Evaluation and Use of Flow Functions in Predicting Vertical Infiltration of Water at Different Soil Bulk Densities

by

A.K. Sharda*

R.P. Gupta**

Introduction

Water moves continuously through the soil mass to remain in equilibrium with the prevailing environmental conditions. The rate at which water moves through soil has been quantified in a number of measurable indices such as infiltration rate, hydraulic conductivity, soil water diffusivity and capillary rise. Amongst the various factors responsible for influencing soil water movement, soil bulk density plays a predominent role as it primarily determines the total porosity and poresize distribution in a soil.

To make an economic and efficient use of our limited water resources available for irrigation, it becomes imperative to know in advance a general pattern of soil water distribution in response to a given quantity of irrigation under a given set of physico-chemical conditions of a soil. To solve this problem, Philip (1955, 1957) introduced the theory of unsaturated flow wherein he developed various flow functions facilitating the computation of soil water movement quantitatively. In our country, however, scantly attention has been paid to this aspect of soil water movement. Keeping this objective in view, an attempt, therefore, has been made in the present investigation to evaluate various flow functions, as proposed by Philip, at different soil bulk densities of a silty clay loam from Jabalpur, and to subsequently use them in the computation of soil water distribution, infiltration rate and cumulative influx; and also to study, in general, the influence of soil bulk density on vertical infiltration. The experiments have been conducted under closely controlled conditions in view of complexities involved in the phenomenon of soil water movement. The experimental results obtained have been compared with the computed ones to elucidate the usefulness of flow functions.

Theory

In the case of vertical infiltration water moves not only due to differences in soil water potential, but also due to the presence of external force field (gravitational), which results into increased flux at the soil surface. The change in water content of a semi-infinite porous medium, when subjected to a continuous flow may be described by the following equation for vertical flow.

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[D \frac{\partial \theta}{\partial x} \right] - \frac{\partial K}{\partial x} \qquad \ldots(1)$$

*Pool Officer, C.S.M.R.S., Near I.I.T. Hostel, New Delhi-110029.

**Project Co-ordinator (Soil Structure Scheme) Division of Agricultural Physics, I.A.R.I., New Delhi.

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subject to following boundary conditions,

$$\theta = \theta_n \quad \text{for } t = 0, x > 0 \qquad \ldots(2)$$

$$\theta = \theta_o \quad \text{for } t > 0, x = 0 \qquad \ldots(3)$$

Where D and K are single valued functions of soil water content θ; and x and t represent vertical distance, and time, respectively.

Philip (1957) defined various residual and approximate residual functions to improve the estimates for the prediction of soil water distribution after a known period of infiltration. Some of the functions introduced by him are denoted by the symbols λ, χ, ψ etc. He suggested the following equation as a general solution of diffusion equation (1) for vertical flow

$$\tfrac{1}{2}m \int_{\theta_n}^{\theta} F_m d\theta = \frac{Pd[F_m]}{d\theta} - R_m \qquad \ldots(4)$$

subject to the conditions

$$\theta = \theta_0, \quad F_m = 0 \qquad \ldots(5)$$

Where R_m is a function based on K_n, K_o, D, λ, χ, ψ,...F_{m-1}, flow functions; K_n and K_o being the hydraulic conductivities corresponding to soil water contents θ_n and θ_o, respectively. λ, χ and ψ are single valued functions of soil water content θ. P, a function of θ, is given by the following equation.

$$P = D\left(\frac{d\theta}{d\lambda}\right)^2 \qquad \ldots(6)$$

F_m is also a function of soil water content θ and is eliminated by a proper transformation (Philip, 1957).

