Stress Transmissibility in Granular Media

by

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Introduction

IN most geotechnical problems, dealing with the behaviour of foundations, the first requisite is the calculation of the stresses produced in the underlying strata by foundation loads for different geological conditions, Burmister (1956), Egorov (1958), Pickett (1938), Poulos and Davis (1974), Sovinc (1961), Westergaard (1938). The distribution of stresses in earth masses, and under some circumstances displacements as well, are generally determined based on classical elastic solutions of Boussinesq (1885) with boundary conditions approximating to those of the actual problem of interest, for lack of anything better. However, comparison of experimental and analytical investigations reveals, that the Boussinesq results may be significantly less than the true stresses in a layer especially near the base where a marked concentration may occur. This is more pronounced in the case of loose cohesionless soils than in stiffer cohesive ones.

Despite the fact that the problems of stress distribution in soils has been studied, both theoretically and experimenally, for more than half a century, and quite extensively during the last two decades, the knowledge of the material properties which affect this phenomenon is still incomplete. However, a more extensive study of the literature reveals that the particulate nature of granular masses is not considered in such great detail as it should have been, in connection with the solution of the boundary value problems, for the obvious reasons of complexity of the resulting solutions, Smoltczyk (1967). These materials consists of randomly arranged irregularly shaped, discrete particles, that are free to be displaced relative to each other. In addition the interparticle forces may range from practically nonexistent to strong electro-static bonds. The interstitial voids may contain air, water, binding material or any combination of these. The inability of classical elastic continuum theory to explain the behaviour of granular masses has been observed in many instances by various investigatiors, *Horne (1965, 1969), Koerner (1970), Proctor (1974), Rowe (1962).* Moreover, each formulation is valid only for the specific conditions upon which it is based.

Scope of the Investigation

Described here are the studies undertaken (Misra, 1976) in an attempt to investigate the influence of the more realistic material properties of the elements constituting the medium, in the stress distribution problems. This involves broadly two purposes: (1) to elucidate the influence of the

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looseness of the sand masses subjected to isolated surface loads and (2) to formulate an analytical model involving this behaviour. Although the experimental results are necessarily restricted by the resources available for this investigation it is expected that the analytical model, developed based on the findings of the experiments, will have wide application for a variety of materials ranging from solids to liquid like substances.

Findings of Earlier Experiments

Because of the fact that the measurement of stresses and displacements, both in the interior of a soil mass and at the contact surface requires a considerable amount of embedded instrumentation, not enough experimental observations are available, Kennard, *et al* (1967). An excellenr review of measured behaviour of sands, subjected to vertical surface loading over a circular area, carried out in recent years by various organisations and individuals at different places, has been reported by Morgan and Gerrard (1971). Although the summary covers a wide range of variables; in the grading and relative density of the fill, in the size, in the magnitude and rate of loading, in methods of compaction, in boundary conditions of the loaded mass, and in the loading history of the fill; as every investigation had specific aim and purpose, it is difficult to draw general conclusions from these experiments. However, it still gives the following useful information; based on which, future investigations could be programmed for bringing into light the more realistic material property influencing this complex problem.

(*i*) The differences between the various measurements of vertical direct stress are, however, significant on the load axis; being higher near the surface but tend to converge with depth and are influenced by the compaction method at least. (*ii*) The measured stresses show a wide variation near the load axis, but converge to a narrow band as the radial offset increases, that is, the actual vertical stresses in real soils are concentrated closer to the axis than the theory predicts. (*iii*) Radial and tangential stresses show a wide variation in measured values and may be grossly over or underestimated by theory. (*iv*) The response of the soil to loading appears to reflect the changes that occur in the structure as a result of the external loading, as well as the initial structure.

Instrumentation and Results

Description of the Set-up

A steel tank of one metre cube internal dimensions, stiffened sufficiently with the help of two rows of steel angle stiffeners to minimise side bulging of the tank, was used for experimentation. The dimension of one metre was considered adequate for footings with diameters of 20.32 cm and less for the semi-infinite condition. One horizontal steel channel across two vertical ones fixed centrically one on each of the two opposite sides of the tank was used to provide bearing for the guide vane of the footing, Figure 1.

Aluminium footing models, circular in shape having rough base, fitted with one 1.27 cm diameter 60 cm long vertical central stud (for guiding vertical settlement of the die) and four aluminium cross-arms placed symmetrically 90 degrees apart (for aiding displacement measurement);

