Phenomenon of Initial Gradient and Its Implications to Seepage Problems

by

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Introduction

Initial gradient is defined as that gradient below which no flow is observed. As per the record goes, the phenomenon of initial gradient was first observed by King (1898) and Stearns (1927) but a formal flow equation incorporating the concept of initial gradient was forwarded by Puzyrevskaya in 1931 and the equation reads as

$$v = K(i - i_0) \qquad i \geqslant i_0 \qquad \ldots(1)$$

in which i_0 is initial gradient, v is the macroscopic seepage velocity, i is the hydraulic gradient and K is the permeability coefficient. Though, the consequences of the existence of initial gradient is far reaching in many practical seepage problems, there seems to be no concerted effort to understand the physics of this phenomenon of initial gradient and whatever meagre information is available in the literature, is not free from contradictions and confusions. In this paper an attempt has been made to collect all available informations (atleast in English literature) with a view to bring out a unifying picture of the state of art on the subject. Consequences of the existence of initial gradient (in the form of analytical solutions) in two physical problems are also brought out.

Genesis of Initial Gradient

The existence of initial gradient in clayey and other fine grained soils, has been attributed to the predominance of surface forces over gravity forces in these fine grained soils and generally, these surface forces are strong enough to counteract a certain portion of applied gradient, giving rise to the phenomenon of initial gradient (Karadi and Nagy, 1961; Kondon, 1967; Miller and Low, 1963). The origin and the type of these surface forces can be best understood by considering the state of water in the clay-water system. Water molecules consist of dipoles formed by negatively charged oxygen ions and positively charged hydrogen ions. Water molecules when in contact with clay minerals get oriented because of the electrical field caused by the surplus energy developed on the surface of clay particles (Karadi and Nagy 1961; Williamson, 1951; Low 1961; Rosenqvist, 1961). The range of these electromolecular forces or more popularly known as surface forces is around 0.25 to 0.5 microns and their magnitude sharply decreases with increasing distance from the surface of the soil particle. The magnitude of this force near the soil particle surface may

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attain a value of 10,000 kg/sq cm but at a distaince of 0.5 micron, it is negligible. As a result, these oriented fluid layers which are very near to clay minerals, get firmly adhered to and are termed as strongly bound water or adsorbed water. Beyond this layer, water molecules are loosely bounded but still *oriented* and are very much different from free water. Schematically, the situation is depicted in Figure 1. This loosely bound water (sometimes called liosorbed water) may exist up to an appreciable distance from the particle surface and in fact the range of porosities or void ratios found in soils for engineering and agricultural uses are such that there is a very good probability that almost all the water therein, is bound loosely or strongly (Macey, 1942; Grim, 1942; Lutz and Kamper, 1959). The direction of the above mentioned surface forces interacting with the fluid element is normal to the clay particle surfaces hence also normal to the fluid motion. The concept of this mutual normality of surface force and fluid motion will be more distinct if a capillary model is envisaged (Klausner and Kraft, 1965*a*). Now, if this concept of normality is true, then it is not very clear how before flow commences, a certain portion of applied gradient (*i.e.* initial gradient) is resisted by these forces which are perpendicular to the direction of applied gradient. One of the hypotheses (Klausner & Kraft 1965*a*, 1965*b*, 1966*a*, and 1966*b*; Basak 1975) is that these normal surface forces make their presence felt by bringing into play *shear forces* (opposing the fluid motion) which become responsible for the existence of initial gradient. The mechanisms involved in the origin of these shear forces and the parameters affecting them need to be explained in more concrete scientific terms and the present literature does not throw much light on these.

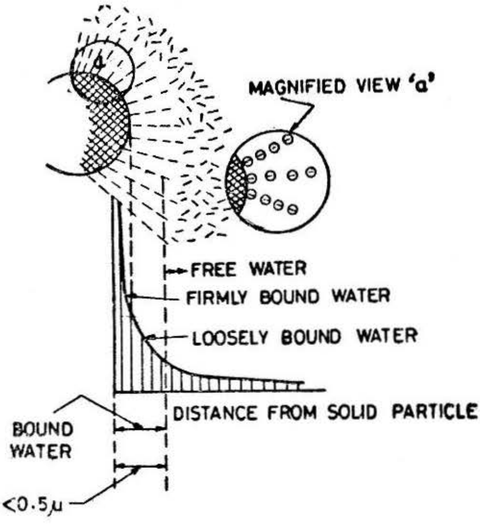

FIGURE 1 A conceptual model of free and bound water (after Citovies 1951)

