Limiting of Immediate Settlement of Shallow Foundations

by

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Introduction

Immediate settlement forms one of the components of the total settlement of a foundation structure. In the design of a footing for compliance with preset settlement values, generally a tentative plan dimension complying with the bearing capacity criterion is chosen to start with. The settlement of such a foundation is then determined, assuming a certain level of load transfer. If the settlement is found to exceed the prescribed limits it is brought within limits by either of one or by a combination of the following methods, namely, increasing the depth of embedment of the foundation, increasing the bearing area of the foundation and adopting appropriate plan shape or dimensions of the foundation. Analyses to quantify the effects of the above three methods on consolidation settlement of footings have been presented by Kaniraj and Ranganatham (1977). Analyses to quantify the effects of the above three methods on immediate settlement of footings are herein presented. The analyses developed for rectangular and square footings are based on Fox's (1948) solutions for the mean elastic settlement of a uniformly loaded area. The medium is assumed to be semi-infinite, homogeneous, isotropic and linearly elastic continuum for the purpose of computing immediate settlement.

Immediate Settlement of Footings

The mean elastic settlement, ρ, of a uniformly loaded area of $2a \times 2b$ embedded at a depth of h below the surface (Figure 1) is given by (Fox, 1948),

$$\rho = \frac{q\,(1+\mu)}{4\pi E\,(1-\mu)} \sum_{s=1}^{5} \beta_s W_s \qquad \ldots(1)$$

In equation 1,

q = load intensity on the loaded area

μ, E = Poisson's ratio and modulus of elasticity of the medium respectively

$\beta_1 = 3-4\mu$

$\beta_2 = 5-12\mu+8\mu^2$

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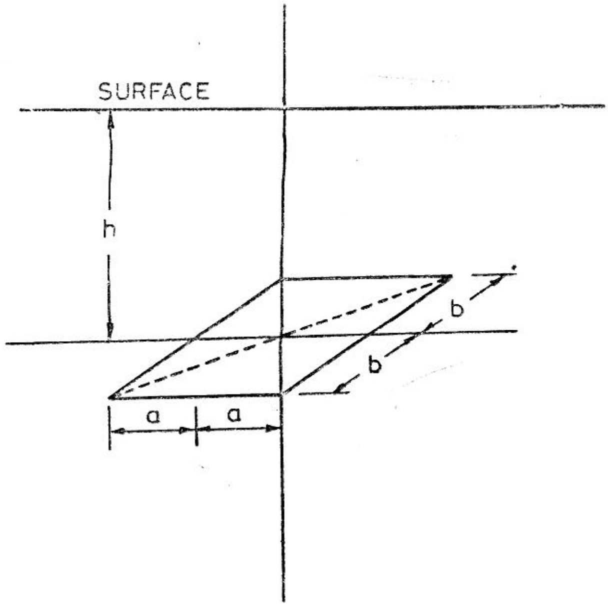

FIGURE 1. Loaded area in the interior of the semi-infinite deposit

$$\beta_3 = -4\mu\,(1-2\mu)$$
$$\beta_4 = -1+4\mu-8\mu^2$$
$$\beta_5 = -4\,(1-2\mu)^2$$
$$W_1 = 2a\;ln\,\frac{r_4+2b}{2a}+2b\;ln\,\frac{r_4+2a}{2b}-\frac{r_4^3-8a^3-8b^3}{12ab}$$
$$W_2 = 2a\;ln\,\frac{r_3+2b}{r_1}+2b\;ln\,\frac{r_3+2a}{r_2}-\frac{r_3^3-r_2^3-r_1^3+r^3}{12ab}$$
$$W_3 = \frac{r^2}{2a}\;ln\,\frac{(2b+r_2)\,r_1}{(2b+r_3)\,r}+\frac{r^2}{2b}\;ln\;\frac{(2a+r_1)\,r_2}{(2a+r_3)\,r}$$
$$W_4 = \frac{r^2\,(r_1+r_2-r_3-r)}{4ab};\;W_5=r\,tan^{-1}\left(\frac{4ab}{rr_3}\right)$$

and $r = 2h\,;\;r_1^2=4a^2+r^2\,;\;r_2^2+4b^2+r^2\,;$

$r_3^2= 4a^2+4b^2+r^2;\;r_4^2=4a^2+4b^2$

In order to express settlement in terms of dimensionless parameters the following substitutions are made in equation 1.