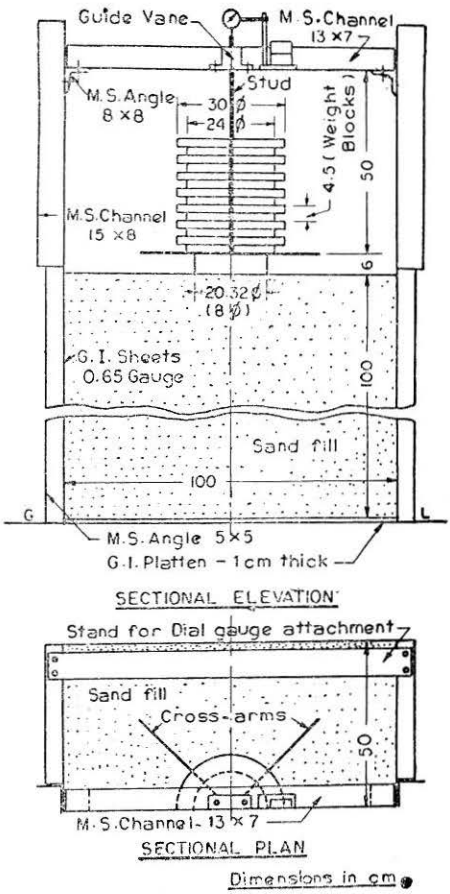

FIGURE 1 Experimental set-up (half in section)

were used in the investigation. Five displacement dial gauges were used with suitable fixtures, one each on the four cross-arms and the fifth on the top of the guide stud, to detect tilting of the footing, if any, during settlement. A thickness of 6 cm for the aluminium casting was considered to be sufficient for the development of uniform pressure at the contact surface due the dead load applied on its top surface.

Air dry uniform sand (having $D_{60} = 0.7$ mm and $D_{10} = 0.62$ mm, Figure 2*a*) with subangular to subrounded particles was used for the test; mainly for easy working conditions, since its behaviour is time-independent and maintenance of uniformity of medium and reproducible densities are possible. The limiting void ratios were $e_{max} = 0.915$ and $e_{min} = 0.602$, (Figure 2*b*), corresponding to dry densities of $\gamma_{min} = 1394$ kg/m³ and $\gamma_{max} = 1667$ kg/m³ respectively.

Hoppers of different openings each having 15 cm wide slots were used for allowing the sand to fall as a rain to build up the required bed, Bieganousky and Marcuson (1976), Kolbuszewski and Jones (1961). Use of, 6 mm slot opening keeping the slot 7 to 8 cm above the sand surface gave

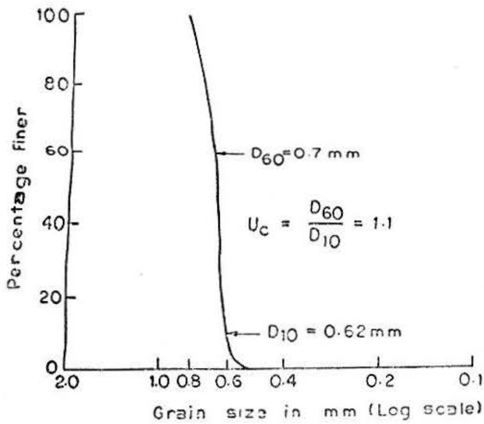

FIGURE 2*a* Grading curve for test sand

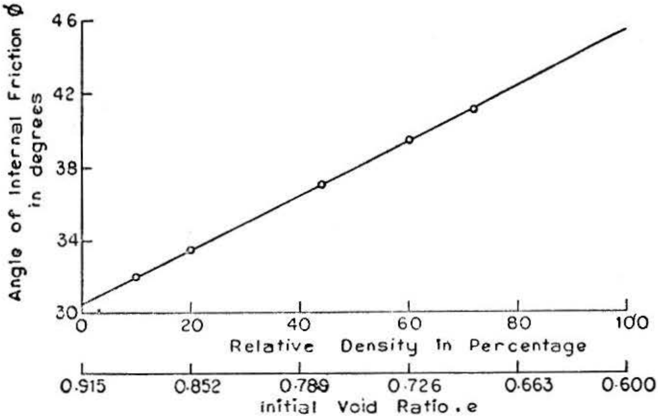

FIGURE 2*b* ϕ versus *e* and ϕ versus R.D. of test sand

a relative density of 35 per cent and, 3 mm slot opening keeping the slot 30 cm above the sand surface gave a relative density of 75 per cent.

Redshaw (1954) pressure cells of the deflecting diaphragm type, having 2.5 cm overall diameter and 1.25 cm thickness (pressure range of 0 to 7 kg/cm^2) were used to measure the vertical stress, σ_{zz} at different depths along the central axis. Many excellent studies of soil stress cells have been reported by Krizek, *et al* (1974) and Triandafilidis (1974) and much detailed information regarding the selection and use of appropriate pressure cells for a given test installation can be found in their works. The possibility of under-registering of pressures, when used in soils, by such devices mainly due to its shape and size was carefully checked as suggested by Redshaw (1954). As the results did not indicate any appreciable difference, the cells were calibrated with alround water pressure using the conventional triaxial pressure chamber.

Preparation for the Experiment

The tank was placed on a firm surface and levelled. The footing was placed in position by pushing its central stud through the guide vane from inside the tank and was held at an elevated position with the help of a temporary cross-beam support across the testing area. The test bed was prepared by allowing the sand to fall as a rain through the slot opening, while the hopper was moved manually over a horizontal plane, all the time maintaining a constant height of the slot above the sand surface.