Another hypothesis for initial gradient advocated mainly by scientists from Soviet block (Bondarenko, 1968; Churyev and Gorokhov, 1970; Nerpin and Deryagin, 1969; Miller and Low, 1963) is that due to *oriented* or quasi-crystalline structure of the pore water, the pore water does not behave like Newtonian fluids but acts as Bingham body with a plastic viscosity and definite yield stress of the order of 10^{-2} to 10^{-3} dynes/sq cm and thereby giving rise to the phenomenon of initial gradient. The proposers of this hypothesis further specify that only liquids with hydrogen bonding *e.g.* H_2O, C_2H_5OH, CH_2OH and a mixture of CH_3 and $CHCl_3$ behave like a Bingham liquid. This hypothesis fails to explain many of the reported values of initial gradient in many clayey soils (with fluid having hydrogen bonding) much in excess of the gradient explainable in terms of the gradient explainable in terms of the yield stress (τ_0) of magnitude reported (*e.g.* 10^{-2} to 10^{-3} dynes/cm²). It also fails to explain substantial amount of data for sand soils showing complete absence of initial gradient (Childs and Tzimus, 1971). It is to be pointed out here, that none of the above existing hypothesis has got wide acceptance in the scientific community, due to inherent contradictions and lack of sufficient experimental data to support them. However, one has to remain contended till a better hypothesis appears.

Some Important Experimental Information about Initial Gradient

One of the earliest non Darcy flow rate-gradient data for clay is due to Stearns (1927). Many of his experimental results showed that the existence of initial gradient is related with percentage of clay fraction present in the sample. His experimental data showing the presence of initial gradient versus percentage of clay fraction of size $> 5\,\mu$ are being replotted and shown in Figure 2. Though there is a wide scatter in data, a trend of

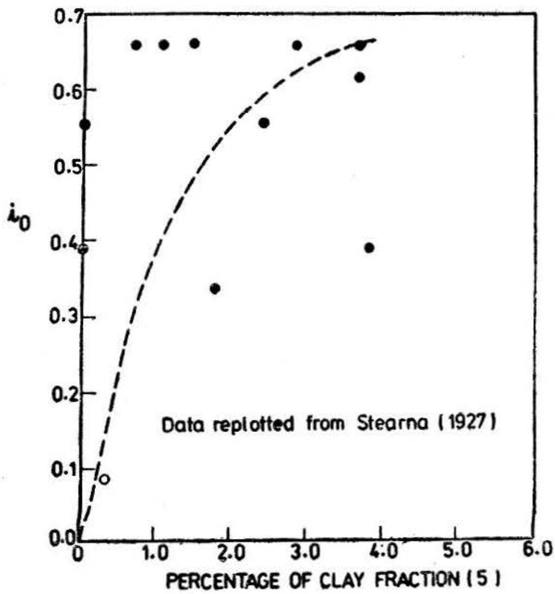

FIGURE 2 Relationship between percentage of clay fraction and initial gradient

increasing initial gradient with increasing percentage of clay fraction is recognisable. This trend is also expected as percentage of clay fraction is a crude measure of surface forces present in the soil-water system. Higher the percentage of clay fraction, higher will be the specific surface and higher will be the surface forces resulting in higher initial gradient. Kondon's (1967) experimental data also gives a similar trend. The variation of initial gradient with void ratio for different types of clays were studied only by few (Li, 1963; Karadi and Nagy, 1961; Roza, 1955). The available results are consolidated in Figure 3. The limited data show a very distinct expected trend of increasing initial gradient with decreasing void ratio.