$$a=AK \qquad \ldots(2a)$$
$$b=BK \qquad \ldots(2b)$$
$$h=HK \qquad \ldots(2c)$$

where A, B and H are non-dimensional parameters and K is a dimensional constant having the unit of length. Following these substitutions equation 1 transforms to,

$$\rho = \frac{K}{N} q\, I_m \qquad \ldots(3)$$

where $N = 4\,\pi\, E\,(1-\mu)/(1+\mu)$ ...(4)

and I_m is the settlement factor expressed by

$$I_m = \sum_{s=1}^{5} \beta_s\, W_s' \qquad \ldots(5)$$

In equation 5,

$$W_1' = 2A\, ln \frac{R_4+2B}{2A} + 2B\, ln\, \frac{R_4+2A}{2B} - \frac{R_4^3 - 8A^3 - 8B^3}{12AB}$$

$$W_2' = 2A\, ln \frac{R_3+2B}{R_1} + 2B\, ln\, \frac{R_3+2A}{R_2} - \frac{R_3^3 - R_2^3 - R_1^3 + R^3}{12AB}$$

$$W_3' = \frac{R^2}{2A}\, ln \frac{(2B+R_2)\, R_1}{(2B+R_3)\, R} + \frac{R^2}{2B}\, ln \frac{(2A+R_1)\, R_2}{(2A+R_3)R}$$

$$W_4' = R^2\,(R_1+R_2-R_3-R)/4AB$$

$$W_5' = R\, \tan^{-1}\left(\frac{4AB}{RR_3}\right)$$

and $R = 2H;\; R_1^2 = 4A^2+R^2;\; R_2^2 = 4B^2+R^2;$

$R_3^2 = 4\,(A^2+B^2)+R^2;\; R_4^2 = 4\,(A^2+B^2)$

The expressions given by Fox (1948) for immediate settlement of surface footings and that of footings embedded at infinite depth can also be expressed in the same form as that of equation 3 with appropriate expressions for settlement factor I_m. The expression for I_m for surface footing is given by,

$$I_m = 8\,(1-\mu)^2\, W_1' \qquad \ldots(6)$$

and that for footings embedded at infinite depth is given by,

$$I_m = (3-4\mu) W_1' \qquad \ldots(7)$$

The above is the analysis for immediate settlement of a rectangular loaded area in general. A procedure is now developed using which the immediate settlement of different loaded areas embedded in the same medium could be compared in terms of comparable settlement factors. The procedure helps to quantitatively asses the effects of depth of embedment, change in the bearing area and that of plan dimensioning on the immediate settlement of footings.

Let X and Y be the footings as shown in Figure 2 embedded in the same medium. Let a_x, b_x be the half sides of the loaded area X. Let q_x be the load intensity on X and h_x be its depth of embedment. Then for X,

