A Comparative Study of the Theoretical Basis for Strength Anisotropy Proposed by Hansen and Gibson and Bjerrum

by

R. Bhaskaran*

Introduction

Bjerrum in 1973 proposed a theoretical description for strength anisotropy in normally consolidated clays apparently as an alternative to the one proposed in 1949 by Hansen and Gibson beeause according to him the range of variation in undrained shear strength predicted by the latter theory is relatively small compared to the experimental findings published in recent years. The author carried out a critical evaluation of the two approaches and has come to a conclusion that the one proposed by Hansen and Gibson in 1942, as modified by Duncan and Seed (1966*a*), is more versatile, simpler and perhaps gives a more reasonable prediction of the directional variation in strength than Bjerrum's theory proposed about 25 year later.

Theoretical Basis for Strength Anisotropy in Normally Consolidated Soil of Hansen and Gibson

Hansen and Gibson (1949) proposed a theoretical basis to show that a clay which is normally and anisotropically consolidated in nature has a strength dependent on the orientation of the failure plane. This theory is based on grossly simplified assumptions and employed the λ *theory* proposed by Skempton (1948). The theory was derived for resistance to shear mobilised along different planes in the ground due to the insitu stress anisotropy. However, since cohesive soils lock within itself the stress anisotropy effect (Poulos and Davis, 1972) the anisotropy will reflect itself even in the tests conducted on extracted samples (Bhaskaran, 1975*b*) provided physical disturbance is a minimum (see also Duncan and Seed, 1966*b*). In any case if tests are conducted simulating ground stress conditions the influence of tress anisotropy will certainly be revealed. Hansen and Gibson's theoretical description for undrained strength which could be mobilised on a failure plane with any orientation, in terms of initial stress conditions and the Hvorslev parameters is based on the effect of reorientation of principal stresses. Duncan and Seed (1966*a*) have given an excellent treatment of the effect of reorientation of principal stresses. Duncan and Seed (1966*a*) have also rewritten Hansen and Gibson's expression in terms of the more familiar pore pressure parameter $\bar{A}_f$ as follows.

$$\frac{c_u}{p}=\frac{c_e}{p}\text{ Cos }\varphi_e+\tfrac{1}{2}\left(1+K_o\right)\text{Sin }\phi_e-\text{Sin }\phi_e\left(2\bar{A}_f-1\right)$$
$$\times\left[\left(\frac{c_u}{p}\right)^2-\frac{c_u}{p}\left(1-K_o\right)\text{Cos }2\left(45^\circ+\frac{\phi_e}{2}-\alpha\right)+\left(\frac{1-K_o}{2}\right)^2\right]^{\frac{1}{2}} \quad ...(1)$$

* Dept. of Civil Engineering, Regional Engineering College, Calicut, India (on deputation to Union Public Service Commission, New Delhi).

This paper was received in September, 1977 and is open for discussion till the end of June 1978.

where

c_u = undrained shear strength

p = consolidation pressure

c_e = true cohesion-Hvorslev parameter

ϕ_e = true angle of shearing resistance-Hvorslev parameter

K_o = co-efficient of earth pressure at rest

$\bar{A}_f$ = Skempton's pore pressure parameter A at failure

α = angle between failure plane and insitu horizontal.

The basic assumption made by Hansen and Gibson (1949) is that Hvorslev parameters c_e and ϕ_e and the compressibility ratio 'λ' (pore pressure parameter $\bar{A}_f$ in the above expression) are independent of the orientation of the failure plane. If the angle of inclination of the failure plane with the base of the specimen is assumed to be independent of the orientation (i.e. direction of action of the major principal stress at failure with respect to the insitu vertical) of the specimen, then this equation can be used to obtain the value of c_u/p corresponding to any particular orientation by substituting in the equation the appropriate value of α.

It would be worthwhile to study the extent to which the variation in the paremeters c_e, ϕ_e and $\bar{A}_f$ influence the directional variation in strength as predicted by this equation. Equation 1 has been rearranged into quadratic form, programmed and solved by means of a computer to obtain the variation in strength with respect to direction for various possible values of c_e/p, ϕ_e and $\bar{A}_f$ for a constant $K_o = 0.5$, assumping that the failure plane makes the same angle with the base of the specimen irrespective of the orientation (i.e. the direction of action of the major principal stress at failure in relation to the insitu vertical) of the specimen. The assumed values of c_e/p, ϕ_e and $\bar{A}_f$ are as given in Table 1. α is assumed as equal to 60°.