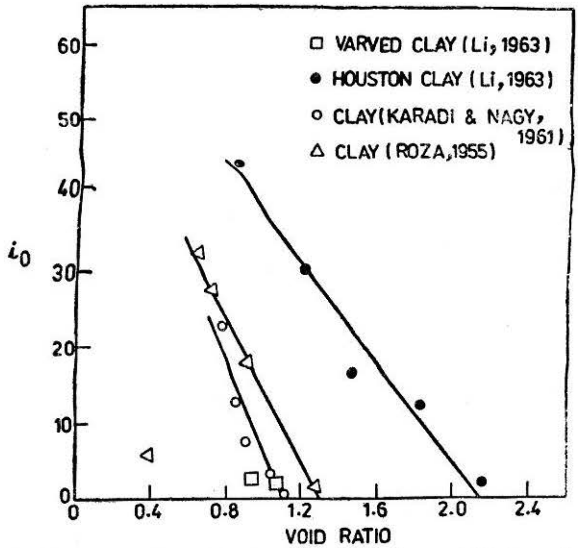

FIGURE 3 Relationship between void ratio and initial gradient

Though it is recognised that the existence of initial gradient is more likely in case of clayey and other surface active materials, few experimental and semi-empirical evidences (Bondarenko, 1966) started appearing in support of initial gradient in case of sandy and other inert materials as well, but these results were contradicted by others (Childs and Tzimus, 1971) later. Hence, it seems, more experimental results are needed to draw any meaningful conclusion about the existence of initial gradient in inert materials.

Initial Gradient at the Shape or Velocity-Gradient Response

Puzrevskaya's flow equation as given in Equation 1 predicts a linear v-i response after i_0 and this is being supported by many. On the other hand, many more experimental results suggest a nonlinear velocity gradient response after i_0 and which meets the straight line portion asymptotically. A summary of the experimental test results on these are made and presented in Table 1. Based on thermodynamic principles, Nerpin and Deriyagin (1969) semi-empirically showed that under some restrictive conditions, Puzyrevskaya's equation is reasonable and gives the best experimental fit. Nerpin and Tchudnovskij (1967) proposed the velocity gradient response as per Equation 2 for soil water system when pore water behaves like a Bingham body.

$$v = \frac{\gamma r^2}{8\eta}\, i \left[1 - \frac{4}{3}\left(\frac{i_0}{i}\right) + \frac{1}{3}\left(\frac{i_0}{i}\right)^4 \right] \quad \ldots(2)$$

Working on the similar line, Kovacs (1967) proposed the equation of the form

$$v = \frac{\gamma r^2}{8\eta}\, i \left[\left(i - \frac{i_0}{i}\right)^2 - \frac{2}{3} \left\{ \frac{i_0}{i}\left(i - \frac{i_0}{i} \right)^{\frac{3}{2}} \tan^{-1}\sqrt{1 - \frac{i_0}{i}} - \log \frac{i_0}{i} - \left(1 - \frac{i_0}{i} \right) \right\} \right] \quad \ldots(3)$$

TABLE 1

Available experimental results

Sl. No.	Authors	Type of soil tested	Moisture condition
1.	King (1898)	sand and sand stone	saturated
2.	Stearns (1927)	clay	saturated
3.	Izbash (1931)	sand	saturated
4.	Puzyrevaskaya (1931)	clay	saturated
5.	Miller and Low (1963)	bentonite paste	saturated
6.	Rawlins Gardner (1963)	silty clay loam	unsaturated
7.	Von Engelhardt and Tunn (1955)	sand stone	saturated
8.	Davidson, *et. el.* (1963)	silty loam	unsaturated
9.	Valarovitch and Tehuraev (1964)	peat	saturated
10.	Kutilek (1964-67)	kaolinite, illite and montmorillonite	saturated
11.	Thames (1966)	silty loam	unsaturated
12.	Nerpin and Tchudnovskij (1967)	clays	saturated
13.	Kovacs (1967)	kaolinite, illite and montmorillonite	saturated
14.	Karadi Nagy (1961)	clay	saturated

in which

$$i_0 = \text{initial gradient} = \frac{\tau_0 s}{n\gamma} \quad \ldots(4)$$

γ = density of permeating fluid

τ_0 = yield stress of permeating fluid

s = specific surface

n = porosity of the medium

r = average pore radius

η = viscosity of permeating fluid

Both equations can be approximated by Puzyrevskaya's equation for $i >> i_0$.

The implications of the existence of initial gradient on two physical problems are discussed in the following sections.

Consequences of Initial Gradient in Physical Problems— Past and Present Works

Consequences of the existence of initial gradient are of potential interest in several disciplines. Ground water movement and drainage in clayey soils, soil water movement to plant roots, consolidation of clayey soils and infiltration of water into soils are some of the areas where the recognition of the role of initial gradient is important.