The estimation of flow function λ may be made by the use of following equation:

$$\int_{\theta_n}^{\theta} \lambda \, d\theta = -2D\frac{d\theta}{d\lambda} \qquad \ldots(7)$$

Subject to the conditions

$$\theta = \theta_o, \quad \lambda = 0 \qquad \ldots(8)$$

$$\theta \rightarrow \theta_n, \quad \lambda \rightarrow \infty \qquad \ldots(9)$$

which implies

$$\theta \rightarrow \theta_n, \ \frac{d\theta}{d\lambda} \rightarrow 0 \qquad \ldots(10)$$

The estimation of flow function χ may be made by the use of following equation

$$\int_{\theta_n}^{\theta} \chi \, d\theta = \left[P\,\frac{d\chi}{d\theta}\right] + \left[K - K_n\right] \qquad \ldots(11)$$

subject to the conditions

$$\theta = \theta_o \,, \chi = 0 \qquad ...(12)$$

and

$$\int_{\theta_n}^{\theta} \chi \, d\theta = \left[P \frac{d\chi}{d\theta} \right]_{\theta=\theta_o} + \left[K_0 - K_n \right] \qquad ...(13)$$

The estimation of ψ function may be made by the use of following equation

$$\frac{3}{2}\int_{\theta_n}^{\theta} \psi \, d\theta = P \frac{d\psi}{d\theta} - Q \qquad ...(14)$$

subject to the conditions

$$\theta = \theta_o \,, \quad \psi = 0 \qquad ...(15)$$

and

$$Q = D \frac{d\theta}{d\lambda} \left(\frac{d\chi}{d\lambda} \right)^2 \qquad ...(16)$$

The distribution of soil water after a given time of infiltration t may be computed by the following equation.

$$x = \lambda t^{1/2} + \chi t + \psi t^{3/2} + \omega t^2 + ... \qquad ...(17)$$

Where x is the length of infiltration. In general, three flow functions λ, χ and ψ are considered to compute the depth of infiltration as the contribution of other flow functions to the total depth of infiltration is negligible.

Materials and Methods

The experiments were conducted on a Jabalpur silty clay loam (Sand=8.0 per cent; silt = 56.4 per cent; clay=35.6 per cent; CEC = 41.8 me/100 g; pH = 7.90; organic carbon = 0.43 per cent) which was air dried and passed through 2 mm sieve before storing it in double walled plastic bags. The water content of air dry soil was about 7 per cent on a gravimetric basis. Three bulk densities, viz. 1.2, 1.3, and 1.4 gm/cc were chosen in accordance with the existing field conditions of the surface layer of soil. The hydraulic conductivity as a function of soil water content at different soil bulk densities was evaluated following the procedure proposed by Gupta and Staple (1964). Soil water diffusivity values and soil water retention characteristic curves, which were employed in the evaluation of hydraulic conductivity at different soil bulk densities, are presented elsewhere (Sharda, 1974, 1977).

For determining the flow functions, the soil was packed in the acrylic columns consisting of 15 rings, each having 3.8 cm diameter and 2 cm height (Figure 1), in 2 cm layers to obtain the desired soil bulk density. A manually rotated wire brush was employed to disturb the surface of soil to a depth of 1-2 mm in order to achieve best possible contact between the sections before soil for next 2 cm section was added. Water was allowed to infiltrate through the inlet end of the soil column at a negative water pressure of 2 cm for one hour. At the end of desired period of infiltration,

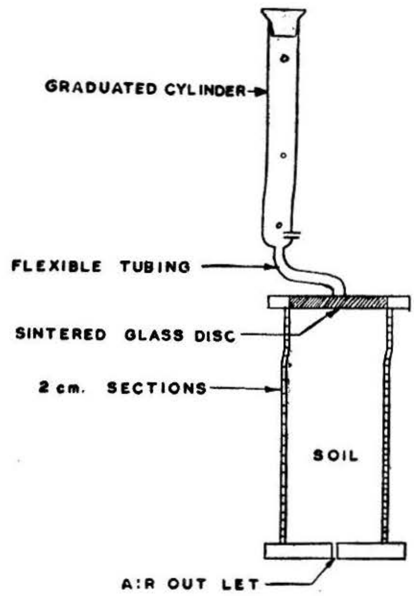

FIGURE 1 : Schematic diagram of apparatus

the water supply was stopped and then 2 cm sections were separated by pushing a thin rigid shim between each pair of rings and water content of the soil in each section was determined gravimetrically. The different flow functions, viz. λ, χ and ψ were evaluated following the procedure of Philip (1955, 1957). Other functions of the series were not evaluated because of their insignificant contribution to the phenomenon of soil water movement. The flow functions were adjusted to take into account the effect of small positive pressure of 0.5 cm (Philip 1957), which was employed to study vertical infiltration as described below.