TABLE 1

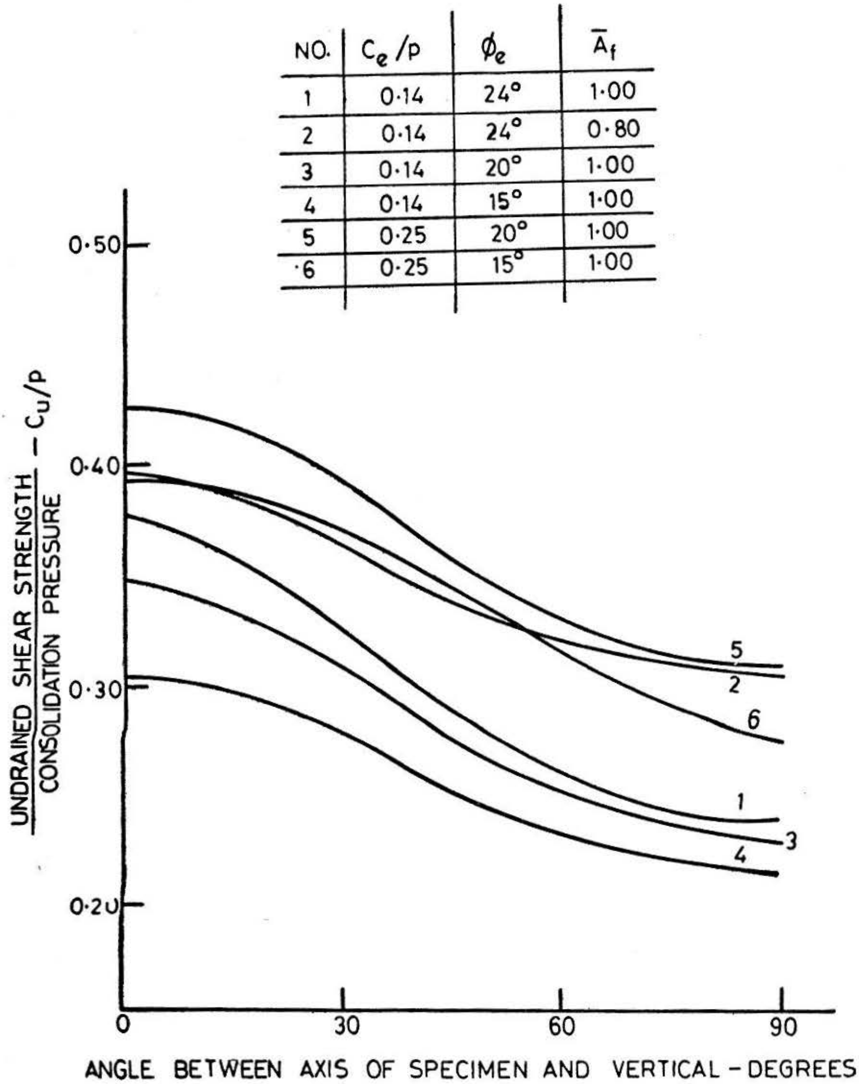
Assumed Values of Parameters and Resulting Strength Ratios

	c_e/p	ϕ_e	$\bar{A}_f$	S_{uh}/S_{uv}
Trial 1	0.14	24°	1.00	0.62
Trial 2	0.14	24°	0.80	0.76
Trial 3	0.14	20°	1.00	0.65
Trial 4	0.14	15°	1.00	0.69
Trial 5	0.25	20°	1.00	0.71
Trial 6	0.25	15°	1.00	0.76

The values of c_e/p adopted are on the higher side considering most test results, however, values as high as 0.4 have also been reported (Lo, 1962). Wide range of values of ϕ_e up to 40° have also been reported in the literature. The variation in the ratio of the undrained shear strength of horizontal specimen to that of the vertical specimen, hereafter called as the strength ratio S_{uh}/S_{uv} predicted by Equation 1 is also given in Table 1. The pattern of variation in strength with respect to direction for the different cases is given in Figure 1.