During last one decade or so there is a considerable interest in solving various physical problems incorporating initial gradient in the flow relation. Using unit hydrograph techniques, Entov (1967) has presented an analytical method for solving two dimensional problems with initial gradient. Ghorghitza (1959), Bondarenco (1968), Polubarinova-Kochina (1969), Valsangkar and Subramanya (1972) and Arumagam (1975) investigated the effect of initial gradient on various physical problems of interest in the field of drainage and irrigation. Florin (1951), Roza and Kotov (1955), Girault (1960), Elnaggar, Karadi and Krizek (1971) and Parlange (1973) studied the effect of initial gradient on the one dimensional consolidation of clayey soils due to vertical drainage. *Recently the author (Basak, 1976a; 1976b; 1977a*: *1977b*; *1977c*) through a series of publications reported the role of initial gradient in various geotechnical engineering problems.

Here, in this paper the role of initial gradient on two more physical problems are brought out. The first problem deals with the effect of initial gradient on the seepage through an embankment overlying an initial gradient soil and the second problem concerns with the seepage through confined aquifer of variable thickness.

Seepage Through Embankments Overlying Initial Gradient Soil

The embankment is considered to be composed of two horizontal layers as shown in the definition sketch (Figure 4). It is assumed that the bottom

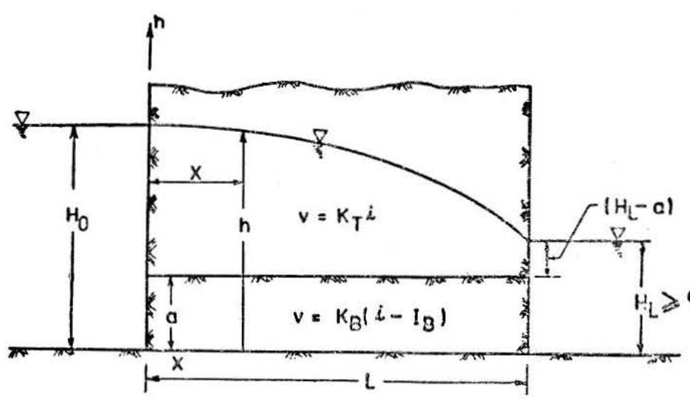

FIGURE 4 Definition sketch of the problem

layer is made of fine grained soils (*e.g.* silt and clay) and consequently shows the presence of initial gradient with velocity gradient response as given in Equation 1*a* whereas the upper layer is assumed to be made of medium to coarse sand and as such follow the Darcian velocity gradient response ($v = Ki$). When both layers follow Darcian flow relation, the solution is reported by Bear (1972). The analysis below deals with the effect of initial gradient of the bottom layer on the overall steady seepage characteristics of the embankment.

Analysis

Figure 4 illustrates the different terms and variables. It is assumed that the two layered embankment considered here is resting on an impervious bottom and the downstream water level, H_L is always more or equal to the thickness a of the bottom layer. As stated earlier, the bottom layer follows flow Equation 1a and the top layer follows the flow Equation 2a.

$$V_B = K_B\,(i - I_B); \quad i \geqslant I_B \qquad \text{...(1}a\text{)}$$

and

$$V_T = K_T\, i \qquad \text{...(2}a\text{)}$$

where

K_B = permeability coefficient for the bottom layer

I_B = initial gradient for the bottom layer

K_T = permeability coefficient for the top layer

and i = hydraulic gradient causing flow.

If Q is the amount of seepage per unit time per unit length from upstream to downstream side, then by Equations (1a) and (2a), one can write,

$$Q = K_B\, a\,(i - I_B) + K_T\,(h - a)\, i \qquad \text{...(3}a\text{)}$$

in which h is the head at any distance x. By Dupuit's assumptions,

$$i = -\frac{dh}{dx} \qquad \text{...(3}b\text{)}$$

By Equations (3a) and (3b)

$$Q = K_B\, a\left(\frac{dh}{dx} + I_B\right) - K_T\,(h - a)\,\frac{dh}{dx} \qquad \text{...(4}a\text{)}$$

