}F\right.$$

$$\left. + \frac{12}{H^3}\,\frac{H}{2}M - \frac{12}{H^3}Z\,M\right\} \qquad ...(9)$$

where $\int_0^H s\,dz = F$ and $\int_0^H s\,z\,dz = M$

Thus $\sigma_x = \sigma_y = \frac{E}{1-\mu}\left\{s + \frac{F}{H}\left(6\frac{Z}{H} - 4\right) + \frac{M}{H^2}\left(6 - 12\frac{Z}{H}\right)\right\}$ ...(10)

and $\epsilon_x = \epsilon_y$ (strains) $= \frac{\sigma_x - \mu\sigma_y}{E} - s$

$$= \frac{F}{H}\left(6\frac{Z}{H} - 4\right) + \frac{M}{H^2}\left(6 - 12\frac{Z}{H}\right) \qquad ...(11)$$

at $Z = 0$

$$\epsilon_x = \epsilon_y = -\frac{4F}{H} + \frac{6M}{H^2} \qquad ...(12)$$

at $Z = H$

$$\epsilon_x = \epsilon_y = \frac{2F}{H} - \frac{6M}{H^2} \qquad ...(13)$$

Shrinkage stresses and strains (full base restraint)

As full base restraint is assumed $\epsilon_x = \epsilon_y = 0$ (at bottom). In order to neutralize ϵ_x or ϵ_y some force must be applied through the subgrade. Let that force be R (a constant force)—which actually has to be a central force R and a moment $RH/2$ (Figure 1*b*).

The stress thus will be $4R/H$ and $\therefore$ strain $= \frac{4R}{HE}(1-\mu)$ and this strain is $\frac{4(1-\mu)R}{EH} = -\left\{\frac{2F}{H} - \frac{6F}{H^2}\right\}$ negative sign for opposing nature, equated to equation 13.

or $R = \frac{E}{2(1-\mu)}\left(-F + \frac{3M}{H}\right)$. This force will produce stress at any Z, which wiil be $\frac{R}{H} + R\frac{H}{2}\left(Z - \frac{H}{2}\right)\frac{12}{H^3}$

substituting the value of $R = R\left(-\frac{2}{H} + \frac{6Z}{H^2}\right)$

Stress introduced $= \frac{E}{H(1-\mu)}\left(-F + \frac{3M}{H}\right)\left(-1 + 3\frac{Z}{H}\right)$ and is tensile in nature

$$\therefore \sigma_x = \sigma_y = \frac{E}{1-\mu}\left\{s + \frac{F}{H}\left(6\frac{Z}{H} - 4\right) + \frac{M}{H^2}\left(6 - 12\frac{Z}{H}\right)\right.$$

$$\left. + \frac{F}{H}\left(1 - \frac{3Z}{H}\right) - \frac{3M}{H^2}\left(1 - \frac{3Z}{H}\right)\right\}$$

$$= \frac{E}{1-\mu}\left\{s+\frac{F}{H}\left(3\frac{Z}{H}-3\right)+\frac{M}{H^2}\left(3-\frac{3Z}{H}\right)\right\} \quad \ldots(14)$$

and
$$\epsilon_x = \epsilon_y = \frac{F}{H}\left(3\frac{Z}{M}-3\right)+\frac{M}{H^2}\left(3-\frac{3Z}{H}\right) \quad \ldots(15)$$

The stresses and strains obtained under the conditions of no base restraint and full base-restraint can be tabulated for comparison (Table 1).

Shrinkage variation through the depth of the slab

Westergaard (1927) assumed that the temperature gradient in a slab is constant throughout its thickness. This however, is occasionally true and depends on the material and the environment. A functional form can be chosen to represent the shrinkage variation through depth.

Some usual functional forms are assumed and the appropriate stress and strain expressions are deduced (Table 2). Any one or more function forms can be combined to fit the field shrinkage condition to evaluate the shrinkage stresses and strains.

Shrinkage stresses using Thomlinson's approach (partial base restraint)

The free stresses as visualized by Thomlinson are given by

$$\sigma_x = \sigma_y = \frac{E}{1-\mu}\left\{s+\frac{F}{H}\left(6\frac{Z}{H}-4\right)+\frac{M}{H^2}\left(6-12\frac{Z}{H}\right)\right\}$$

and
$$\epsilon_x = \epsilon_y = \frac{F}{H}\left(6\frac{Z}{H}-4\right)+\frac{M}{H^2}\left(6-12\frac{Z}{H}\right)$$

Introducing base-restraint

Let a force R/unit length act at the bottom on both sides (Figure 1*c*).

