Transient Seepage Through Soils Having Threshold Gradient

by

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Introduction

IN the field of irrigation and drainage one has to often compute the quantity of seepage through loamy and clayey soils. It is now well recognized that for many clayey soils and to some extent in sandy and silty soils, the seepage starts only when the hydraulic gradient exceeds a certain value called the threshold gradient or initial gradient. The velocity gradient response for such soils can be represented by the following equation:

$$v = K(i-I), \qquad i > I \qquad \ldots(1)$$

where v is the macroscopic seepage velocity; K is the proportionality constant or permeability coefficient, i is the hydraulic gradient and I is the threshold gradient or initial gradient. Kovacs (1967) reports the existence of initial gradient in case of flow of non-Newtonian fluids such as bentonite suspensions, oil water emulsions etc; through porous media. In experiments conducted by Li (1963) with Houston clay (clay content of 72 per cent with void ratio of 0.85), he found the initial gradient around 50. In case of flow of water through clayey soils, Polubrinvoa-Kochina (1962) found that the magnitude of initial gradient may vary from 15 to 20. For dense clays Scott (1963) quotes the value of initial gradient to be between 20 and 30. In experiments conducted by Roza (1958) measured values of initial gradient ranged from 0.2 to 0.5 for silts, and 12 to 18 for clays. Karadi and Nagy (1961) and Kodan (1967) have reported that the initial gradient increases with the decrease in void ratio or water content for fully saturated soils. The experimental data showing the variation of initial gradient with water content is shown in Table 1.

Initial gradient also depends upon such factors as the mineral composition and absorption complex of the soil, temperature etc.

Predominance of surface forces over the gravity forces in the case of clayey and other fine grained soils is said to be the cause of existence of initial gradient. These forces being strong enough to counter balance a certain portion of the applied hydraulic gradient are called the initial gradient (Miller and Low, 1963). It is known that higher the specific surface, the higher will be the surface forces. As such it is natural to expect increase

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TABLE I

Variation of Initial Gradient of Clayey Soil with Water content (After Karadi & Nagy 1961)

Water content per cent	Initial or threshold gradient I
29.1	22.8
32.2	12.6
34.5	7.3
39.3	3.1
42.1	0

in the magnitude of initial gradient with the decrease in the grain size and porosity.

The consequences of existence of initial gradient are of great interest in several areas such as suitable spacing of trenches or tile drains; soil water movement to plant roots; and consolidation of clayey soils.

Recognising the existence of initial gradient, many researchers in the past few years have investigated various physical problems. Polubrinova-Kochina (1969), Valsangkar and Subramanya (1972), Arumagam (1975) studied the effect of initial gradient on various physical problems of interest in the field of irrigation and drainage. Florin (1951), Roza and Kotov (1958), Girault (1960), Elnaggar et al. (1971) and Parlange (1973) investigated the role of initial gradient on the one dimensional consolidation of clayey soils by vertical drainage. Entov (1967) using unit hydrograph has presented the analytical method for solving two dimensional problem with initial gradient.

The problem of gravity drainage for a homogeneous soil medium due to Darcian flow has been solved by Kezdi (1974). The above problem for non-Darcy Flow ($v = Ki^m$; m being the nonlinear index), has been solved by Soni and Basak (1976). In this paper the same gravity drainage problem has been solved for a single and multilayered soil medium with initial gradient.

Drainage by Gravity in Soils with Initial Gradient

Consider a homogeneous soil medium of depth d, as shown in Figure 1; soil is having initial gradient. In problems of drainage by gravity the important question arises of the rate at which the process takes place. This problem can be dealt with the simplifying assumption that in a soil medium the drained zone is separated by a distinct boundary from the zone of capillary saturation.

Further it is assumed that the surface of ground water is at level with the ground at time $t = 0$. It is assumed that the drainage by gravity is

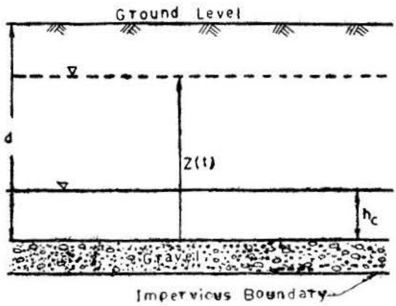

FIGURE 1. Definition sketch

occurring only in the vertical direction. As mentioned earlier, the problem of gravity drainage in case of homogeneous soil, when the flow follows Darcy's law has been solved by Kezdi (1974). It is intended to find out the effect of initial gradient on transient movement of free water surface.