Sufficient care was taken to place the pressure cell accurately over the sand layer at the required position, such that, when in position, its depth was given from the mid-thickness of the cell. Maximum attention was given to its horizontal placing without disturbing the surrounding sand; as it was felt that even a little error in its horizontality might affect the accurate measurement of the pressure, more than, when its placing was out by the same degree with respect to either or both of the radial and depth co-ordinates.

After installing the cells the sand pouring was continued and the test bed was completed. Then the footing was slowly guided down to the sand surface by its own weight, with the help of properly lubricated guide vane and the stud, without imparting force in any direction whatsoever. The horizontality of the footing was checked. Sufficient care was taken not to disturb the test bed surface while placing the footing in position.

Load Application and Observations

After recording the initial readings of the cells and the dial gauges, the dead weight was placed over the footing and the readings were recorded. The weight was placed over the footing very carefully, without imparting the slightest of jerks, to avoid any disturbances of the set-up. The placement was very much similar to that used in the well-known odometer tests, except that, in this case the weight was placed directly over the footing instead of over a hanger plate.

The settlement readings recorded by the diametrically opposite dial gauges were first compared as independent pairs and if found reasonably equal, the average settlement as recorded by the two pairs of diametrically opposite gauges and the central gauge were then compared. If these were equal, with permissible difference, the results were accepted and the experiment was continued with the next higher load. Such checks were made at each stage before proceeding with the next higher load. If at any stage the difference was observed to be more, the whole experiment was discarded at that stage and repeated afresh starting from the filling of the tank.

For every set, pressures were measured for the different depths with a single cell embedded in the entire fill at the specified depth, as well as with number of cells emedded at those depths simultaneously (a maximum of six were used at a time). It is observed that the presence of more cells has very little effect on the functioning of the individual cells, especially when used at a vertical spacing of 7.5 cm or more.

At the end of each experiment the uniformity of the relative density of the fill was checked with the help of a dynamic penetrometer at different locations spread all over the test area. Besides this the sand forming the test mass was weighed at the end of each experiment to verify the uniformity of density of the medium prepared for the different sets.

Test Programme

The present investigation was carried out with four sizes of circular footings varying from 20.32 cm to 10.16 cm on two fills; one dense having $\gamma_{dry} = 1589.3$ kg/m^3, $D_r = 75$ per cent and the other loose with $\gamma_{dry} = 1478.4$ kg/m^3, $D_r = 35$ per cent ; for the following two cases.

Case I—Semi-infinite medium : At six different points located at depths of 1, 2, 3, 4, 5 and 6 times the radius of the footing, measured from the surface.

Case II—A finite layer underlain by a rigid interface : (*i*) For five different thicknesses of the finite layer (1, 2, 3, 4 and 5 times the radius of the footing); and (*ii*) for each thickness, pressure was measured at points located at depths of 1, 2, 3, 4 and 5 times the radius of the footing depending on the thickness of the finite layer.

Analysis of Results

The primary object of this experimental investigation is to study the influence of looseness of the medium on the stress transmission. Because of the fact that, the factors responsible for the development of a stress field in a medium affect all the stress components alike; the vertical stress σ_{zz} was measured, guided mainly by the experimental convenience.

Even though it was observed that the presence of more than one cell in the medium has very little effect on the accuracy of measurement of pressures, especially when they are placed at 7.5 cm or more apart, those results obtained with only one cell were given more weightage. The results with several cells were used mainly for comparison. If for any point the observations, with one cell and several cells together, were found to differ considerably, the experiment was repeated for that set with only one cell in position.

Only the best representative results of vertical pressures, out of the several observations taken by such repetitions, for each point have been reported. The σ_{zz} values along the load axis, for both dense and loose fills, are presented in a non-dimensional graphical form in Figures 3 and 4, (a = radius of footing, p = intensity of surface pressure), for the semi-infinite case and finite layers respectively. The results have been compared with that of Boussinesq's and Froehlich's (with concentration index of 6 for sand) theories.

Findings of the Experiments

Although some error is expected in the results of such experiments, especially in loose materials like sand, this investigation reveals the following qualitative, if not quantitative, conclusions:

1. The results endorse the agreement with the experimental findings of other investigators, Morgan and Gerrard (1971), in respect to