It can be seen from Figure 1 that though there is a variation in the absolute values of c_u/p at any orientation for the different cases, the pattern of variation predicted is the same with maximum strength for vertical specimens and minimum strength for horizontal specimens and specimens at intermediate orientations having intermediate values of c_u/p.

It may be seen from Table 1 that a decrease in $\bar{A}_f$ decrease in ϕ_e and increase in c_e/p result in an increase in the strength ratio S_{uh}/S_{uv}. The ratios S_{uh}/S_{uv} ranging from 0.62 to 0.76 observed for the different cases in the Table is very nearly equal to what has been reported for the different



NO.	C_e/p	ϕ_e	$\bar{A}_f$
1	0·14	24°	1·00
2	0·14	24°	0·80
3	0·14	20°	1·00
4	0·14	15°	1·00
5	0·25	20°	1·00
6	0·25	15°	1·00

FIGURE 1 : Directional strength variation - Normally consolidated soil (theoretical) Influence of C_e/p, ϕ_e and $\bar{A}_f$ $(K_o = 0.5)$

normally and lightly over consolidated soils by different investigators (Bhaskaran, 1975*a*, 1975*b*). For example, Khera and Krizek (1969) have reported a value of this ratio of equal to 0.59 for a laboratory prepared sample of Grundite. D'Appolonia (1972) has observed a value of this ratio equal to 0.6 for the Boston blue clay in the normally consolidated state and Lo (1965) and Parry and Nadarajah (1974) have observed a strength ratio ranging from 0.64 to 0.80 and 0.83 respectively for the Welland clay and the Fulford deposit respectively which are lightly over consolidated (over consolidation ratio approximately equal to 2.0). The comparison is remarkable considering the fact that the parameters c_e/p, ϕ and $\bar{A}_f$ may very well be different in these clays from what has been used in the analysis.

Bjerrum's theoretical approach to strength anisotropy

Bjerrum (1973) proposed a theoretical description of anisotropy which can be expected in a normally consolidated clay, based on a hypothesis proposed earlier by Bjerrum and Kenney (1967) that strength anisotropy depends upon the magnitude and direction of action of the externally applied shear stress with respect to the magnitude and direction of action of the shear stress on different planes insitu. If a clay is brought to failure under undrained conditions along a plane which carried a shear stress, the shear stress at which the clay fails will only amount to the sum of effective friction already mobilised by the initial stresses and the available effective cohesion.

The critical shear stress on a certain plane to cause failure has been stated to be as follows:

$$\tau_{cr}/p_o = \tau_\alpha/p_o + (\sigma_\alpha/p_o \tan\phi_e - \tau_\alpha/p_o)\, D_M + \kappa p_c/p_o \qquad \ldots(2)$$

for the active case

$$\tau_{cr}/p_o = (\sigma_\alpha/p_o \tan\phi_e - \tau_\alpha/p_o)\, D_M + \kappa\, p_c/p_o \qquad \ldots(3)$$

for the passive case (see editor's foot note in the paper of Bjerrum reprinted by Norwegian Geotechnical Institute-NGI Pn. No. 100).

where

τ_{cr} = critical shear stresses at which failure takes place on a plane inclined at α to the horizontal

p_o = effective overburden pressure

σ_α = normal stress on any plane inclined at α to the horizontal

ϕ_e = angle of effective internal friction

τ_α = shear stress on any plane inclined at α to horizontal

D_M = a parameter describing how large a fraction of effective frictional resistance mobilised in excess of τ_α

κ = c_e/p_e

p_c = preconsolidation pressure arising out of 'aging'

Bjerrum has clearly stated that this expression is valid only to predict the strength variation in clays which have cohesive properties large enough to give them a brittle behaviour in undrained shear, so that they fail at small strain before there is any tendency for significant volume change. He has given test data from simple shear tests, using specimens reconsolidated to the same shear stress as insitu for three typical clays failed in the 'active' and 'passive' cases (shear stress in the same direction as the original shear stress and shear stress increased in the opposite direction to original shear stress) and compared it with the theoretical curve (the parameter D_M in the theoretical expression being assumed such that the Equation 2 gave a correct prediction for shear strength on a horizontal plane where $\tau_\alpha = 0$ and $\alpha = 0$). Excellent agreement was noticed for the active case but the agreement in the passive case was unsatisfactory. This was explained as due to the fact that failure in the passive case took place at larger strains.