$$a_x = A_x K_x \qquad \ldots(8a)$$

$$b_x = B_x K_x \qquad \ldots(8b)$$

$$h_x = H_x K_x \qquad \ldots(8c)$$

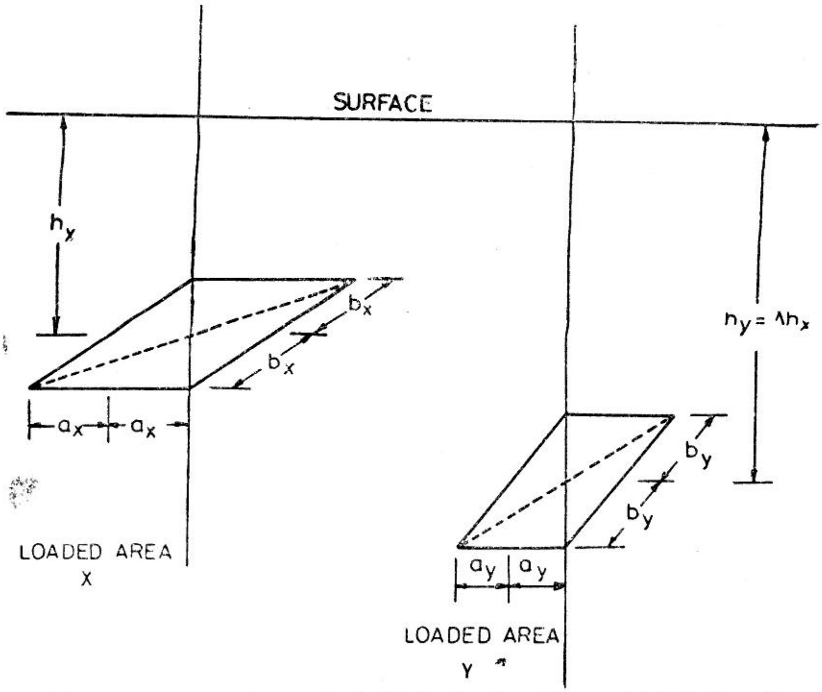

FIGURE 2. Loaded areas X and Y in the interior of the semi-infinite deposit

and $$\rho_x = \frac{K_x}{N} q_x I_{mx} \qquad \text{...(9)}$$

where ρ_x is the mean settlement of X. Similarly let a_y and b_y be the half sides of the loaded area Y whose area is $(1+p)$ times that of X, where p is given as,

$$p = (\text{area of } Y - \text{area of } X)/\text{area of } X \qquad \text{...(10)}$$

and p can be either positive or negative. Let q_y be the load intensity on Y and h_y be its depth of embedment. Then for Y,

$$a_y = A_y K_y \qquad \text{...(11}a\text{)}$$

$$b_y = B_y K_y \qquad \text{...(11}b\text{)}$$

$$h_y = H_y K_y \qquad \text{...(11}c\text{)}$$

The mean immediate settlement of Y, ρ_y is given by,

$$\rho_y = \frac{K_y}{N} q_y I_{my} \qquad \text{...(12)}$$

The I_m values of X (i.e. I_{mx}) and Y (i.e. I_{my}) according to equations 9 and 12 can be directly compared to know the relative settlement of X and Y only when $K_x = K_y$ and $q_x = q_y$. Equation 12 can be expressed as,

$$\rho_y = \frac{K_x}{N} q_x I'_{my} \qquad \text{...(13)}$$

whereby I'_{my} is the settlement factor for Y which is directly comparable to I_{mx} in equation 9. Equation 13 is derived in the following manner. From the bearing areas of X and Y,

$$a_y b_y = a_x b_x \, (1+p) \qquad \ldots(14)$$

Substituting for a_x, a_y, b_x and b_y in equation 14 from equations 8 and 11

$$A \; B \; K_y^2 = A \; B \; K_x^2 \, (1+p) \qquad \ldots(15)$$

which gives,

$$K_y = K_x \sqrt{\frac{A_x \, B_x}{A_y \, B_y} \, (1+p)} \qquad \ldots(16)$$

From the known loads on the two respective loaded areas,

$$q_x \, a_x \, b_x = M \, q_y \, a_y \, b_y = M \, q_y \, a_x \, b_x \, (1+p) \qquad \ldots(17)$$

where $M = P_x/P_y$, P_x and P_y being the total load on X and Y respectively. From equation 17 it follows that,

$$q_y = \frac{q_x}{M \, (1+p)} \qquad \ldots(18)$$

Substituting for K_y (from equation 16) and q_y from Eqn. 18 in Eqn. 12,

$$\rho_y = \frac{K_x}{N} \, \frac{q_x}{M} \sqrt{\frac{A_x B_x}{A_y B_y \, (1+p)}} \, I_{my} \qquad \ldots(19)$$