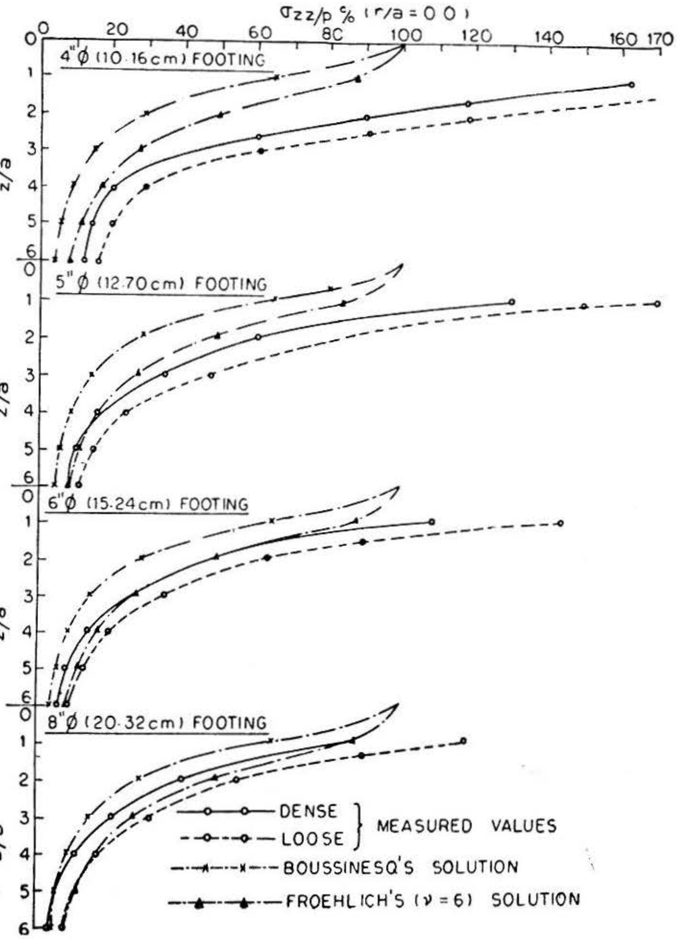

FIGURE 3 Vertical direct stress down load axis (semi infinite mass)

(*i*) heavy concentration of stresses around the loading area-observed to be significantly higher than that the existing theories could predict at points near the surface and reducing much rapidly as the depth increases, (*ii*) development of non-uniform contact pressure with the decrease in the size of the footing; though the exact nature of the non-uniformity could not be predicted accurately from these small plate tests, and (*iii*) tendency for rotation of the footing about the vertical axis increasing with the magnitude of loading-observed to be more for the loose fill.

2. By far the most conspicuous finding of all is that, looseness of the medium gives rise to higher vertical stress especially near the loading area, irrespective of the thickness of the fill and size of the footing. It is believed that, this is the first of its kind reported and might well be the most striking one to understand the basic fundamental character that demarcates solids from granular discrete particles; hence, a partial explanation of the divergences between the results of the existing theories and the reality.

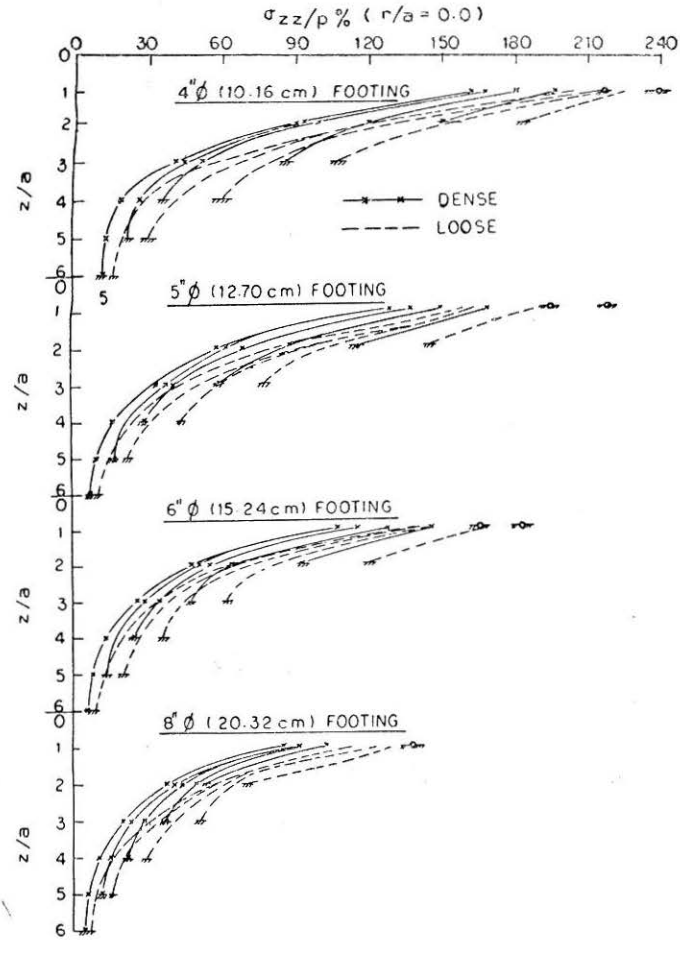

FIGURE 4 Vertical direct stress down load axis (finite layers)

Mathematical Model

General

The formulation of mechanical constitutive relations for granular media presents itself as a complex problem, one which has yet to be definitely resolved. The need for such laws is apparent when one attempts to relate the theoretical and experimental results for stresses in granular soil masses. Clearly neither the linearly elastic nor the ideally plastic idealisations are adequate under general circumstances. The behaviour of granular masses subject to external forces and displacements is governed by the individual forces and displacements occurring at each particle contact; neglecting elastic compression of particles. An external force disturbs the balance of positions within the system. This results in finite movements of particles, retarded by mobilised friction.