Non-dimensionalising and re-arranging the terms in Equation (4a) one can write

$$q_* + \frac{K_B}{K_T}\,\frac{a}{H_0}\, I_B = -\frac{H_0}{L}\,\frac{dy}{dX}\left[\left(y - \frac{a}{H_0}\right)\left(1 - \frac{K_B}{K_T}\right)\right] \qquad \text{...(5)}$$

where

$$q_* = \frac{Q}{K_T\, H_O} \qquad \text{...(6)}$$

$$X = \frac{x}{L}, \quad L = \text{length of embankment} \qquad \text{...(7)}$$

$$y = \frac{h}{H_0} \qquad \text{...(8)}$$

The boundary conditions of the problem are

$$x = 0,\ h = H_0 \quad \text{or} \quad X = 0,\ y = 1 \qquad \text{...(9)}$$

and

$$x = L,\ h = H_L \quad \text{or} \quad X = 1,\ y = \frac{H_L}{H_0} = y_0 \qquad \text{...(10)}$$

Integrating Equation (5) with the boundary conditions (Equation 9) the equation for phreatic surface is obtained as

$$X = \tfrac{1}{2}\left[\frac{H_0/L}{q_* + \dfrac{K_B}{K_T}\dfrac{a}{H_0} I_B}\right](1-y)\left[(1+y) - \frac{2a}{H_0}\left(1 - \frac{K_B}{K_T}\right)\right] \qquad \text{...(11)}$$

In the absence of initial gradient I_B, Equation (11) boils down to the known solution for Darcian flow due to Bear (1972). Typical phreatic surfaces for known values of embankment properties $\left(i\,e.,\ \frac{H_0}{L},\ K_B/K_T,\ \frac{a}{H_0}\right)$ and non-dimensional discharge (q_*) for various values of initial gradient I_B as given by Equation 11 are drawn and shown in Figure 5. Inserting boundary

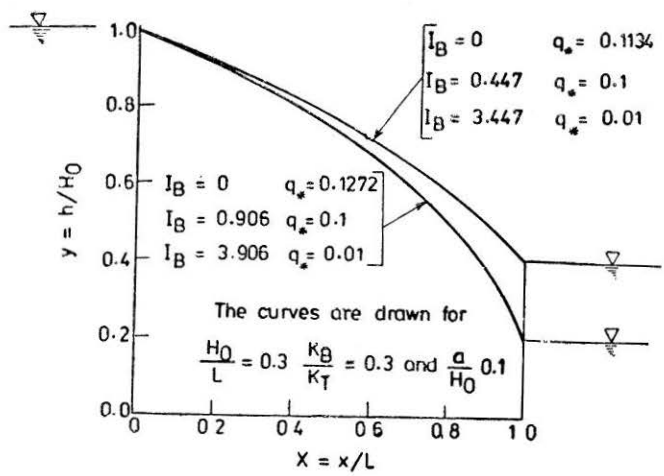

FIGURE 5 Typical phreatic surface showing the effect of initial gradient and subsequent discharge

condition (Equation 10) in Equation 11, expression for non-dimensional discharge (q_*) is obtained as

$$q_* = \tfrac{1}{2}\frac{H_0}{L}(1-y_0)\left[(1+y_0) - 2\frac{a}{H_0}\left(1 - \frac{K_B}{K_T}\right)\right] - \frac{K_B}{K_T}\frac{a}{H_0} I_B \qquad \text{...(12)}$$

Equation 12 can also be written as

$$q_* = \frac{K_B}{K_T}\frac{a}{H_0}(I_{av} - I_B) + I_{av}\left[\left(\frac{I_{av}}{2\,(H_0/L)} + \frac{H_L - a}{H_0}\right)\right] \qquad \text{...(13)}$$

where

$$I_{av} = \frac{H_0 - H_L}{L} = \frac{H_0}{L}(1-y_0) = \text{average gradient} \qquad ...(14)$$

The discharge as given by Equation 13 can be thought of made of two parts, the first part being discharge through the underlying initial gradient soil and the second part constitutes the discharge through the overlying porous medium without any initial gradient hence, Equation 13 can be rewritten as

$$q_* = q_{*B} + q_{*T} \qquad ...(15)$$

where

$$q_{*B} = \frac{K_B}{K_T}\frac{a}{H_0}(I_{av} - I_B) \qquad ...(16)$$

and

$$q_{*T} = I_{av}\left[\frac{I_{av}}{2(H_0/L)} + \frac{H_L - a}{H_0}\right] \qquad ...(17)$$

The non dimensional discharge q_{*B} and q_{*T} as given by Equations 16 and 17 are plotted against the average gradient, I_{av} for various values of initial gradients, I_B, permeability ratio, $\frac{K_B}{K_T}$, and the embankment geometry, $\frac{H_0}{L}$, for fixed values of $(H_L - a)/H_0 = 0.1$ and $\frac{a}{H_0} = 0.1$ and is shown in Figure 6.