Denoting,

$$I'_{my} = \frac{1}{M} \sqrt{\frac{A_x B_x}{A_y B_y \, (1+p)}} \; I_{my} \qquad \ldots(20)$$

which is the mean settlement factor as in equation 13 which is directly comparable to I_{mx}. The relationship between the depths of embedment h_x and h_y is given by,

$$h_y = \lambda h_x \qquad \ldots(21)$$

where $\lambda > 0$. Substituting for h_x and h_y from equations 8 and 11 and on simplification,

$$H_y = \lambda \; H_x \sqrt{\frac{A_y B_y}{A_x B_x \, (1+p)}} \qquad \ldots(22)$$

In order to find the relative settlements of the two loaded area X and Y, I_{mx} and I'_{my} according to equations 9 and 20 respectively are calculated and the ratio of these values give the relative settlement of X and Y.

Results

Calculations for I_m values have been carried out with a view to quantify the effects of depth of embedment, the change in the bearing area and that of plan dimensioning, separately, on settlement of foundations. I_m values are computed keeping the non-dimensionalised width parameter A (A_x, A_y) to be unity in all the cases. The results of the three different cases of the study are as presented below.

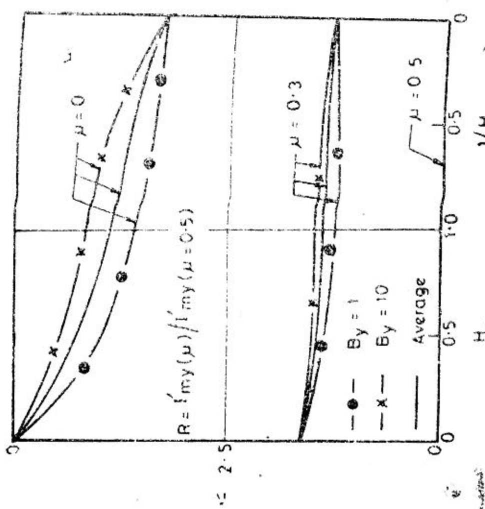

FIGURE 4. Variation of R with depth of embedment parameter, H_y

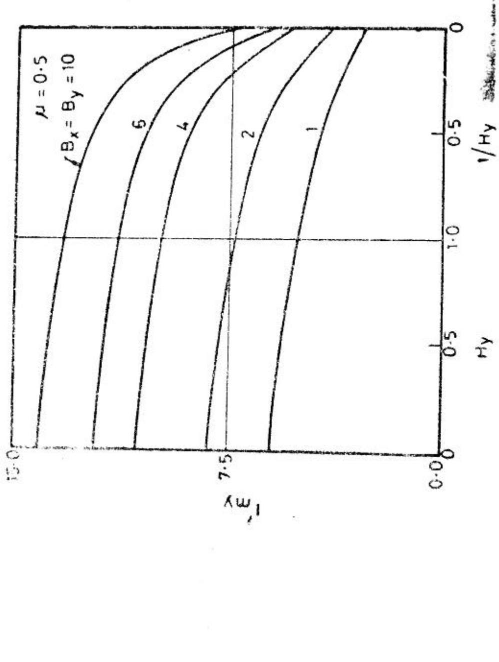

FIGURE 3. Variation of mean settlement factor, I'_{my}, with depth of embedment parameter, H_y

(*a*) *Effect of Depth of Embedment*: In this case the settlement of two equally loaded areas of identical plan dimensions but embedded at different levels within the medium are considered. The relationships between two such loaded areas and their parameters are defined as follows.