For studying vertical infiltration, soil was packed in the acrylic columns, assembled by 20 rings or sections of the same diameter and height as mentioned above, to obtain the desired bulk density. Water was allowed to infiltrate vertically downward through the inlet end in each soil column at a positive water pressure of 0.5 cm for a known period. Measurement of time in all the experiments commenced at the instant when contact was established between the water body (sintered glass disc attached to a graduated water filled burette) and the soil columns (Figure 1). The amount of water infiltrated into the soil column was noted at each 10 minutes interval till the completion of the experiments. At the end of desired period of vertical infiltration the water supply was stopped. The position of the wetting front was immediately noted at four different places and then 2 cm sections were separated as mentioned above. The water content of soil in each section was determined gravimetrically. The experiments were conducted for different periods of infiltration. Each experiment was conducted at least twice, provided that the difference between the two consecutive runs did not exceed 5 per cent. Otherwise, a third replicate was taken. Abnormal results which occurred in some of the cases were discarded. All experiments were conducted at room temperature of 25 to 30°C. The computed soil water profiles, infiltration rates and cumulative influx were obtained using the various flow functions (Kirkham and Powers, 1971).

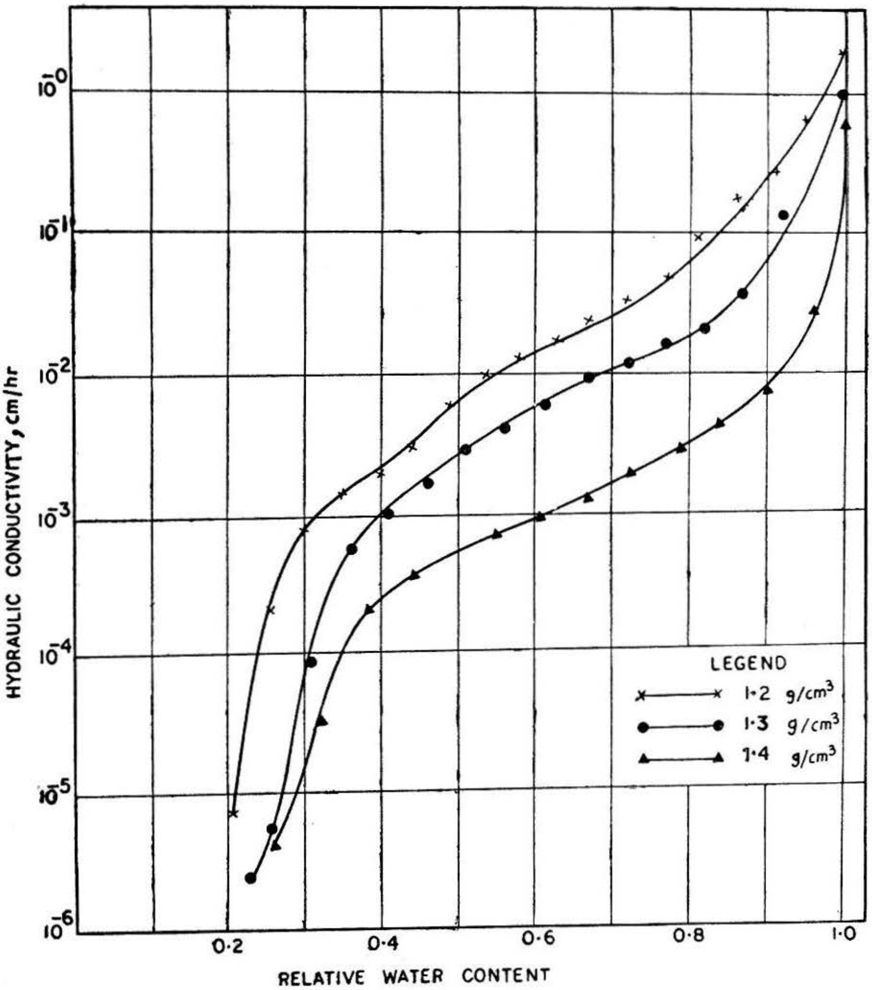

FIGURE 2: Hydraulic conductivity at different soil bulk densities

Results and *Discussion*

Figure 2 presents the hydraulic conductivity of soil as a function of relative water content (*RWC*) at different soil bulk densities. The values of void ratio at different soil bulk densities are presented in Table 1. The use of parameter *RWC* allows better comparison of hydraulic conductivity of soil at different bulk densities than the volumetric water content. *RWC* is given by the following relationship.