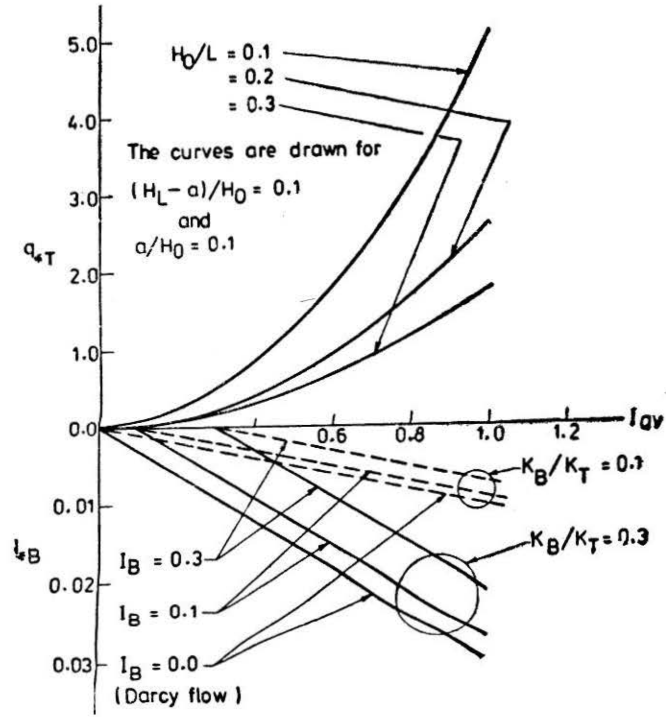

FIGURE 6 Non-dimensional discharge through top and bottom layers as a function of initial gradient I_B, permeability ratio K_B/K_T and various embankment geometry parameters

It is to be noted that the discharge through the underlying initial gradient soil is nil as long as $I_{av} \leqslant I_B$.

The conclusions and discussions of this problem will be made after solution of the second problem is presented.

Seepage Through Confined Aquifer of Variable Thickness

Figure 7 illustrates problem of uniformly sloping confined aquifer of thickness D_0 on the downstream side and D_L on the upstream side. At steady state, downstream water level is H_e and that of upstream is H. The aquifer has a length L and is resting on a impervious horizontal base. The steady state solution for the problem with constant aquifer thickness (*i.e.* $D_L/D_0 = 1$) obeying Darcian flow is available in standard text books (Harr, 1962). The effect of initial gradient on the same problem but again with constant aquifer thickness was also reported earlier (Valsangkar and Subramanya, 1972). It is intended to study here the effect of initial gradient, i_0 and the geometry parameter, D_L/D_0 on steady state discharge and piezometric pressure distribution.

Analysis

Choosing the origin on the downstream toe as shown in Figure 7.

$$v_x = q/D_x \qquad ...(18)$$

where

$$D_x = D_0 + \frac{D_L - D_0}{L} x \qquad ...(19)$$

in which v_x is the macroscopic velocity of flow and D_x is the thickness of aquifer at any distance x from the origin. Assuming Dupuit's assumption of horizontal flow line, the equation with initial gradient can be written as

$$v_x = k\left(\frac{dh}{dx} - i_0\right) \qquad ...(20)$$

Combining Equations 18, 19 and 20, the differential equation in non dimensional form is obtained as

$$\frac{H}{L}\frac{dy}{dX} = i_0 + \frac{q^*}{1+\left(\frac{D_L}{D_0}-1\right)X} \qquad ...(21)$$