Concept of the Model

The essential difference in the fundamental mechanics governing the deformation behaviour of discrete granular materials from that of solids lies in the basic fundamental character that demarcates the two; namely, the marked tendency of sliding between particles. Based on this consideration, the concept of relating stress distribution in granular soils to the ratio E/G given by the author (1975, 1976); in which it is considered that $E/G > 2\ (1 + \mu)$ for materials other than solids, (where E = Young's modulus, G = shear modulus and μ = Poisson's ratio); very effectively explains the concentration of stresses especially in loose particulate materials.

The material parameter E/G will have different values for different material structures, such as, solids, solidslike materials (firm rocks and deposites), loose particulate materials (dry or saturated to different degrees), sensitive grain structures (quick sand, sensitive clays, soft soils like marginal lands etc.) and liquid like materials. It is expected that the softer/looser medium the higher will be the material parameter E/G; because of the poor shearing resistance of such materials.

The vertical stress σ_{zz} equations resulting from this concept, Misra (1976), Misra and Sen (1975, 1976), are reproduced in the final form, in Appendix I.

Influence of the Paramet.r E/G

Some of the pertaining numerical results included here, as supplement to the writer's earlier works (1975, 1976), amply demonstrate the utility of the E/G ratio concept. In the absence of established values of this parameter for different soil structures; results have been presented for arbitarily selected values of E/G over a wide range, to understand its influence on the stress distribution problems. For the different values of Poisson's ratio the trend of the results being similar and the quantitative values not much different, the results for the two extreme values of μ are given for a broader presentation, though it is a fact that $\mu = 0$ does not correspond to reality.

Numerical values of α^2 and β^2 for different values of the parameter E/G and μ are presented in Figure 5. The effect of the parameter E/G is reflected through the coefficients α and β in the stress and deformation equations 1 through 4, given in Appendix I. The similarities between these and the classical equations are significant; equation 1 is one of the closed form solutions reproduced here in this context (Jumikis, 1964).

The values for vertical direct stress σ_{zz} at various radii on a horizontal plane at a given depth ($z/a = 2$) are shown in Figures 6 and 7. The inability of certain natural materials to transmit the vertical force in the horizontal direction is clearly indicated by this model, as seen in Figures 6 and 7. The analysis shows a wide variation of vertical stress near the load axis (increasing with increase in looseness) converging in a narrow band to zero as the radial offset increases.

The influence of the material parameter E/G on σ_{zz} along the vertical axis is presented in Figures 8 and 9. The trend of these curves indicates that for very high values of E/G, nearing infinity, the vertical direct

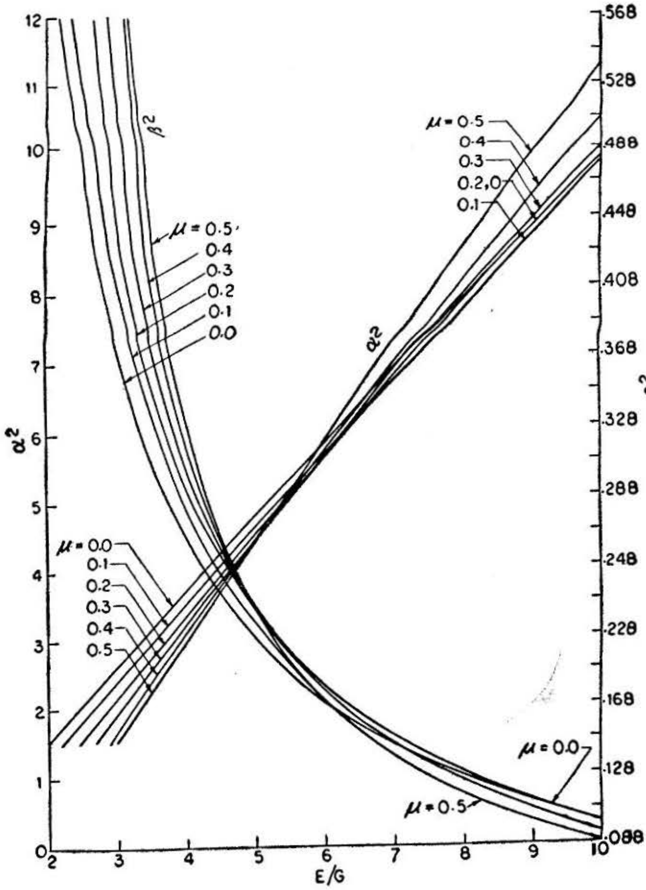

FIGURE 5 α^2 (β^2) versus E/G

stress along the load axis is the same at all depths and equals to the applied pressure on the surface; a phenomenon typical of liquid like materials. The results are also compared with that of Boussinesq's and Froehlich's theories. Although the results of the present analysis show the same general pattern; the deviation of σ_{zz} values from the classical solution, with the increase in the values of E/G, is quite large and is on the higher side.