As the drainage proceeds the upper level of the saturated zone drops continuously, but the piezometric head at the changing boundary condition remains constant and negative and equal to capillary head, h_c throughout. Thus when the top of the saturated zone is at a height Z the hydraulic head is $(Z-h_c)$ and the hydraulic gradient

$$i = \frac{Z-h_c}{Z} \qquad \ldots(2)$$

It is clear from Equation 2 that the gradient in the case of flow under gravity shall never exceed unity. The macroscopic velocity of flow will be given by

$$v = -\frac{dZ}{dt}\eta\,(1-S) \qquad \ldots(3)$$

where η = effective porosity;

S = degree of saturation *after* completion of drainage.

The velocity of flow in the case of homogeneous soil medium with initial gradient I, shall be governed by the Equation 1 *i.e.*

$$v = K\,(i-I) \qquad i > I \qquad \ldots(4)$$

Combining Equations 2, 3 and 4 one obtains:

$$\frac{dZ}{dt} = -\frac{K}{\eta(1-S)}\left[\frac{Z-h_c}{Z}-I\right] \qquad \ldots(5)$$

Equation 5 represents the governing differential equation for the problem of drainage by gravity in initial gradient soils.

Integrating Equation 5 and solving for the boundary condition;

at $\qquad t = 0, \qquad Z = d; \qquad \ldots(6)$

one obtains

$$\frac{Kt}{\eta(1-S)} = \frac{d-Z}{(1-I)}+\frac{h_c}{(1-I)^2}\log\left[\frac{h(1-I)-h_c}{Z(1-I)-h_c}\right] \qquad \ldots(7)$$

Rearranging Equation 7 and writing it in non-dimensional form

$$\tau = \frac{\bar{h}-\bar{Z}}{(1-I)} + \frac{1}{(1-I)^2} \log \left[\frac{1-\bar{h}(1-I)}{1-\bar{Z}(1-I)}\right] \qquad \ldots(8)$$

in which

$$\left.\begin{aligned} \tau &= \frac{K\,t}{\eta(1-S)h_c} \\ \bar{h} &= \frac{d}{h_c} \\ \bar{Z} &= \frac{Z}{h_c} \end{aligned}\right\} \qquad \ldots(9)$$

Multilayered medium

Soil medium consisting of different layers each having different permeability coefficient and different initial gradient can be converted into a hydraulically equivalent system of a single layer with equivalent permeability coefficient and equivalent initial gradient. Under such condition one need to know the expressions for equivalent permeability coefficient and equivalent initial gradient in the directions parallel and perpendicular to the stratification.

Equivalent Permeability Coefficients

In case of flow through soils with initial gradient the linear velocity response curve shall not pass through the origin but gets shifted to the right of the origin with the starting point of the curve lying on the x-axis at $i = I$. So the slope of the $v-i$ curve, i.e. K remains unaltered. Therefore the permeability coefficient of the soil is not affected by the absence or presence of initial gradient. The expression for equivalent permeability coefficients in case of multilayered soils with initial gradient is exactly the same as in the case of Darcy flow (Harr, 1962) i.e.

$$K_{HD} = \frac{1}{d} \sum_{i=1}^{n} (K_i\, d_i) \qquad \ldots(10)$$

and

$$K_{VD} = \frac{d}{\sum_{i=1}^{n} \left(\frac{d_i}{K_i}\right)} \qquad \ldots(11)$$

where K_{HD} and K_{VD} are the equivalent permeability coefficients in horizontal and vertical directions respectively for n-layered system with the depth of soil medium as d.

Equivalent initial gradient in vertical direction

In this case the same quantity of flow shall occur in each layer, whereas the total head loss h, is the sum of head losses in each layer. Considering a unit area perpendicular to the direction of flow one can write:

$$q_v = K_1\left(\frac{h_1}{d_1}-L_1\right) = K_2\left(\frac{h_2}{d_2}-I_2\right) = \ldots = K_n\left(\frac{h_n}{d_n}-I_n\right) \quad \ldots(12)$$

in which q_v = rate of flow discharge in vertical direction; and h_1, h_2 ..., h_n are the head losses in different layers. Considering the flow in n-layered system collectively, and recalling that the equivalent permeability coefficient in this case shall be the same as in Darcian case, one can write

$$q_v = K_{VD}\left(\frac{h}{d}-I_V\right) \quad \ldots(13)$$

in which I_V is the equivalent initial gradient through n-layered system in vertical direction. Noting that $h = h_1+h_2+\ldots h_n$ and making use of Equations 12 and 13 one gets,

$$K_{VD} = \frac{q_V\, d}{q_V\left(\frac{d_1}{K_1}+\frac{d_2}{K_2}+\ldots+\frac{d_n}{K_n}\right)+\left(I_1d_1+I_2d_2+\ldots I_nd_n\right)-I_V\cdot d} \quad \ldots(14)$$