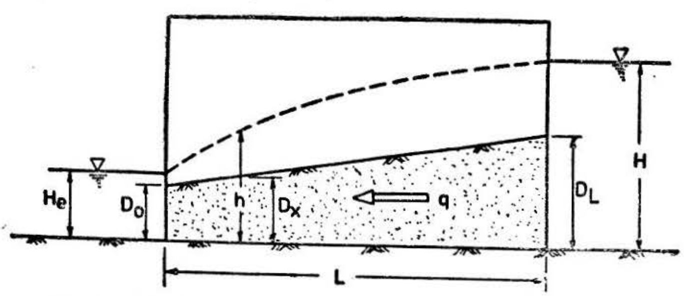

FIGURE 7 Definition sketch for the confined acquifer problem

where

$$X = x/L \qquad ...(22)$$

$$y = h/H \qquad ...(23)$$

and

$$q_* = q/KD_0 \qquad ...(24)$$

The boundary conditions of the problem are

$$x = 0, h = H_e \text{ or } X = 0, y = \frac{H_e}{H} = y_0 \qquad ...(25)$$

and

$$x = L, h = H \text{ or } X = 1, y = \frac{H}{H} = 1 \qquad ...(26)$$

The equation for piezometric surface can be obtained by integrating Equation 21 along with the boundary condition (Equation 25) and reads as

$$y = y_0 + \frac{1}{H/L}\left[i_0\, X + q_* \left\{ \frac{\log\left[1 + \left(\frac{D_L}{D_0} - 1\right) X\right]}{\left(\frac{D_L}{D_0} - 1\right)} \right\} \right] \qquad ...(27)$$

For constant aquifer thickness, Equation 27 in the limit $\frac{D_L}{D_0} \to 1$, leads to known equation (Valsangkar and Subramanya, 1972)

$$y = y_0 + \left\{ \frac{i_0 + q_*}{H/L} \right\} X \qquad ...(28)$$

Typical piezometric surfaces (Equation 27) for initial gradient, $i_0 = 0$, 1 and 2, for geometry parameter, $D_L/D_0 = 2$ with $y_0 = 0.3$ and non dimensional discharge $q_* = 1.0$ are drawn and shown in Figure 8. The linear piezometric surface (Equation 28) for constant aquifer thickness with $q_* = 1.0$ and $y_0 = 0.3$ is also shown in Figure 8.

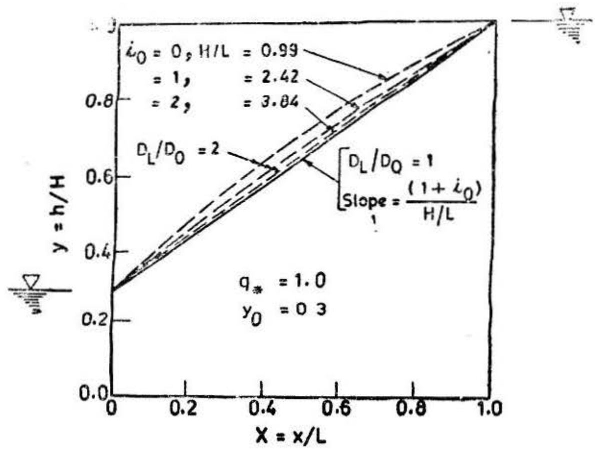

FIGURE 8 Typical piezometric surface for various initial gradients and D_L/D_0 ratios

The expression for non dimensional discharge q_* is obtained by inserting second boundary condition (Equation 26) into Equation 27 and is given by Equation 29.

$$q_* = \left\{ \frac{\frac{D_L}{D_0} - 1}{\log \frac{D_L}{D_0}} \right\} \left\{ \frac{H}{L} (1-y_0) - i_0 \right\} \qquad \ldots(29)$$

The Equation 29 can be rewritten as

$$q_* = T\,(I_{av} - i_0) \qquad \ldots(30)$$

where

$$T = \frac{\frac{D_L}{D_0} - 1}{\log \frac{D_L}{D_0}} \qquad \ldots(31)$$

and $$I_{av} = \frac{H}{L}(1-y_0) = \frac{H-H_e}{L} = \text{average gradient} \qquad \ldots(32)$$

The coefficient T may be termed as *transmission coefficient* for variable aquifer thickness. If the thickness of aquifer does not change then transmission coefficient T becomes equal to unity $\left(\text{in the limit } \frac{D_L}{D_7} \to 1\right)$. The variation of non-dimensional discharge q_* with average gradient I_{av} for various values of initial gradient i_0 and geometry parameter, $\frac{D_L}{D_0}$ as given by Equation 29 to 32 are plotted and shown in Figure 9.