It is observed in all the cases that, for E/G values of 3.0, 4.0, 5.0 and 6.0, the results show quite good agreement with the values for the corresponding Froehlich's concentration indices, except close to the loading area (Figures 8-9); the Froehlich's hypothesis giving rise to greater concentration of the vertical stress close to the surface than the present concept. This is probably related to the fact that a constant property, either concentration index or the material parameter E/G, all through the depth is not a true model.

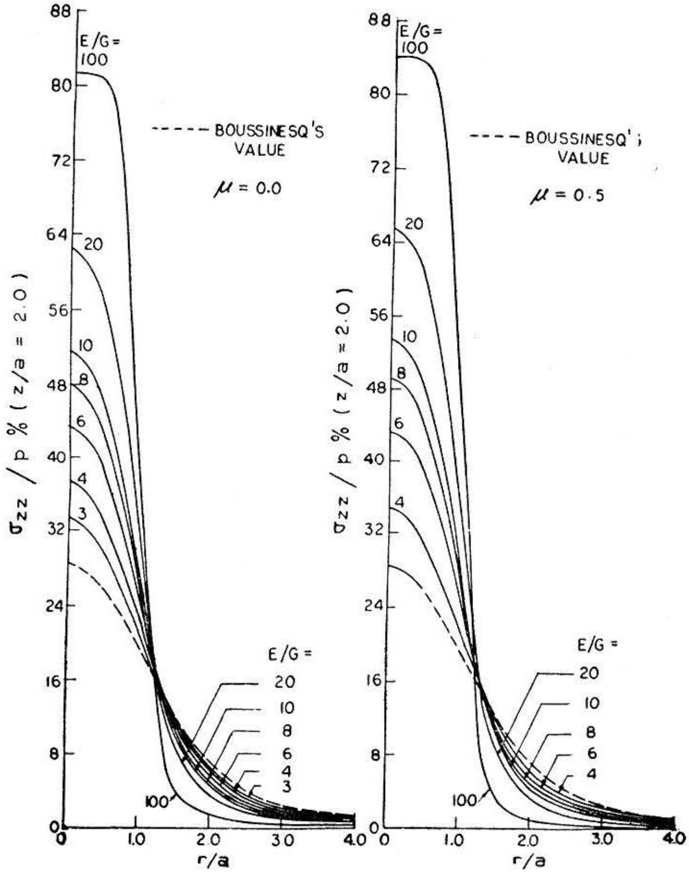

FIGURE 6 **Vertical direct stress versus offset (uniform contact pressure)**

The variation of the vertical axial stress with the thickness of the medium, for the two values of E/G (4 and 20—chosen for clarity of presentation) are presented in Figure 10. It includes the nature of the underlying rigid strata, namely rough and smooth. A value of 4 for E/G represents a near solid like material (when $\mu = 0.5$) and the resulting graph is in good agreement with the results based on the classical elastic approach.

Summary and Conclusions

An attempt has been made to identify some of the significant factors affecting the stress transmissibility in granular media and to present a simplified analytical model for evaluating stresses and deformations which will take these factors into account. It gives a partial explanation of the divergences between the results of the existing theories and reality.

This study has indicated that near the loading area, the vertical stresses are higher the looser is the sand, and even in a dense state are larger than obtained by Boussinesq. It is observed to be significantly higher at points

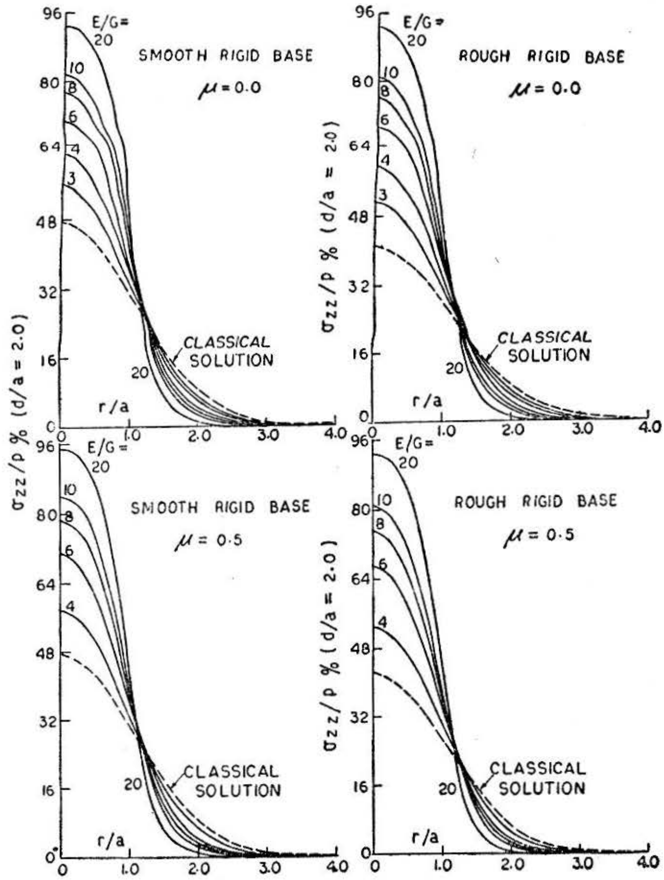

FIGURE 7 **Vertical direct stress at the rigid surface versus offset (uniform contact pressure)**

near the surface and reducing much rapidly as the depth increases; indicating thereby the fact that confinement improves the performance of materials, which are even weak in resisting shear deformation. It appears that a tendency for rotation of the footings about the vertical loading axis is always associated with isolated model footing tests. It may be pointed out that for such type of experiments guided settlement are needed for better results.