$$RWC = \frac{\theta - \theta_i}{\theta_s - \theta_i} \qquad \ldots(18)$$

Where θ is the volumetric water content corresponding to a certain value of hydraulic conductivity of soil, θ_i the initial water content of soil and θ_s is the water content at saturation. Data presented in Table 1 can be used to convert *RWC* to volumetric water content.

TABLE 1

Sl. No.	Soil bulk density (gm/cc)	Void ratio	Percentage water (cc/cc)		Length of infiltration (cm)	
			θi	θ_s	2 hours	4 hours
1.	1.2	1.21	8.4	50.9	23.0	33.8
2.	1.3	1.04	9.1	48.1	15.2	23.1
3.	1.4	0.89	9.8	44.6	9.8	15.4

A more than two fold reduction in hydraulic conductivity occurred with the increase in bulk density by 0.1 gm/cc over 1.2 gm/cc. A further 40 per cent reduction occurred with the additional increase in bulk density by 0.1 gm/cc. The influence of soil bulk density persisted throughout the water range of soil studied. However, the effect of decrease of *RWC* on hydraulic conductivity was more pronounced at the same soil bulk density. Gumbs and Warkentin (1972) also observed similar effects in the case of clay soil samples. The distribution of pores of different sizes in an unit volume of soil play a dominant rate in controlling the flow processes in a porous medium. The maximum decrease in hydraulic conductivity value near saturation could be attributed to a rapid decrease in the volume of non-capillary pore space with increase in soil bulk density.

Figure 3 shows the plots of flow functions λ, χ and ψ vs. relative water content. These flow functions were evaluated by using the initial soil water contents (volumetric basis) of 9, 10 and 10.5 per cent at soil bulk densities 1.2, 1.3 and 1.4 gm/cc respectively. The maximum values of soil water content chosen were 51.0, 48.0 and 44.5 per cent, corresponding to soil water content at zero soil water tension, at the bulk densities 1.2, 1.3 and 1.4 gm/cc, respectively. It is interesting to note that all the three functions are very sensitive to the changes in soil water content near the zone of saturation at all the three bulk densities. However, in the drier soil water range all the flow functions tend to assume a constant value. λ function was observed to have a negative curvilinear relationship with the change in soil water content whereas both χ and ψ functions at first increased rapidly with decrease in soil water content, then decreased rather slowly with further changes in the soil water content to assume a distinct constant value at each bulk density. Similar changes in the magnitude of χ and ψ values with changes in soil water content were observed by Philip (1957) in the case of a Yolo light clay loam.

The distribution of water in soil, as computed by the flow functions, viz. λ, χ and ψ, and the experimentally determined soil water profiles for different periods of vertical infiltration at bulk densities 1.2, 1.3 and 1.4 gm/cc are shown in Figures 4 and 5. Though a large number of experiments were conducted for various periods of vertical infiltration, the results in respect of 2 and 4 hours of infiltration are being presented for the sake of brevity at each bulk density. For the computation of soil water profiles, the initial and maximum soil water contents chosen were the same as in the evaluation of flow functions. In Figures 4 and 5, points are the experimental values and the continuous lines are the computed soil water profiles. The