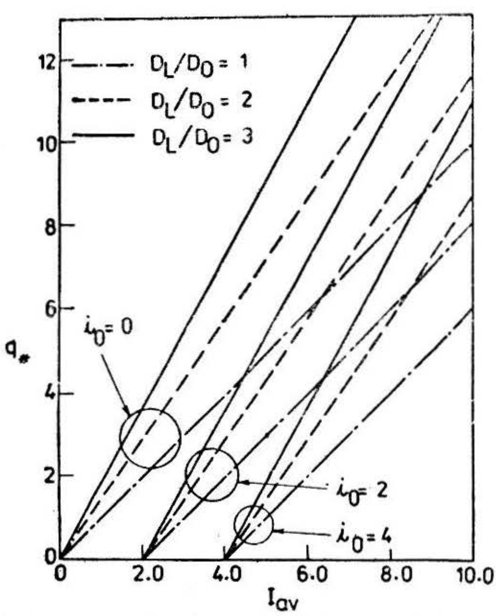

FIGURE 9 Non-dimensional discharge through the aquifer for various values of initial gradient, i_0 and geometry parameter, D_L/D_0

Equation 30 suggests that percentage error in discharge estimation for non recognition of initial gradient is directly proportional to the ratio i_0/I_{av}.

Discussions and Conclusions

From the perspective view on the subject of initial gradient as presented in the paper, the following conclusions can be drawn :

(1) The existence of initial gradient for clay and other surface active soils are more or less established whereas for sand and other inert material, further evidence are necessary to draw any meaningful conclusions.

(2) Available experimental data suggests that the magnitude of initial gradient increases with the decrease in void ratio (Figure 3).

(3) At a particular void ratio, the initial gradient is observed to increase with the increase in the percentage of clay fraction present in the porous media (Figure 2).

(4) There seems to be a great need of a sound mathematical model leading to a realistic velocity-gradient response of surface active soils showing the presence of surface forces and consequently the existence of initial gradient.

(6) The existing two hypotheses, explaining the phenomenon of initial gradient needs further close examination from experimental and theoretical point of view.

It is believed that the state of art presented and the conclusions drawn thereof would be of help to practicing engineers and research workers and would make them more aware about this abnormal phenomenon of initial gradient.

The consequences of the existence of initial gradient on two physical problems are analytically studied and the conclusions follow.

(A) Seepage Through Embankment Overlying an Initial Gradient Soil

The analysis shows that as the initial gradient of the underlying soil layer increases, a particular phreatic surface will yield less and less discharge (Equation 11 and Figure 5). In other words, to get the same discharge with higher initial gradient of the underlying soil, phreatic surface will have to go up. Typically for a phreatic surface represented by $y_0 = 0.2$, $H_0/L = 0.3$ and $a/H_0 = 0.1$ (Figure 5), the non-dimensional discharge q_* reduces by ten times for an initial gradient increase of approximately four times for permeability ratio of $K_B/K_T = 0.3$.

Equations 13, 15, 16, 17 and Figure 6 show that non-recognition of initial gradient of the underlying soil will overestimate the discharge through the embankment. Higher the initial gradient, higher will be the error in discharge estimation. Analysis also shows (Figure 6) that for all values of initial gradient of the underlying soil and the parameters a/H_0 and $(H_L—a)/H_0$, the increase in permeability ratio, K_B/K_T and the geometry factor, H_0/L will cause an increase the total seepage (q_*).

(A) Seepage Through Confined Aquifer of Variable Thickness

The analysis (Equations 27 and 28) shows that irrespective of the presence or absence of initial gradient, as long as the thickness of the aquifer remains same $\left(i.e. \frac{D_L}{D_0}=1 \right)$, the piezometric surface is linear, otherwise it is always curved. It is also observed that higher the initial gradient, lower will be piezometric head (Figure 8).

Similar to earlier problem, here also it is seen that non-recognition of initial gradient will overestimate the discharge (Equation 30, 31, 32 and Figure 9) and the flow will not start till the average gradient is more than the initial gradient. The percentage error in discharge estimation for non-recognition of initial gradient is directly proportional to the ratio of initial gradient to average gradient (*i.e.* i_0/I_{av}).

Essentially, the primary objective of the investigation has been to bring out the necessity of studying the concept of initial gradient in more detail and the necessity of its inclusion in the analysis of various physical problems where the likelihood of meeting initial gradient of some magnitude is strong. This is more so because the results can be at considerable variance with results based on purely Darcian flow without initial gradient. The analytical solutions of the two seepage problems considered and the representative numerical results thereof are believed to be useful in estimating the magnitude of error that may result in neglecting the presence of initial gradient.

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