The stresses and strains resulting from this partial sub-grade restraining force are

$$\sigma_{x'} = \sigma_{y'} = \frac{R}{H}\left(6\frac{Z}{H}-2\right)$$

and
$$\epsilon_{x'} = \epsilon_{y'} = \frac{1-\mu}{E}\frac{R}{H}\left(6\frac{Z}{H}-2\right)$$

Total stresses and strains

The total stresses and strains are obtained by combining the free stresses and the stresses contributed by the partial sub-grade restraint *i e.*

$$\sigma_x^t = \sigma_y^t = \frac{E}{1-\mu}\left\{s+\frac{F}{H}\left(6\frac{Z}{H}-4\right)+\frac{H}{H^2}\left(6-12\frac{Z}{H}\right)\right\}$$
$$+\frac{R}{H}\left(6\frac{Z}{H}-2\right) \quad \ldots(16)$$

and
$$\epsilon_x^t = \epsilon_y^t = \frac{F}{H}\left(6\frac{Z}{H}-4\right)+\frac{M}{H^2}\left(6-12\frac{Z}{H}\right)$$
$$+\frac{1-\mu}{E}\frac{R}{H}\left(5\frac{Z}{H}-2\right) \quad \ldots(17)$$

Full base restraint

The partial base restraining force R can increase to a value R_{max} at full base restraint

$$\text{When } \epsilon_x^t = \epsilon_y^t = 0 \qquad \text{at } Z = H$$

These conditions yield

$$R_{max} = \frac{EH}{2(1-\mu)}\left(\frac{3M}{H^2} - \frac{F}{H}\right)$$

Partial base restraint

Introducing a factor K_r, which has a value, zero for no base restraint and unity for full restraint and it may have intermediate values as well.

Taking $R = K_r R_{max}$ and substituting in Equations 16 and 17 we get

$$\sigma_x^m = \sigma_y^m = \frac{E}{1-\mu}\left[s + \frac{F}{H}\left\{(6-3K_r)\frac{Z}{H} - (4-K_r)\right\} + \frac{M}{H^2}\left\{(6-3K_r)-(12-9K_r)\frac{Z}{H}\right\}\right] \quad ...(18)$$

and

$$\epsilon_x^m = \epsilon_y^m = \frac{F}{H}\left[(6-3K_r)\frac{Z}{H} - (4-K_r)\right] + \frac{M}{H^2}\left[(6-3K_r) - (12-9K_r)\frac{Z}{H}\right] \quad ...(19)$$

Special cases

$$\epsilon_x \text{ (bottom) for } K_r = 0 \quad = \frac{2F}{H} - \frac{6M}{H^2}$$

$$\epsilon_x \text{ (bottom) for } K_r = 1 \quad = 0$$

$$\epsilon_x \text{ (bottom) for } K_r = 0.75 = \frac{F}{2H} - \frac{3M}{2H^2}$$

$$\epsilon_x \text{ (bottom) for } K_r = 0.50 = \frac{F}{H} - \frac{3M}{H^2}$$

$$\epsilon_x \text{ (bottom) for } K_r = 0.25 = \frac{3F}{2H} - \frac{9M}{2M^2}$$

Giving $$\epsilon_{max} = \frac{2F}{M} - \frac{6M}{H^2} \text{ and } \epsilon_{min} = 0$$

Strain release factor

Let us define a factor $F_r = \frac{\epsilon_{max} - \epsilon_x}{\epsilon_{max}}$ thus the strain release factor. On evaluating F_r for various values of K_r it is found that they are equal, indicating thereby that the relationship between the stresses and strains for no base restraint and full base restraint is linear, and that stresses and strains forcing partial base restraint can be linearly interpolated.

Evaluation of shrinkage stresses

Material properties

For numerical analysis material properties for soil-cement are taken as recommended by George (1970) pertaining to a similar study:

$$E = 4650 \text{ kg/cm}^2, \ \mu = 0.3 \text{ and } K = 5.55 \text{ kg/cm}^3$$

Panel dimensions

Standard panel dimensions as give by Bradbury (1938) are taken for calculation purposes.

length = 10 m breadth = 6 m thickness = 20 cm

Shrinkage variation

The variation of shrinkage through the depth of the slab has been taken from Haroon (1971) and is as per Figure 2 (50 hour shrinkage).