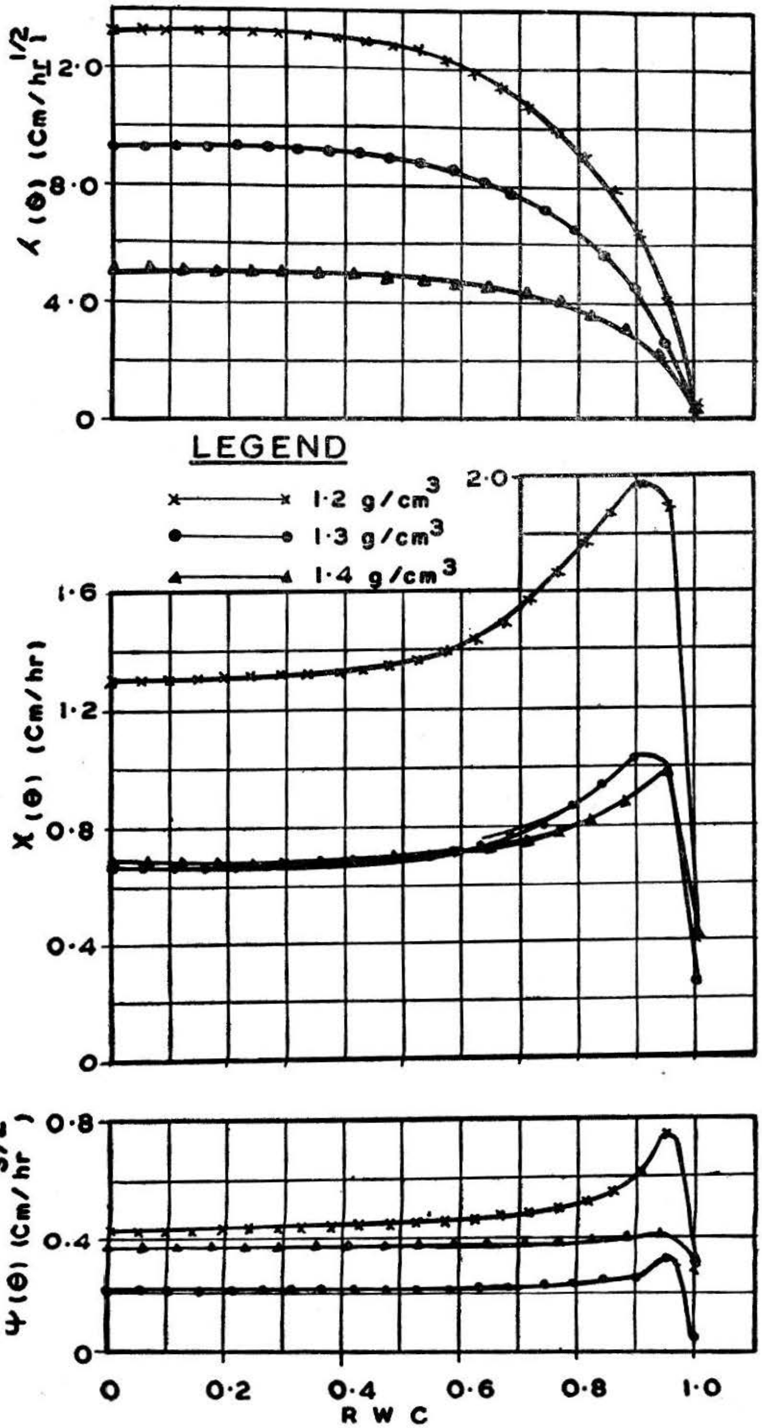

FIGURE 3: Flow-functions—$\lambda(\theta)$, $\chi(\theta)$ and $\psi(\theta)$ at different soil bulk densities