A critical examination of the approaches of Hansen and Gibson and Bjerrum

Bjerrum proposed this theory apparently because, as stated by him (Bjerrum, 1973), Hansen and Gibson's theory predicts lower anisotropy than what was observed from insitu tests either using vanes of different shapes or insitu direct shear tests and tests on samples reconsolidated anisotropically in the laboratory. It is worth observing that there is disagreement in the ratio of maximum to minimum strength observed for the three types of tests reported by Bjerrum. For instance, referring to his Table 3, (Bjerrum, 1973), it may be seen that the ratio of maximum to minimum strength obtained by the vane test, insitu direct shear test and triaxial test (compression and extension) are 1.5, 3.75 and 2.25 respectively for the same Manglerud quick clay.

It will be shown little later, that both the pattern and range of variation in strength with respect to direction observed in at least two clays reported by Bjerrum, using the technique of reconsolidation of specimens to the same insitu stress conditions in the direct sample shear apparatus, can be obtained by means of Hansen and Gibson theory.

In order to examine clearly Bjerrum's theoretical approach it may be worthwhile to examine the basis on which it has been derived. Bjerrum and Kenney (1967) had suggested, on the basis of field direct shear tests carried out, that anisotropy in strength is dependent upon the magnitude and direction of action of the shear stress causing failure in relation to the magnitude and direction of action of the shear stress existing on these planes. In the field shear test conducted they had observed a wide variation in the strength mobilised on 45° planes under two conditions, namely when shear was applied downwards and when shear was applied upwards. When shear is applied upwards the major principal stress at failure will be approximately horizontal and when shear is applied downwards the major principal stress will be approximately vertical. Therefore, purely from consideration of principal stresses at failure, the strength is higher when the major principal stress at failure acts in a vertical direction and lower when the major principal stress acts in a horizontal direction. This is in fact exactly the pattern of variation predicted by Hansen and Gibson (1949) for normally consolidated soil. It may also be stated that the mechanism of failure (and the stress conditions during shear) is far too complex in the field shear apparatus considering that it is complex even in the laboratory direct shear test where failure is induced on a horizontal plane.

Taking the case of the results of the experiments conducted by Soydemir, which has been used by Bjerrum (1973) in support of his theoretical approach, it may be seen that if orientation of principal stresses are considered Soydemir's data may describe a pattern of variation predicted by Hansen and Gibson's theory. The major principal stress at failure in a direct simple shear test is likely to be inclined at approximately 60° to horizontal (Duncan and Dunlop, 1969). Therefore, from consideration of the direction of action of major principal stress at failure, the major principal stress at failure will act in a vertical direction (in terms of insitu vertical direction) in the 30° specimens of Soydemir (active case) and in the horizontal direction (insitu horizontal direction) in the 60° specimens (passive case), tested in the direct simple shear apparatus. Figure 15 of Bjerrum (1973) reveals that the 60° and 30° specimens respectively have given the lowest and the highest values of strength just as what is predicted by Hansen and Gibson's theoretical approach.

It can now be shown that Hansen and Gibson's theoretical prediction of the ratio of maximum by minimum strength for at least two of the three clays tested by Soydemir (as reported by Bjerrum, 1973) is close to the experimental ratios, than that predicted by Bjerrum's theoretical approach.

Table 2 gives the values of maximum and minimum values of τ_{cr}/p_o and their ratios (experimental or theoretical values) for the Ska-Edeby clay, Drammen plastic clay and Drammen lean clay presented in Figure 15 of Bjerrum (1973). Table 3 gives the values of the maximum and minimum values of strength in terms of c_u/p and their ratios for various clays with certain assumed values of c_e/p, ϕ_e, K_o and $\overline{A_f}$ obtained from Equation 1.