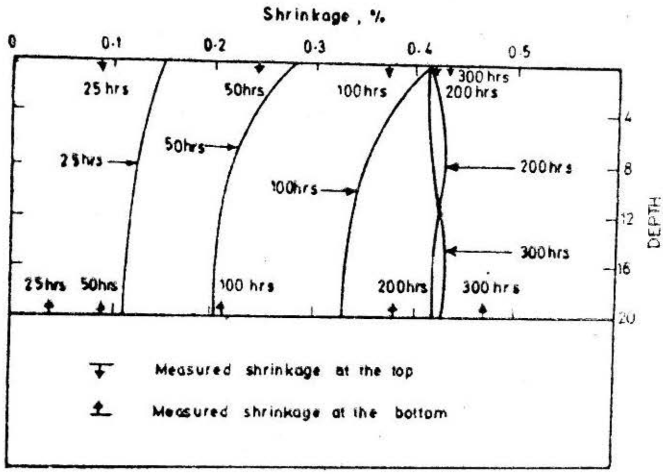

FIGURE 2

Shrinkage stresses

Using the above data shrinkage stresses are computed by Westergaard's approache and internal stresses are worked out as visualized by Thomlinson They are tabulated in Table 3. Sample calculations are presented in Appendix I.

TABLE 1 : Stresses and strains

	No base restraint	*Full base restraint*
Stress at any depth Z $\sigma_x = \sigma_y$	$\frac{E}{1-\mu}\left\{ s + \frac{F}{H}\left(6\frac{Z}{H}-4\right) + \frac{M}{H^2}\left(6-12\frac{Z}{H}\right)\right\}$	$\frac{E}{1-\mu}\left[s + \frac{F}{H}\left(3\frac{Z}{H}-3\right) + \frac{M}{H^2}\left(3-\frac{3Z}{H}\right)\right]$
Strain at any depth Z $\epsilon_x = \epsilon_y$	$\frac{F}{H}\left(6\frac{Z}{H}-4\right)+\frac{M}{H^2}\left(6-12\frac{Z}{H}\right)$	$\frac{F}{H}\left(\frac{3Z}{H}-3\right)+\frac{M}{H^2}\left(3-\frac{3Z}{H}\right)$
Top stress $Z = 0$	$\frac{E}{1-\mu}\left(s-\frac{4F}{H}+\frac{6M}{H^2}\right)$	$\frac{E}{1-\mu}\left(s-\frac{3F}{H}+\frac{3M}{H^2}\right)$
Bottom stress $Z = H$	$\frac{E}{1-\mu}\left(s+\frac{2F}{H}-\frac{6M}{H^2}\right)$	$\frac{E}{1-\mu}(s)$
Top strain	$-\frac{4F}{H}+\frac{6M}{H^2}$	$-\frac{3F}{H}+\frac{3M}{H^2}$
Bottom strain	$+\frac{2F}{H}-\frac{6M}{H^2}$	0

TABLE 2 : Stesses and strains

Shrinkage function	*Stresses*					*Strains*				
	No base restraint			*Full base restraint*		*No base restraint*			*Full base restraint*	
	Factor x	*Top*	*Bottom*	*Top*	*Bottom*	*Factor x*	*Top*	*Bottom*	*Top*	*Bottom*
Constant $s = A_0$	$\frac{A_0 E}{(1-\mu)}$	0	0	$-\frac{1}{2}$	$+1$	A_0	-1	$+1$	$-\frac{3}{2}$	0
Linear $s = A_1 Z$	$\frac{A_1 EH}{(1-\mu)}$	0	0	$-\frac{1}{2}$	$+1$	$A_1 H$	0	-1	$-\frac{1}{2}$	0
$s = A_2 Z^2$	$\frac{A_2 EH^2}{(1-\mu)}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{4}$	$+1$	$A_2 H^2$	$\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{4}$	0
$s = c\, e^{D^{Z}/H}$	$\frac{EC}{D^2(1-\mu)}$	$6+4D+D^2$ $-6e^D+2De^D$	$-6X-2D$ $+6e^D$ $-4De^D$ $+D^2e^D$	$3+3D+D^2$ $-3e^D$	D^2e^D	$\frac{C}{D^2}$	$6+4D-6e^D$ $+2De^D$	$-6-2D+6e^D$ $-4De^D$	$3+3D$ $-3e^D$	0