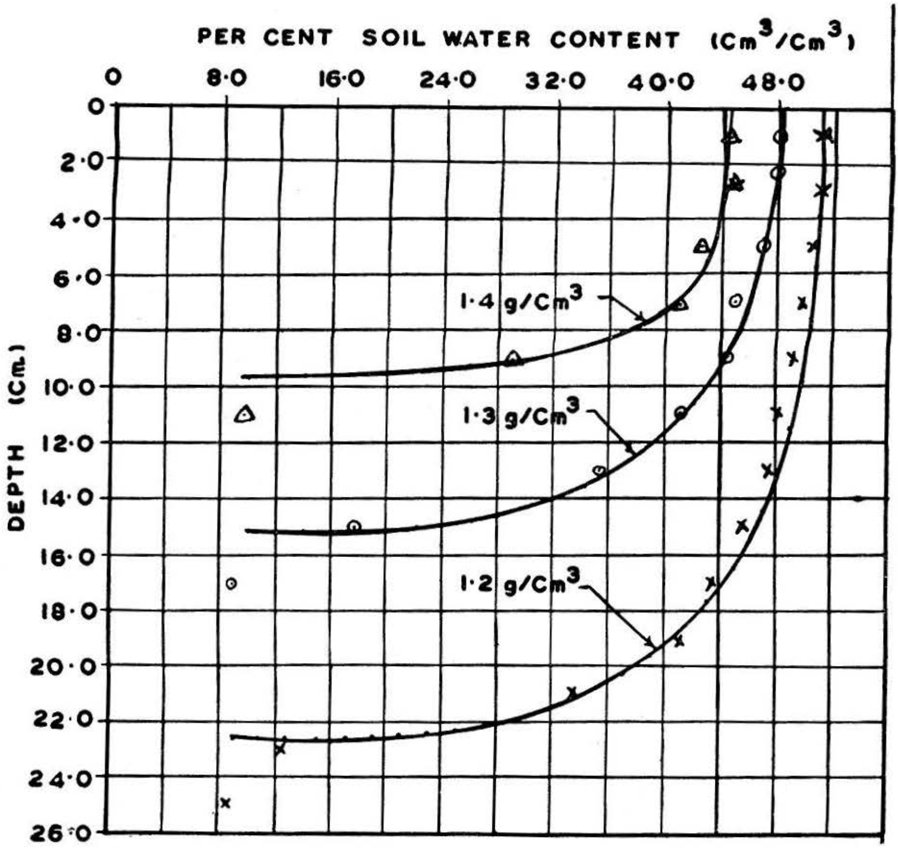

FIGURE 4: Experimental and computed soil water distribution for two hours of infiltration at different soil bulk densities

experimental points represent the soil water content at the centre of each ring. The experimental lengths of infiltration of 2 and 4 hours of vertical infiltration at different soil bulk densities are presented in Table 1. A reasonably good agreement was observed between the experimental soil water profiles and the computed ones, thereby confirming the magnitude of hydraulic conductivity and flow functions as evaluated at different soil bulk densities. The slight variations which might occur between the experimental and computed soil water profiles are largely due to errors involved in experimental sampling of actual soil water content determinations. Moreover, movement of water through a soil is a complex phenomenon. Several processes occur as water moves into air dry soil. Because of the heterogeneous nature of the size, shape, composition and arrangement of soil particles, the shape of the water—air interface must undergo continual change (Nielson et al, 1962). These varying contact angles which also depend upon rate of movement could account for such errors. However, within the limits of experimental errors, it is safe to conclude that the flow functions may be used as a satisfactory means for the evaluation of soil water profiles.