TABLE 2

Maximum and Minimum Values of τ_{cr}/p_o and Their Ratios (Theoretical and Experimental from Figure 15 of Bjerrum, 1973)

	Critical shear stress/p_o					
	Theoretical			Experimental		
	Max.	Min.	Ratio	Max.	Min.	Ratio
Ska-Edeby clay $D_M = 0.60$, $\kappa = 0.14$ $K_o = 0.75$, $\phi_e = 11°$	0.30	0.21	1.43	0.30	0.25	1.20
Drammen plastic clay $D_M = 0.60$, $\kappa = 0.10$ $K_o = 0.60$, $\phi_e = 16°$	0.38	0.15	2.54	0.38	0.22	1.73
Drammen Lean clay $D_M = 0.35$, $\kappa = 0.01$ $K_o = 0.55$, $\phi_e = 27°$	0.28	0.08	3.50	0.28	0.08	3.50

Bjerrium has clearly stated that anisotropy is greater the lower the K_o value, the lower the κ p_c/p_0 and the larger the tan ϕ_e which is borne out by the test results. An examination of Table 3 reveals that Hansen and Gibson theory also predicts a greater anisotropy, the lower the K_o value, the lower the value of c_e/p and the higher the value of tan ϕ_e.

TABLE 3

Maximum and Minimum Values of c_u/p and Their Ratios as per Hansen and Gibson Theory (Equation 1)

c_e/p	ϕ_e	K_0	$\bar{A}_f$	c_u/p Maximum	c_u/p Minimum	c_u/p Ratio
0.00	15°	0.5	1.00	0.168	0.103	1.63
		0.7	0.80	0.208	0.170	1.22
	24°	0.5	1.00	0.286	0.144	1.99
		0.7	0.80	0.306	0.248	1.23
	30°	0.5	1.00	0.333	0.166	2.00
		0.7	0.80	0.361	0.292	1.24
0.14	15°	0.5	1.00	0.305	0.211	1.45
		0.7	0.80	0.326	0.287	1.14
	24°	0.5	1.00	0.378	0.235	1.61
		0.5	0.80	0.396	0.299	1.32
		0.6	1.00	0.379	0.264	1.44
		0.7	1.00	0.379	0.293	1.29
		0.7	0.80	0.409	0.351	1.17
	30°	0.5	1.00	0.414	0.247	1.67
		0.7	0.80	0.454	0.385	1.18

It is particularly noticed from Table 3 that $\bar{A}_f$ has a significant influence on strength anisotropy and higher anisotropy is observed higher the $\bar{A}_f$ value particularly at lower values of K_0.

Comparing the experimental values of the ratios reported by Bjerrum (Table 2) with the values of the ratio predicted by Hansen and Gibson's theory (Table 3) it can be seen that for Ska-Edeby clay ($K_0 = 0.75$, $\phi_c = 11°$ and $\kappa = 0.14$) the ratio of maximum to minimum strength of 1.20 observed is in close agreement with the predicted value of the ratio for a clay with $\phi_e = 15°$, $K_0 = 0.7$ and $\bar{A}_f = 1.00$, of 1.14. Information regarding $\bar{A}_f$ for this clay has not been furnished but it is likely that it is higher than 1.0 (as usually all sensitive clays have high $\bar{A}_f$ values). Therefore, in spite of the lower ϕ_e of the Ska-Edeby clay the higher $\bar{A}_f$ values would have resulted in a prediction by Hansen and Gibson's approach closer to the experimental value of the ratio of 1.20.

Similarly for the Drammen plastic clay also the predicted value of the strength ratio by Hansen and Gibson's approach is closer to the experimental values than the ratios predicted by Bjerrum's approach. Only for the case of Drammen lean clay is the predicted value of strength ratio by Bjerrums theoretical approach in agreement with the experimental values. Here again if the value of $\bar{A}_f$ is available it might have been possible to show that the experimental value of the ratio could be predicted by Hansen and Gibson's theory also.

