Alignment Chart for the Design of Sand Drains

by

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Introduction

In science and technology, nomograms are recognized for ease of operation and for the time saved in the repeated solution of mathematical formulae. The type of nomogram, known as alignment chart, although by no means a new method for solving equations graphically, is being appreciated more fully. An alignment chart is prepared in this investigation for getting the solution for the consolidation problem connected with three-dimensional flow. The chart is prepared for a constant load, constant permeability and no smear case.

Formulation of Design Equations

The Terzaghi's (1943) solution for the one-dimensional consolidation under instantaneonsly applied load is

$$U_v\,(T) = 1 - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}\, e^{-\frac{(2n+1)^2}{4}\pi^2\, T_{vv}} \qquad \ldots(1a)$$

$$T_{vv} = \frac{C_{vv}\, t}{H^2} \qquad \ldots(1b)$$

where U_v = degree of consolidation for vertical drainage and vertical consolidation

T_{vv} = time factor for vertical drainage and vertical consolidation

C_{vv} = coefficient of consolidation for vertical drainage and vertical consolidation

t = time

H = vertical drainage path distance

Solutions for radial consolidation was developed by Barron (1948). The

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solution for the equal strain case for constant load, variable permeability, no smear case is presented by Schiffman (1960) making the following assumptions:

(*i*) the surface load activating the sand-drain action is time-independent;

(*ii*) the drain is perfect and the soil surrounding the drain is completely undisturbed (i.e. no smear); and

(*iii*) the permeability is time-dependent according to $\overline{K}(t)=\alpha\,\bar{u}\,(t)+K_f$ where $\overline{K}$ is the average coefficient of permeability, α the coefficient of variation and $\bar{u}$ the average excess pore pressure.

The solution of the consolidation problem under the above condition is

$$\frac{\bar{u}}{\bar{u}_o} = \frac{\overline{W}_r}{\overline{W}_r + \mu\,(1-\overline{W}_r)} \qquad \text{...(2a)}$$

where $\overline{W}_r = e^{-\frac{8\,T_{vr}}{F(n)}}$ ...(2b)

$$\mu = \frac{K_o}{K_r} \qquad \text{...(2c)}$$

$$F(n) = \frac{n^2}{n^2-1}\,log\,(n) - \frac{3n^2-1}{4n^2} \qquad \text{...(2d)}$$

$$T_{vr} = \frac{C_{vr}\,t}{4d^2} \qquad \text{...(2e)}$$

in which $s=b/a$ and $n=d/a$ ...(2*f*)

d = radius of influence of drain

b = radius of smeared zone

a = radius of drain

T_{vr} = time factor for radial consolidation

C_{vr} = coefficient of radial consolidation

For constant load, constant permeability and no smear case, equation (2) reduces to

$$U_r = 1-\bar{w}_r = 1-e^{-\frac{8T_{vr}}{F(n)}} \qquad \text{...(3)}$$

The total average consolidation U is given by

$$U = 1-(1-U_v)\,(1-U_r) \qquad \text{...(4)}$$

Substituting equations (1) and (2) in equation (4), the total degree of average consolidation can be obtained.

$$U = 1-\left[\frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}e^{-\frac{(2n+1)^2}{4}\pi^2\,T_{vv}}\right]\left[\frac{\overline{W}_r}{\overline{W}_r+\mu\,(2+\overline{W}_r)}\right] \qquad \text{...(5)}$$

$$\text{Let } V = \frac{8}{\pi^2} \sum_{n=0} \frac{1}{(2n+1)^2} e^{-\frac{(2n+1)^2}{4}\pi^2 T_{vv}}$$

and V is constant for a particular value of T_{vv}.

$$\therefore U = 1 - V\left[\frac{\overline{W}_r}{\overline{W}_r + \mu\,(1-\overline{W}_r)}\right]$$

$$\overline{W}_r = e^{-\frac{8\,T_{vr}}{F(n)}} = \frac{(1-U)\mu}{V + (\mu-1)(1-U)} \qquad \ldots(6)$$