$$A_x = A_y = 1 \qquad \ldots(23a)$$

$$B_x = B_y \qquad \ldots(23b)$$

$$M = 1 \qquad \ldots(23c)$$

$$p = 0 \qquad \ldots(23d)$$

$$q_x = q_y \qquad \ldots(23e)$$

$$H_y = \lambda\ H_x \qquad \ldots(23f)$$

From the above and from equation 20 it follows that,

$$I'_{my} = I_{my} \qquad \ldots(24)$$

or in other words I_{my} itself is directly comparable to I_{mx} in this case. This also means that I_{my} (or I_{mx}) values calculated with different values of H_y (or H_x) are themselves indicative of the influence of the depth of embedment of footings. In view of this calculations for I'_{my} $(=I_{my})$ are carried out keeping A_y as unity and varying H_y from 0 to ∞ and B_y from 1 to 10. Figure 3 shows the variation of I'_{my} with H_y with each curve in the figure representing the variation for one value of B_y for $\mu=0{\cdot}5$. In order to cover the range from 0 to ∞ in H_y values the x-axis is taken in two parts as H_y from 0 to 1 in the first and as $1/H_y$ from 1 to 0 in the second corresponding to H_y values from 1 to ∞. It is easily seen from Figure 3 that an increase in the depth of embedment causes a decrease in I'_{my} consequently decreasing the immediate settlement value.

In order to investigate the effect of μ on I'_{my} values for the same range of parameters as above calculations for I'_{my} are carried out varying μ from 0·5 to 0. It is found that the value of μ has considerable influence on I_m values. This influence is indicated by the value of ratio R defined as,

$$R = (I'_{my})_{\mu} / (I'_{my})_{\mu=\cdot 5} \qquad \ldots(25)$$

The variation of R with H_y for different values of μ is shown in Figure 4. In this figure the average variation of R with H_y (for B_y up to 10) is shown in solid lines. The values of R along the limiting broken lines deviate from the average line by a maximum of plus or minus 5 per cent. Thus the average curve represents the actual variation mostly with and accuracy of greater than 95 per cent. It is seen from Figure 4 that when μ is changed from 0·5 to 0 I_m value increases maximum by 300 per cent for surface footings. However, it does not represent an equal increase in the immediate settlement as well since the other term N is also influenced by the valve of μ. Hence, in order to know the relative settlement of footings when μ is varied a ratio R_1 is defined as follows.

$$R_1 = \frac{(I_m/N)\ \mu}{(I_m/N)_{\mu=0\cdot 5}} \qquad \ldots(26)$$

The value of R_1 does not deviate much from 1. The maximum increase in immediate settlement because of change in μ from 0·5 to 0 is seen to be only

33 per cent for surface footings. This will also be much less for embedded footings.

(*b*) *Effect of Change in the Bearing Area* : The relative settlement of two equally loaded areas with one of them having an area of $(1+p)$ times that of the other but the same sides ratio and depth of embedment is considered here. Let X and Y be the two loaded areas, Y having an area of $(1+p)$ times that of X. The relationships between the parameters of X and Y will then be defined as follows:

$$A_x = A_y = 1 \qquad \text{...(27a)}$$

$$B_x = B_y \qquad \text{...(27b)}$$

$$M = 1 \qquad \text{...(27c)}$$

$$\lambda = 1 \qquad \text{...(27d)}$$

$$q_y = q_x/(1+p) \qquad \text{...(27e)}$$

$$H_y = H_x/\sqrt{1+p} \qquad \text{...(27f)}$$

I'_{my} is given by,

$$I'_{my} = I_{my}/\sqrt{1+p} \qquad \text{...(28)}$$

Calculations for I_{mx} are carried out varying B_x from 1 to 10 and H_x from .25 to 10. Different bearing areas for loaded areas Y could be chosen by varying the value of p and p has been varied from 0 to 1. The settlement of Y is compared to that of X and at $p=0$ X and Y are identical in all respects and their settlements are equal $[(I_{mx} = I_{my\,(p=0)} = I'_{my\,(p=0)}]$. For other values of p, the equivalent parameters (q_y and H_y) for the calculation of I_{my} are given by equation 27 I'_{my} is obtained from equation 28. Figure 5 shows the variation of I'_{my} with p for $H_x=2$ and $\mu=.5$. Each curve in the figure represents the variation for one particular value of B_y $(=B_x)$. In order to quantify the effect of change in bearing area on settlement of foundations, value of ratio R' defined as,