Conclusions

The above stated facts indicate that Hansen and Gibson's theory, in spite of its simplicity and limitations is reasonably reliable for predicting the directional variation in strength in normally consolidated soils. Bjerrum's theory on the other hand is strictly applicable only to clays which will fail at small strains. Besides the parameter D_M in the Bjerrum's theoretical equation has to be determined experimentally by conducting a direct simple shear test on a specimen with $\tau_\alpha = 0$ and $\alpha = 0$. It is far easier to use Hansen and Gibson's expressions in the modified form as given in Equation 1, because only material properties are involved—the results are also represented in terms of the direction of action of principal stresses at failure with respect to the initial consolidation stresses.

References

BHASKARAN, R. (1975*a*), "Variability in Strength and Deformation Characteristics of Anisotropic Clays", *Analysis of Soil Behaviour and its Application to Geotechnical Structures, Proc. Tech. Session of Symp. held at Univ. N.S.W., Australia,* Editors S. Valliappan, S. Hain and I.K. Lee, pp. 289-301.

BHASKARAN, R. (1975*b*). "A Study on Anisotropic Strength and Stress—Deformation Behaviour of Cohesive Soils", *Ph.D. Thesis,* Ind. Inst. of Tech., Madras, India.

BJERRUM, L. (1973). "Problems of Soil Mechanics and Construction on Soft Clays, State-of-the-Art-Report", *Proc. 8th Int. Conf. Soil Mech. and Fdn. Engrg.,* Moscow, Vol. 3, pp. 111-159.

BJERRUM, L and KENNY, T.C. (1967). "The Effect of Structure on the Shear Behaviour of Normally Consolidated Quick Clays, *Proc. Geotech. Conf.,* Oslo, Vol. 2, pp. 19-27.

D'APPOLONIA (1972). Discussion on Bearing Capacity of Anisotropic Cohesive Soil by E.H. Davis and Christian, J.K., *Jl. of Soil Mech. and Fdn. Div., Proc. ASCE,* Vol. 98, SM 1, pp. 126-132.

DUNCAN, J.M. and SEED, H.B. (1966 *a*). "Anisotropy and Stress Reorientation in Clay", *Jl. of Soil Mech. and Fdn. Div., Proc. ASCE,* Vol. 92, SM 5, pp. 21-50.

DUNCAN, J.M. and SEED, H.B. (1966 *b*). "Strength Variation Along Failure Surface in Clay", *Jl. of Soil Mech. and Fdn. Div., Proc. ASCE* Vol. 92, SM 6, pp. 81-104.

DUNCAN, J.M. and DUNLOP, P. (1969). "Behaviour of Soils in Simple Shear Test", *Proc. 7th Int. Conf. Soil Mech. and Fdn. Engrg.,* Mexico, Vol. 1, pp. 101-109.

HANSEN, J.B. and GIBSON, R.E. (1949). "Undrained Shear Strength of Anisotropically Consolidated Clays", *Geotechnique,* Vol. 1, No. 3, pp. 189-204.

KHERA, R.P. and KRIZEK, R.J. (1969). "Strength Anisotropy in Cohesive Soils", *Journal of Soil Mech. and Fdn. Engrg., Ind. Nat. Soc. of Soil Mech. and Fdn. Engrg.* Vol. 8, No. 1, pp. 41-55.

LO, K.Y. (1962). "Shear Strength Properties of a Sample of Volcanic Material of the Valley of Mexico," *Geotechnique* Vol. 12, p. 303.

LO, K.Y. (1965). "Stability of Slopes in Anisotropic Soils", *J. of Soil Mech. and Fdn. Div., Proc. ASCE,* Vol. 91, SM 4, pp. 85-106.

PARY, R.H.G., and NADARAJAH, V. (1974). "Anisotropy in a Natural Soft Clayey Silt", *Engineering Geology,* Amsterdam, Vol. 8, pp 287-309.

POULOS, H.G. and DAVIS, E.H. (1972). "Laboratory Determination of Insitu Horizontal Stress in Soil Masses", *Geotechnique,* Vol. 22, No. 1, pp. 177-183.

SKEMPTON, A.W. (1948). "A Study of the Immediate Triaxial Test on Cohesive Soils", *Proc. 2nd International Conf. Soil Mech. and Fdn.,* Rotterdam, Vol. 2, pp. 50-53.