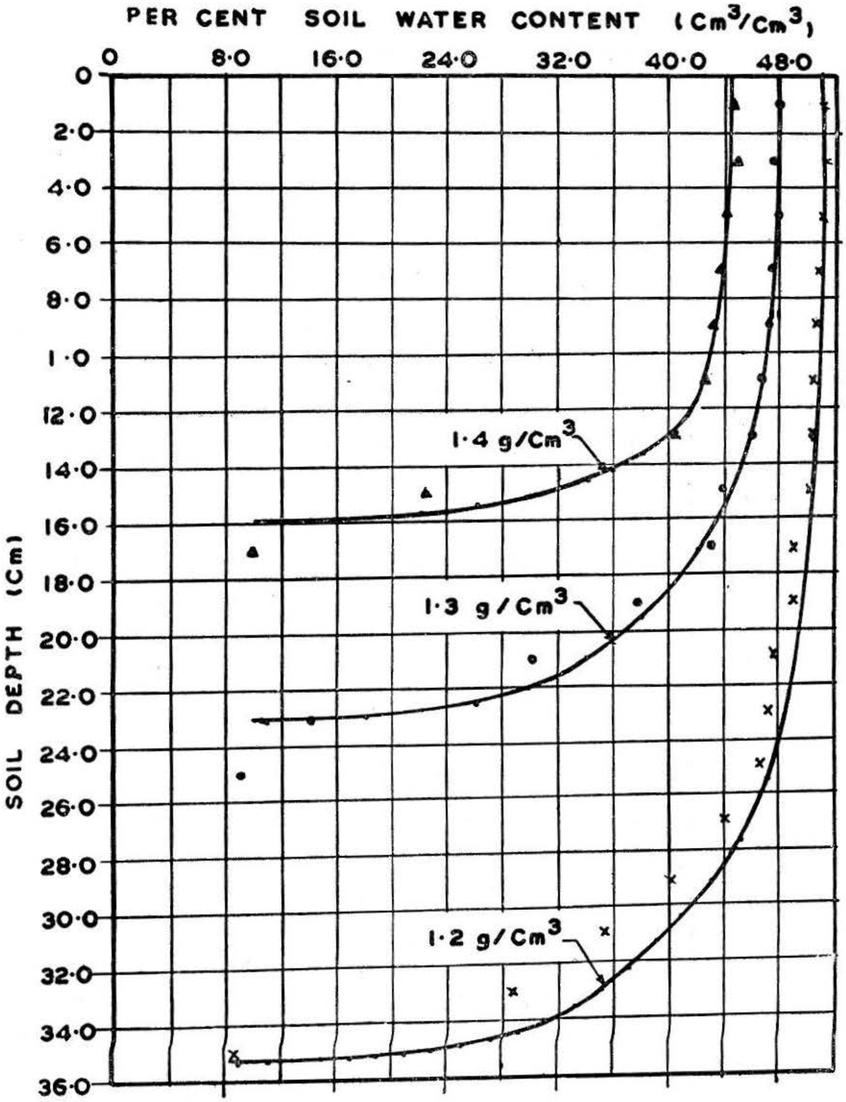

FIGURE 5: Experimental and computed soil water distribution for four hours of infiltration at different soil bulk densities

A marked reduction in the total lengths of infiltration was observed in relation to soil bulk density for different periods of vertical infiltration, e.g. a reduction of 30 per cent in the total length of infiltration occurred with each increment of 0.1 gm/cc over 1.2 gm/cc for 2 and 4 hours of vertical infiltration. It is, again, can be explained on the basis of the fact that an increase in soil bulk density is always accompanied by a corresponding decrease in the non-capillary pore space and an increase in capillary pore-space, thereby resulting in the increased tortuosity in the flow channels of a porous medium. However, lengths of infiltration were found to be commensurate with the times of infiltration at the same bulk density.

Figures 6 and 7 depict computed commulative influx and infiltration rates (solid lines) and the experimental values (points) of the same at different soil bulk densities. The experimental values of cumulative influx were within 5 per cent of the computed ones. Similarly, the experimental rates of infiltration were within 5 to 10 per cent of the computed ones at different bulk densities and for all periods of vertical infiltration. Both cumulative influx and infiltration rates were markedly influenced by soil bulk density. The cumulative influx of water reduced by 40 per cent with each increment of 0.1 gm/cc over 1.2 gm/cc. Similarly, the infiltration rates near the end of four hour of infiltration decreased by 40 per cent with the increase in bulk density from 1.2 gm/cc to 1.3 gm/cc and further decreased by 30 per cent with an additional increase in bulk density by 0.1 gm/cc, thereby reflecting the inherent importance of size, shape and composition of pores in controlling the flow processes in a porous medium.