TABLE 3 Comparison of stress at top surface through various formulae

Westergaard computed stresses kg/cm²				*Thomlinson internal stresses kg/cm²*				
Infinite slab uniform shrinkage	*Infinity slab linear variation*	*Long and narrow slab*	*Finite slab*	*Restraint*	*Constant*	*Linear*	*Parabolic*	*Exponential*
14.95	3.3715	2.3706	4.38	No base restraint	0	0	1.185	0.1012
				Full base restraint	7.48	6.2875	4.985	6.2574

Conclusions

The Westergaard's approach yields expressions for shrinkage stresses under various conditions of slab dimensions and restraint taking E, μ, and subgrade restraint into account which are simple and reasonable as a first approximation.

Thomlinson's modification is realistic as non-linear shrinkage variation through slab depth is the usual type of variation. The internal stresses worked out in this study have to be combined with stresses induced due to restraints against longitudinal movement and warping to get a complete picture of the nature of shrinkage stresses. The comparison between the shrinkage stresses obtained by using Westergaard's equations and those obtained by using Thomlinson's approach becomes difficult as the table given by the the latter (HMSO, 1963) has been compiled showing only maximum stresses at various depths and thus not at one time.

Self weight of the slab has also not been considered. Their consideration however, is very important as the increase in panel dimensions and self weight of the pavement slab prevents hogging, thereby causing stress reversal.

The actual shrinkage strain through the depth of drying slab is a complex phenomenon, but for practical purposes the variation can be simulated to a combination of polynomial and exponential functions, for which Table 2 has been prepared.

Subgrade restraint has been taken into account but the restraining force is assumed to be constant. It is observed from the fact that K_r^2 is not coming that there is a linear relationship between the stresses and strains for no base-restraint and full-base restraint and that of any partial base-restraint, stresses and strains can be interpolated.

This part study is only a preliminary work which is being extended by introducing base-restraint in the Thomlinson's approach.

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Appendix I

Sample calculation

I. Westergaard's approach

(*a*) Infinite slab (uniform shrinkage)

$$\sigma_x = \sigma_y = \frac{sE}{1-\mu} = \frac{0.225\times 4650}{100\times 0.7} = 14.946 \text{ kg/cm}^2$$

(*b*) Infinite slab linear variation

$$\sigma_0 = \frac{sE}{2(1-\mu)} = \frac{0.101\times 4650}{10(1.-3)\times 2} = 3.37 \text{ kg/cm}^2$$

(*c*) Long and narrow slab

$$l = \sqrt[4]{\frac{4650\times 20^3}{12\,(1-.09)\,5.55}} = 27.99$$

$$\lambda = \frac{600}{27.99\sqrt{8}} = 7.578$$

$$\sigma_y = 3.37125\left[1-\frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda+\sinh 2\lambda}(\tan\lambda+\tanh\lambda)\right] = 3.37 \text{ kg/cm}^2$$

(*d*) Finite slab

$$\varphi = \frac{1000}{27.99\sqrt{8}} = 12.63$$

$$\lambda = 7.578$$

$$\sigma_0 = 3.371$$

$$F_x = \frac{2\cos\varphi\cosh\varphi}{\sin 2\varphi+\sinh 2\varphi}[\tan\varphi+\tanh\varphi]$$

$$= 6.948\times 10^{-6}$$

$$\sigma_x = 3.371\,[1+.3-6.948\times 10^{-6}-.3\times 1.259\times 10^{-3}]$$

$$= 4.381 \text{ kg/cm}^2$$

II. Thomlinson's approach

(*a*) Parabolic variation (no base restraint)

$$\sigma_x = \frac{A_2\, EH^2}{6(1-\mu)}$$

$$= \frac{2.676 \times 10^{-6} \times 4650 \times 20^2}{6(1-.3)}$$

$$= 1.185 \text{ kg/cm}^2$$

(*b*) Constant shrinkage variation (full base restraint)

$$\sigma_x = \frac{A_0 E}{(1-\mu)} \left(-\tfrac{1}{2}\right)$$

$$= \frac{2.25 \times 10^{-3}}{(1-.3)} \left(-\tfrac{1}{2}\right)$$

$$= 7.473 \text{ kg/cm}^2$$

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