$$R' = I'_{my}/I_{mx} = I'_{my\,(p)}/I'_{my\,(p=0)} \qquad \text{...(29)}$$

for corresponding values of H_x and H_y and q_x and q_y are computed. This ratio gives the relative settlement of Y with respect to X as a result of change in the area of X. The variation of R' with p is shown in Figure 6. It is observed that all the values of R' (for the chosen range of H_x and B_x) lie within the range indicated by the broken lines in the figure. The average variation of R' with p is indicated by the solid line in the figure. For the same reasons as explained for Figure 4 the average curve represents the variation with an accuracy of greater than 95 per cent. It is evident from this curve that immediate settlement is reduced by 25 per cent when the load bearing area is doubled.

(*c*) *Effect of Plan Dimensioning*: The study here involves the limiting of settlement by change in only the plan shape (accordingly the dimensions of the footing). The scope of the study is limited to rectangular and square plan dimensions only. The relationships between the parameters of X and Y are as follows.

$$A_x = A_y = 1 \qquad \text{...(29a)}$$

$$B_x \neq B_y \qquad \text{...(29b)}$$

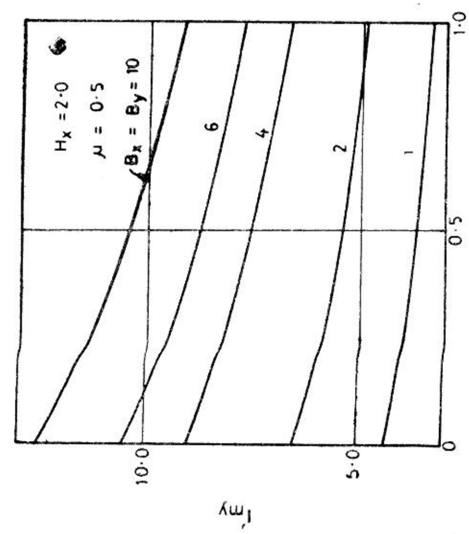

FIGURE 5. Variation of mean settlement factor, I'_{my}, with p

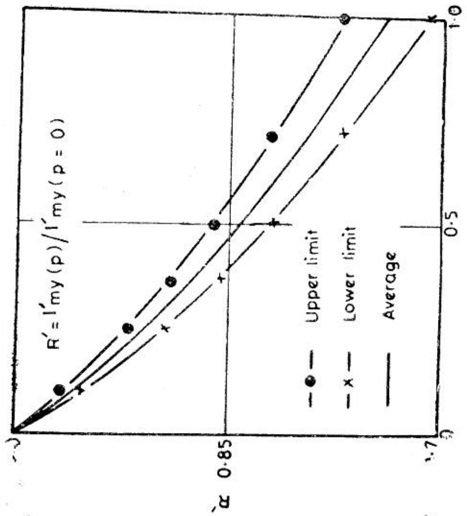

FIGURE 6. Variation of R' with p

$$p=0 \quad \ldots(29c)$$

$$M=1 \quad \ldots(29d)$$

$$\lambda=1 \quad \ldots(29e)$$

$$q_x=q_y \quad \ldots(29f)$$

$$H_y=\sqrt{\frac{B_y}{B_x}}\,H_x \quad \ldots(29g)$$

and I'_{my} is given by,

$$I'_{my}=\sqrt{B_x/B_y}\,I_{my} \quad \ldots(30)$$

I_{mx} values are calculated keeping A_x and B_x as unity (ie X is a square) and varying H_x from 0·25 to 10. Similary in the calculation of I_{my}, A_y is kept as unity and B_y is varied from 1 to 10. The values of H_y corresponding to H_x are determined by equation 29g. Thus a square loaded area (X) is transformed into rectangles (Y) with length to breadth ratio varying up to 10:1 and the settlement of these rectangles is compared against that of the square. The settlement of Y with sides ratio 1:1 (ie $B_y=1$) is equal to the settlement of X. Figure 7 shows the variation of I'_{my} with B_y with each curve in the figure showing the variation for one particular value of H_x. In order to quantify the effect of plan dimensioning on settlement, values of ratio R'' defind as,