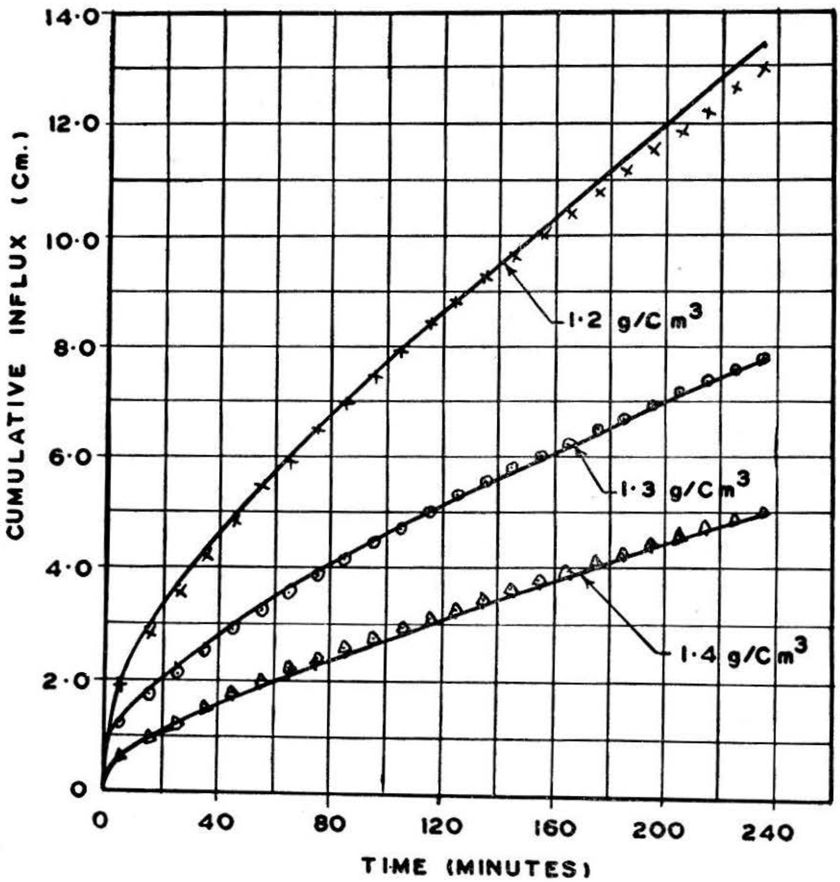

FIGURE 6: Cumulative influx at different soil bulk densities

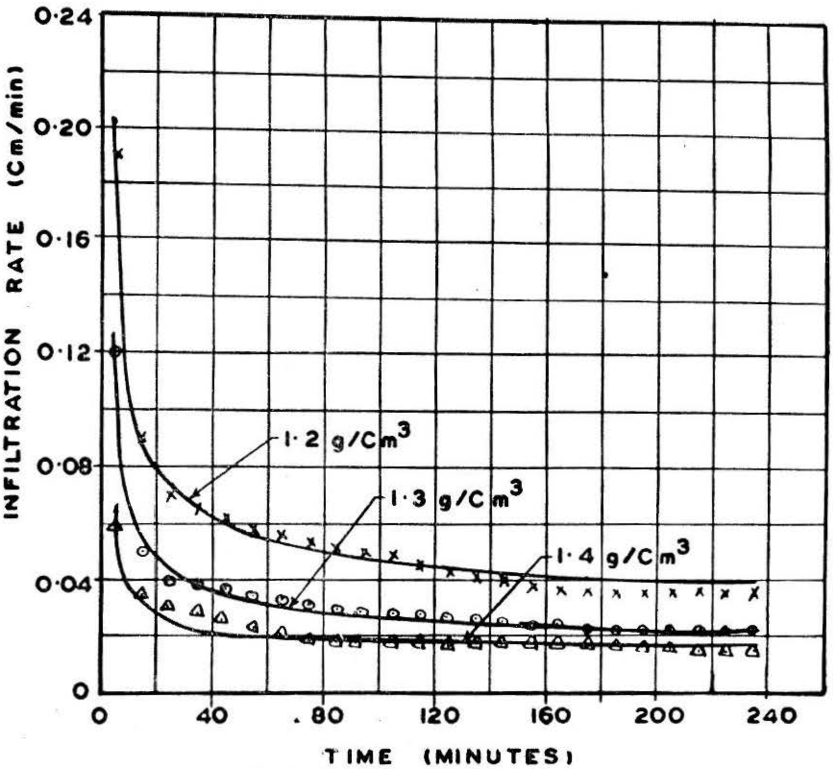

FIGURE 7: Infiltration rates at different soil bulk densities

Conclusions

The vertical infiltration in the case of a Jabalpur silty clay loam was studied at different soil bulk densities under a small positive water head (0.5 cm). Various flow functions viz. λ, χ and ψ, including hydraulic conductivity of soil, were evaluated following the procedure of Philip (1955, 1957). It was observed that the experimental data was in good agreement with the computed ones, thereby suggesting that water distribution in a soil profile could be reasonably ascertained from the knowledge of predetermined flow functions of the soil. These flow functions were also employed to compute cumulative influx and infiltration rates at different bulk densities of soil, which agreed well with the experimental data.

The overall influence of the soil bulk density was to reduce the total lengths of infiltration, cumulative influx and infiltration rates to a considerable extent. However, the total length of infiltration were found to be proportionate with times of infiltration at the same soil bulk density.

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