Equation (6) can be rearranged as

$$-\frac{2\,\epsilon\,\lambda^2\,T_{vv}}{F(n)} = \log_e \frac{(1-U)\,\mu}{V+(\mu-1)(1-U)} \qquad \ldots(7)$$

where $\epsilon = \frac{C_{vr}}{C_{vv}}$ and $\lambda = \frac{H}{d}$

$$\therefore \lambda = \sqrt{-\frac{F(n)}{2\,\epsilon\,T_{vv}} \log_e \frac{(1-U)\,\mu}{V+(\mu-1)(1-U)}} \qquad \ldots(8)$$

For constant load, constant permeability and no smear case equation (8) reduces to

$$\lambda = \sqrt{\frac{-F(n)}{2\,\epsilon\,T_{vv}} \log_e \left(\frac{1-U}{V}\right)} \qquad \ldots(9)$$

Based on equation (9), design charts were presented by Jeebala Rao *et al* (1971) for getting the solution of sand drain problem for 90 per cent consolidation of the clay layer. The main disadvantage of these charts is similar such curves have to be prepared for different percentages of total consolidations. Another difficulty is several charts have to be seen for obtaining a solution. To circumvent the above limitation, a single nomograph is presented in this investigation for design purposes.

Equation (9) can be represented in a nomographic form, from which λ can be estimated for the given values of a, ϵ, T_{vv}, U and V.

Procedure for Constructing Nomograph for the Design of Sand Drains

The range of values, of practical interest, of all the variables on the right hand side of equation (9) are first obtained. Two parallel lines for the $F(n)$ and $\log_e\left(\frac{1-U}{V}\right)$ scales as shown in Figure 1 are drawn. The maximum and minimum values of $F(n)$ and $\log_e\left(\frac{1-U}{V}\right)$ are obtained. Subdivide both the lines, within the limits obtained, logarithmically. Both the scales should ascend in the same direction in case their product is to be obtained. The size of each scale is independent of the other and can be as large as the space allows for the given limits.

$$\text{Let } Y = F(n).\log_e\left(\frac{1-U}{V}\right) \qquad \text{...(10)}$$

The Y scale will always be on a line parallel with and between $F(n)$ and $\log_e\left(\frac{1-U}{V}\right)$ scale lines. Select a value of Y within the range of its scale and determine two sets of values of $F(n)$ and $\log_e\left(\frac{1-U}{V}\right)$ yielding the selected values of Y in the equation. Draw two lines connecting the corresponding determined values of $F(n)$ and $\log_e\left(\frac{1-U}{V}\right)$ on their respective scales. The intersection of these two lines locates the Y scale line and also the selected value of Y upon the Y scale line.

Draw lines connecting the upper values of the $F(n)$ and $\log_e\left(\frac{1-U}{V}\right)$ scales and between the lower ends of these scales. The corresponding values of Y for the top and bottom of the Y scale can then be determined from equation (10). The Y scale is sub-divided logarithmically between these two Y values.

$$\text{Let } X = \frac{Y}{2\epsilon} = \frac{F(n).\log_e\left(\frac{1-U}{V}\right)}{2\epsilon} \qquad \text{...(11)}$$

The X scale can be fixed similar to the previous mentioned procedure, the only difference being that the ϵ scale should be in the descending direction (opposite to the direction of Y scale)

$$\text{Let } \lambda^2 = \frac{X}{T_{vv}} = \frac{F(n).\log_e\left(\frac{1-U}{V}\right)}{2\epsilon T_{vv}} \qquad \text{...(12)}$$

After obtaining the position and limits of X scale, in a similar way fix λ^2 scale knowing the range of values of T_{vv}. Finally λ scale is fixed on the right hand side of the λ^2 scale. Thus the nomograph for design of sand drains is complete.

In the nomograph (Figure 1) curves are plotted between V and $\log_e\left(\frac{1-U}{V}\right)$ for various degrees of consolidation and therefore corresponding to the value V, the values of $\log_e\left(\frac{1-U}{V}\right)$ can be obtained for the given degree of consolidation.