$$R''=I'_{my}/I_{mx}=I'_{my(B_y)}/I'_{my\ (B_y=1)} \quad \ldots(31)$$

have been calculated. The variations of R'' with B_y is shown in Figure 8. It is observed that R'' values calculated for all the chosen range of parameters lie within the limits as indicated by the broken lines in Figure 8. The average variation is shown by the solid line in the figure which represents the variation with an accuracy of greater than 95 per cent. It is clear from figures 7 and 8 that the settlement of a loaded area decreases as it is made more and more oblong. It is seen from Figure 8 that the settlement of a square loaded area decreases on the average by 25 per cent when it is changed into a rectangle with sides ratio of 1:10. However, if it is changed into a rectangle with sides ratio of 1:3 (the usually adopted maximum ratio) the average reduction in settlement is only of the order of 7 per cent.

Discussion

The foregoing sections describe the change in immediate settlement of a footing due to variation in depth of embedment, bearing area and plan dimensions, treating each of the method in isolation. However, the use of the analysis and results presented in Figures 3 through 8 are not limited to particular cases but can also be used when the variation occurs by a combination of the above three methods ($\lambda\neq1$; $p\neq0$ and $a_x: b_x\neq a_y:b_y$) together with unequal loading of the areas ($M\neq1$) and the characteristics of the medium also being different ($E_x\neq E_y$; $\mu_x\neq\mu_y$). This is best illustrated with a numerical example problem.

Example problem: It is required to compare the immediate settlement of two footings X and Y with the following data.

Footing X: $P_x=80$ Tonnes; $a_x\times b_x=4m\times 4m$; $h_x=2m$;

$\mu=0{\cdot}5$; Modulus of elasticity$=E_x$

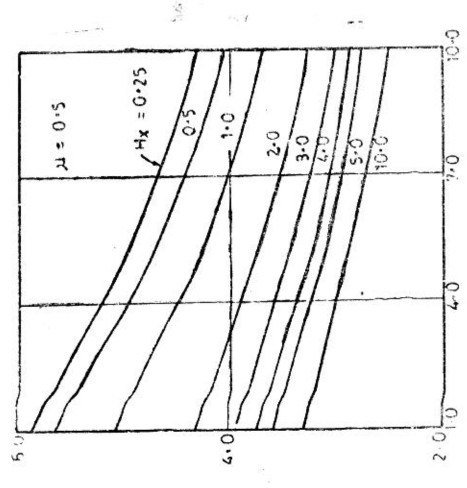

FIGURE 7. Variation of mean settlement factor, I'_{my}, with breadth parameter, E_y

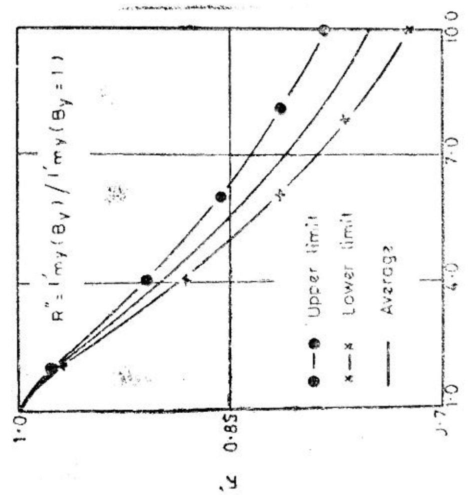

FIGURE 8. Variation of R'' with breadth parameter, E_y

Footing Y: $P_y=240$ Tonnes; $a_y \times b_y = 4m \times 8m$; $h_y=4m$;

$\mu=0.3$; Modulus of elasticity$=E_y$

The problem is solved through the following steps:

Step 1: Footing X is taken as the reference footing. In order to find I_{mx} the parameters of X then become: $A_x=1$; $B_x=1$ and $H_x=1$. I_{mx} value can be obtained from Figure 3 (value of I'_{my} for $H_y=1$ and $B_y=1$; ie $I_{mx}=5.1$) or from Figure 7 (value of I'_{my} for $B_y=1$ and $H_\lambda=1$; ie $I_{mx}=5.1$. The value of I_{mx} could have been obtained from Figure 5 also (I'_{my} value for $p=0$ and $B_y=1$) if the figure had been drawn for $H_x=1$. Now the problem remains to find the value of the settlement factor of footing Y which is directly comparable to I_{mx} found above.

Step 2: The directly comparable settlement factor I'_{my}, provided the medium is the same is given by equation 20. For evaluation of I_{my} in equation 20 the parameters B_y and H_y are as follows: $B_y=2$ and H_y from equation 22 with $\lambda=2$ and $p=1$ is obtained as 2. With these values I_{my} is found to be 6.45 (value of I'_{my} in Figure 3 with $H_y=2$ and $B_y=2$ or value of I'_{my} in Figure 5 with $p=0$ and $B_y=2$). From equation 20 the value of I'_{my} is obtained as 9.675 with $M=1/3$.

Step 3: The I'_{my} computed from Step 2 is not directly comparable to I_{mx} computed in Step 1 since the media are different, in which case the relative settlement is obtained from equation 26. Hence,

$$\frac{\rho_x}{\rho_y} = \frac{(I_{mx}/N)\ _{\mu=\cdot 5}}{(I'_{my}/N)\ _{\mu=\cdot 3}}$$

I_{my}' for $\mu=0.3$ is obtained from Figure 4 as,

$$I'_{my\ (\mu=\cdot 3)} = 1.78 \times 9.675 = 17.21$$

Substituting for values of N the ratio ρ_x/ρ_y works out to .481 (E_y/E_x). If $E_y \simeq E_x$, then it is seen that $\rho_y \simeq 2\rho_x$.

Conclusions

Analyses in order to quantify the effects of depth embedment, the change in the bearing area and that of plan dimensioning on the immediate settlement of shallow foundations have been developed. The settlement of a loaded area is expressed in terms of non-dimensional settlement factors and the settlement of two different loaded areas are compared by computing the comparable settlement factors. The effect of each, namely, depth of embedment, change in bearing area and change in plan dimensions (shape), on immediate settlement has been investigated separately. It is found that the settlement of a loaded area decreases as its depth of embedment increases, as the bearing area increases and as the loaded area is made more and more oblong. The use of the analysis and charts for a general problem where there is simultaneous variation of all the above three parameters together with variations in load and characteristics of the medium is demonstrated with a numercial example problem.

References

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Notations

a = half the breadth of loaded area

a_x, a_y = half the breadth of loaded areas X and Y respectively

A = non-dimensionalised breadth parameter

A_x, A_y = non-dimensionalised breadth parameters for X and Y

b = half the length of loaded area

b_x, b_y = half the length of loaded areas X and Y

B = non-dimensionalised length parameter

B_x, B_y = non-dimensionalised length parameters for X and Y

E = modulus of elasticity of soil medium

h = depth of embedment of loaded area

h_x, h_y = depth of embedment of loaded areas H and Y

H = non-dimensionalised depth of embedment parameter

H_x, H_y = non-dimensionalised depth of embedment parameters for X and Y

I_m = dimensionless mean settlement factor

I_{mx}, I_{my} = dimensionless mean settlement factors for X and Y

I'_{my} = mean settlement factor of Y directly comparable to that of X

K_x, K_y = dimensional constants with units of length

M = P_x/P_y

p = (area of Y—area of X)/area of X

P_x, P_y = total load acting on loaded areas X and Y respectively

q = intensity of load acting on loaded area

q_x, q_y = intensity of load acting on loaded areas X and Y

X, Y = two loaded areas

μ = *Poisson's ratio*

ρ = mean elastic settlement of the loaded area

λ = h_y/h_x