The concept of relating stress distribution in granular soils to values of E/G and of μ seems to be endorsed by the agreement of the resulting graphs with the empirical data and by our own intuitive expectations. The analysis is applicable to a wide range of foundation materials ranging from solid-like at one end to liquid-like structures at the other end of the spectrum.

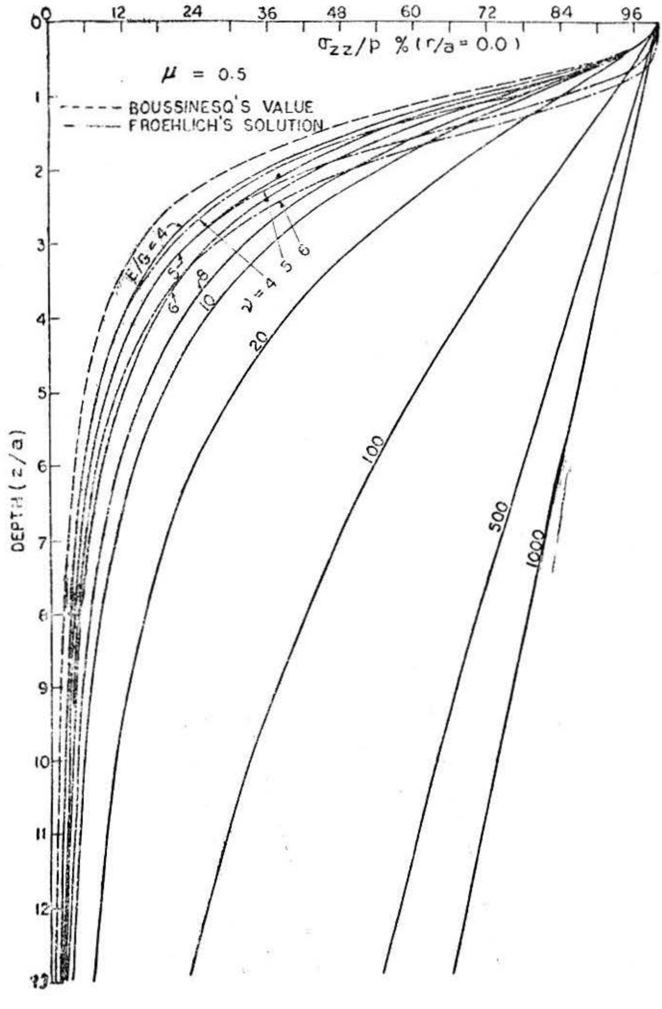

FIGURE 8 Vertical direct stress on load axis (uniform contact pressure)

The experimental and analytical results of this investigation while endorsing the findings of other investigators, make conspicuous the effect of looseness softness of the medium in the transmission of stresses and deformations. The analytical model is able to predict remarkably well the behaviour of materials which are known to be weak in resisting shear deformation. The parameter E/G very effectively accounts for the looseness of the medium.

The hypothesis based on the material parameter E/G very ably explains the observed fact in the actual soils, namely; in such materials the effects are more pronounced near the loading area; getting concentrated around the loading axis.

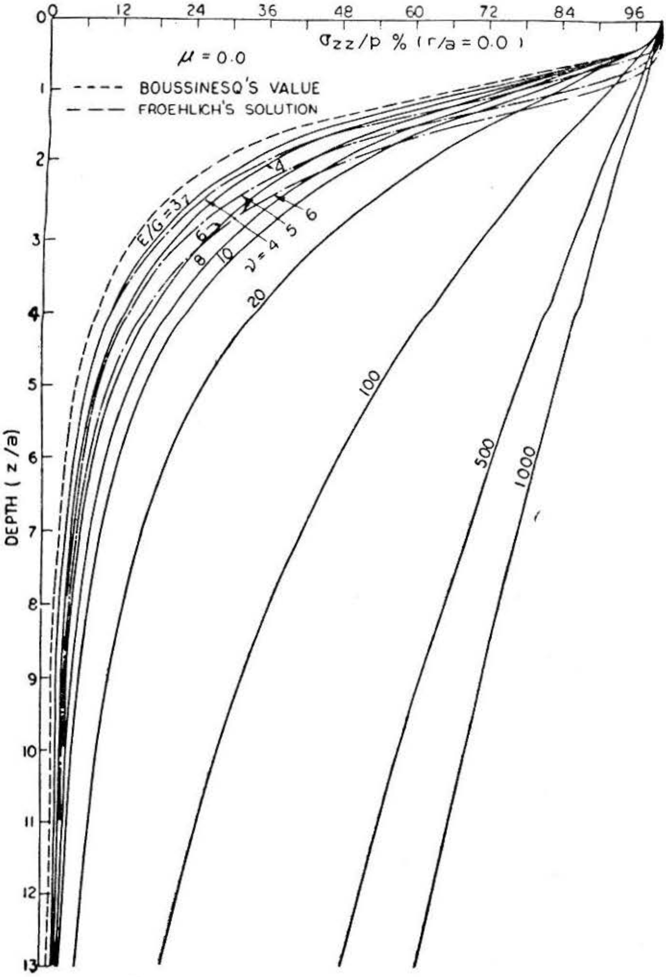

FIGURE 9 Vertical direct stress on load axis (uniform contact pressure)

The practical value of this work, with respect to its ultimate application in engineering practice, will depend upon the development of suitable field tests for the measurement of E, G (or E/G) and μ. This may be a difficult task, but satisfactory procedures probably will be worked out when the utility of the results of this concept becomes fully associated.

Appendix I—Theoretical Expressions

The development of the equations given in this Appendix is dealt with in references cited at the end of the paper.

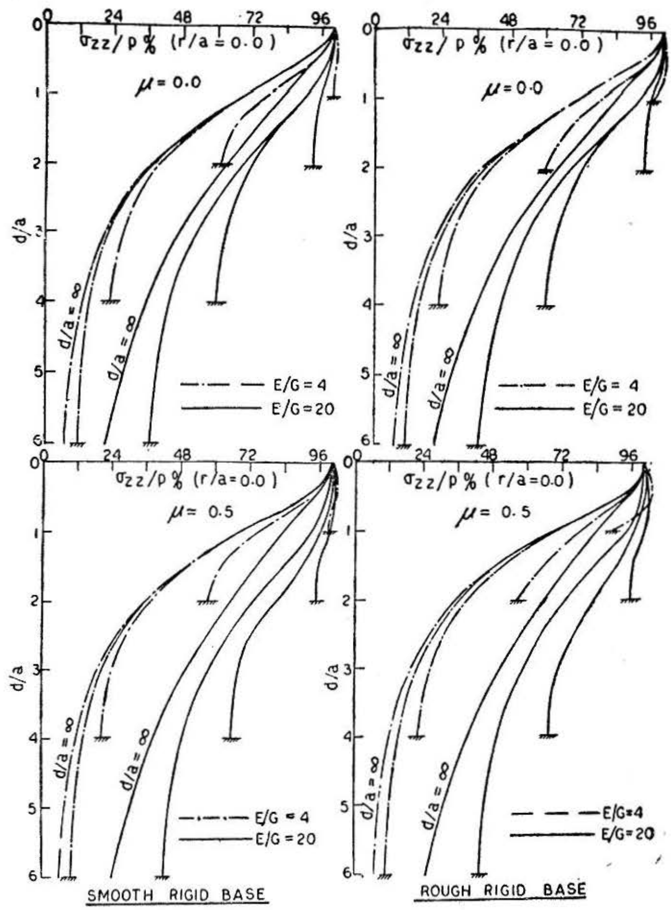

FIGURE 10 Vertical direct stress on load axis—finite layer (uniform contact pressure)

Case A

Point load P on the surface of a semi-infinite mass, Misra and Sen (1975).

$$\sigma_{zz} = \frac{P}{2\pi(\alpha-\beta)z^2}\left[\{\beta^2+(r/z)^2\}^{-3/2} - \{\alpha^2+(r/z)^2\}^{-3/2}\right] \qquad \ldots(1)$$

Case B

Uniformly loaded (intensity p) circular die (radius a) on the surface of;

Case I : Semi-infinite mass, Misra and Sen (1975).

$$\sigma_{zz} = \frac{ap}{(\alpha-\beta)}\int_0^\infty [\alpha e^{-\beta\lambda z} - \beta e^{-\alpha\lambda z}]\, J_0(\lambda r)\, J_1(\lambda a)\, d\lambda \qquad \ldots(2)$$

Case II : Finite layer underlain by a rough rigid base, Misra and Sen (1976).

$$\sigma_{zz} = \frac{ap}{2}\int_0^\infty g_8(\lambda)\, J_0(\lambda r)\, J_1(\lambda a)\, d\lambda \qquad ...(3)$$

Case III : Finite layer underlain by a smooth rigid base, Misra and Sen (1976).

$$\sigma_{zz} = \frac{ap}{2}\int_0^\infty f_3(\lambda)\, \mathrm{J}_0(\lambda r)\, J_1(\lambda a)\, d\lambda \qquad ...(4)$$

where,

α^2 and β^2 are the roots of the equation $x^2+(K'-2)\,x+1=0$, in which, $K'=(1-K)/(1-\mu)$ and $K=(E/G)/(1+\mu)-1$ $g_8(\lambda)$ and $f_3(\lambda)$ are trigonometric functions involving thickness and material properties of the layers, and are defined in respective references.

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