From Equation 14 the expression for equivalent initial gradient can be obtained as

$$I_v = \frac{\sum_{i=1}^{n} (I_i\, d_i)}{d} \quad \ldots(15)$$

when initial gradient for each layer is the same i.e. $I_1 = I_2 = I_n = I$ then Equation 15 reduces to

$$I_V = I \quad \ldots(16)$$

Expressions for initial gradient in horizontal direction can be derived in a similar manner and the final result is given by Equation 17.

$$I_H = \frac{\sum_{i=1}^{n} (I_i\, d_i\, K_i)}{K_{HD}\, d} \quad \ldots(17)$$

When the multi-layered system has been made hydraulically equivalent to homogeneous medium, the solution for vertical drainage is identical with the single layer system i.e. Equation 8 but for the fact I and K is replaced by I_V and K_{VD}.

Equation 8 has been plotted for various values of I_V and for a particular value of $d/h_c = 10$ and is shown in Figure 2.

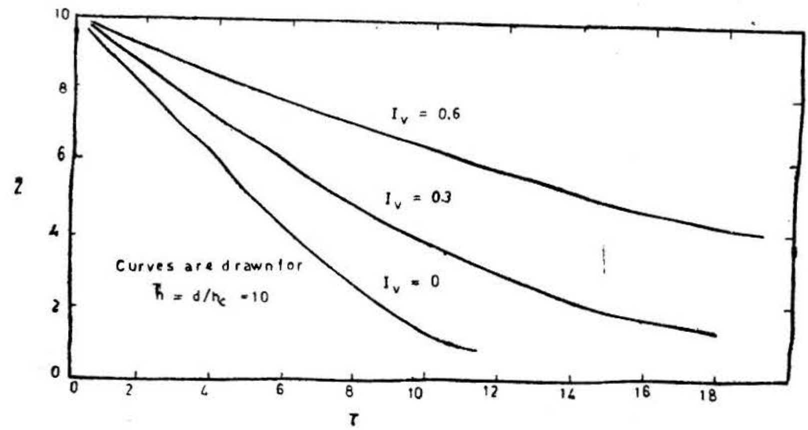

FIGURE 2. Progress of drainage in initial gradient soils

Conclusion

In the case of drainage of saturated soils by gravity it is seen from Equation 8 that as the values of I increase, the process of drainage becomes slower. In other words, if the effect of initial gradient is not accounted for, the time for the drainage to take place upto a certain level is underestimated. Further it is seen from Equation 8 that in case of initial gradient soils the time required to reach the complete drainage in case of gravity drainage is finite, whereas in the absence of initial gradient the solution given by Kezdi predicts infinite times for 100 per cent drainage.

The equivalent initial gradient through layered soil system in the direction of stratification and perpendicular to stratification can be determined from Equations 15 and 17 respectively. These values of initial gradient would be useful in many flow problems involving multilayer soil system.

Notations

a, b = coefficients in Forchheimer's relation

d = total thickness of the porous media

d_1, d_2 = thickness of the individual layers

h = total loss of head of water

h_1, h_2 = head losses through individual layers

h_c = capillary head

I_V = equivalent initial gradient in vertical direction for layered media

I_H = equivalent initial gradient in horizontal direction for layered media

I_1, I_2 = initial gradient for various layers

i = hydraulic gradient = $\partial h / \partial x$

K_1, K_2 = permeability coefficients of different layers

K = proportionality constant or permeability coefficient

K_{HD} = equivalent permeability coefficient in horizontal direction for Darcian flow

K_{VD} = equivalent permeability coefficient in vertical direction for Darcian flow

q_H = rate of flow discharge in horizontal direction

q_v = rate of flow discharge in vertical direction

v = macroscopic seepage velocity

L = length of soil in the horizontal direction

S = degree of saturation after the completion of drainage

t = time

m = index

n = index

η = effective porosity.

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