Illustration

For example, consider that a suitable well diameter and spacing are to be found to obtain 90 per cent consolidation in a time of 110 days in a soil having consolidation characteristics $C_{vv} = 3.726 \times 10^{-4}$ cm²/sec and $C_{vr} = 1.344 \times 10^{-3}$ cm²/sec and of depth $(2H) = 1.5m$. There is free double drainage in the vertical direction.

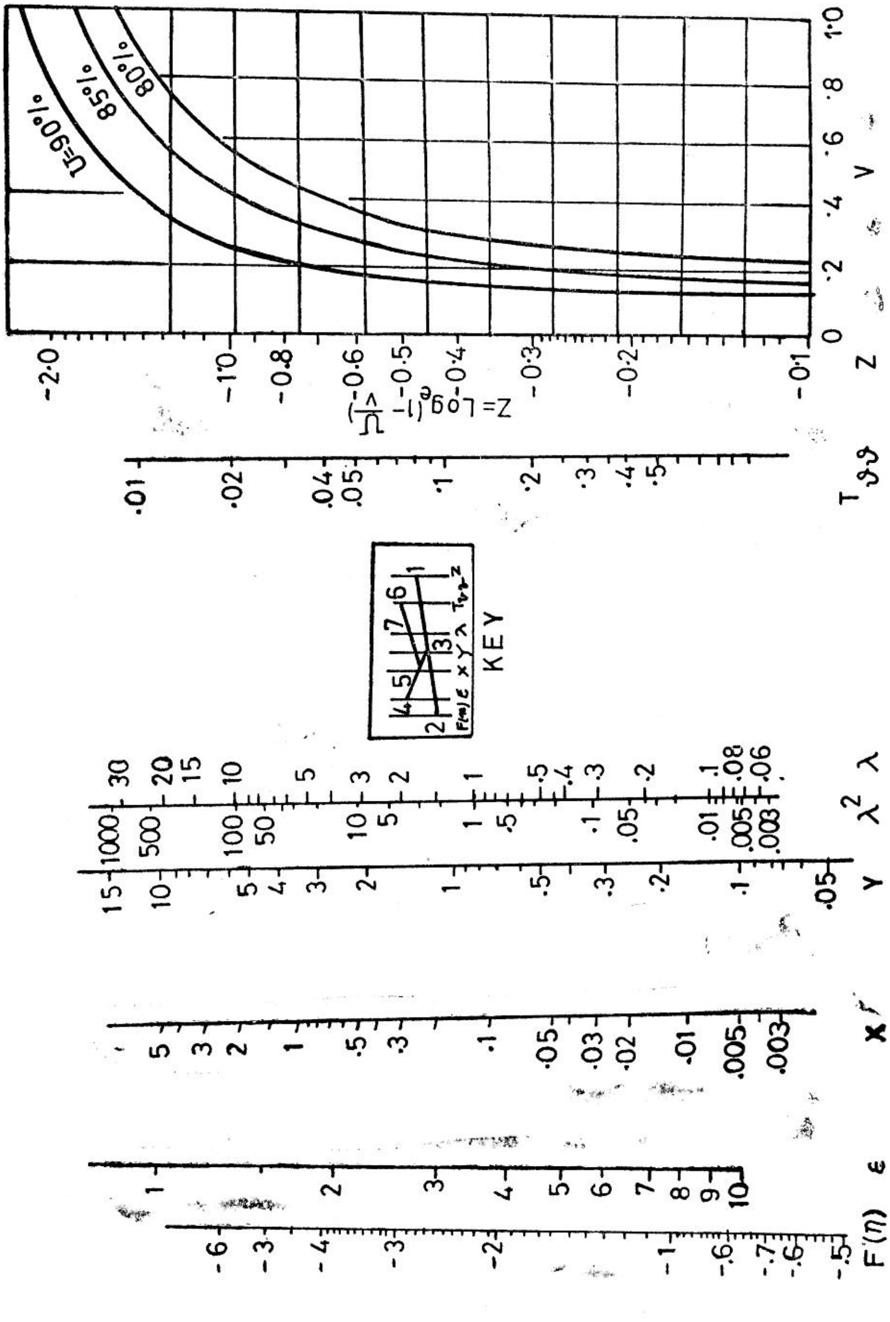

FIGURE 1. Nomogram for the design of sand drains

From equation 1(*b*), $T_{vv} = 0.6295$. It is known from the above that $\epsilon = \frac{C_{vr}}{C_{vv}} = 3.6$. From the obtained value of $T_{vv}=0.6295$, the value of V ($=0.18$) is obtained making use of Figure 2 for the case of double drainage. Corresponding to the value of V, the value of $\log_e\left(\frac{1-U}{V}\right)$ is observed as 0.6. In designing for the spacing of sand drain let it be assumed that the value of n is 5. The corresponding value of $F(n)$ is 0.936. Connect 0.6 on the "$\log_e \frac{1-U}{V}$" scale with 0.936 on the $F(n)$ scale and mark the intersection of this line on "Y" scale. Again connect this point with 3.6 on the ϵ scale which intersect the X scale. Now give connection between the intersection point on X scale and 0.6295 on the T_{vv} scale and mark the intersection on λ^2 scale. Finally the value of λ ($=0.34$) can be noted from the corresponding λ^2 value. From the value of λ ($=0.34$) obtained, the value of n can be obtained corresponding to the values of $H=0.75m$. $a=0.15m$. The value of n in this case is found to be 14.7.

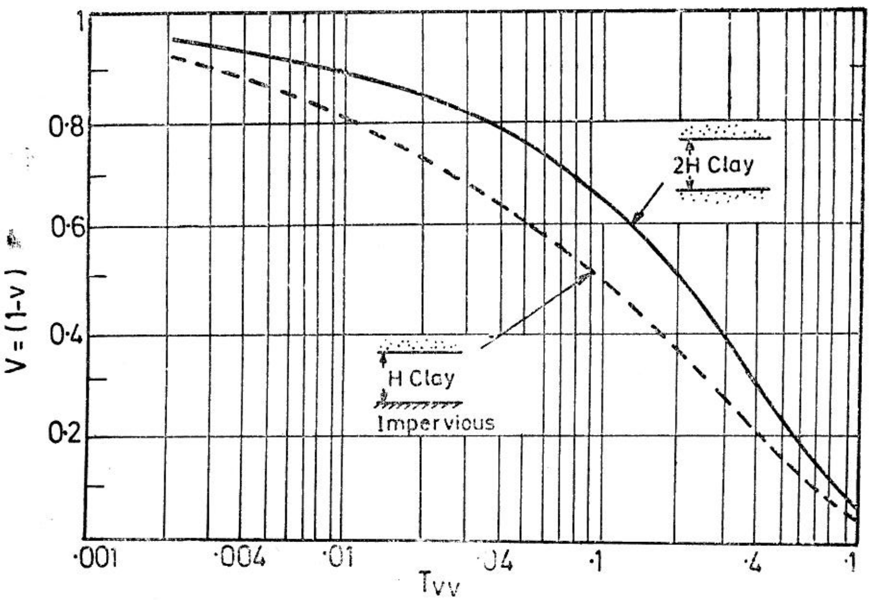

FIGURE 2. Graph between V and T_{vv}

Repeat the above steps for assumed (specified) values of $n=10$ and 15 and obtain the corresponding values of n making use of the nomograph and the results are tabulated as follows:

Specified value of n	Value of λ from chart	value of n obtained
5	0.34	14.70
10	0.45	11.13
15	0.50	10.00

Plot a curve as shown in Figure 3 between the specified values of n and those obtained from alignment chart.

From Figure 3, the design value of n is 11. The corresponding value of d is 1.65m. For square and triangular patterns, the spacing between the wells can be shown respectively as follows:

$$d = 0.564\ S$$

$$d = 0.525\ S$$

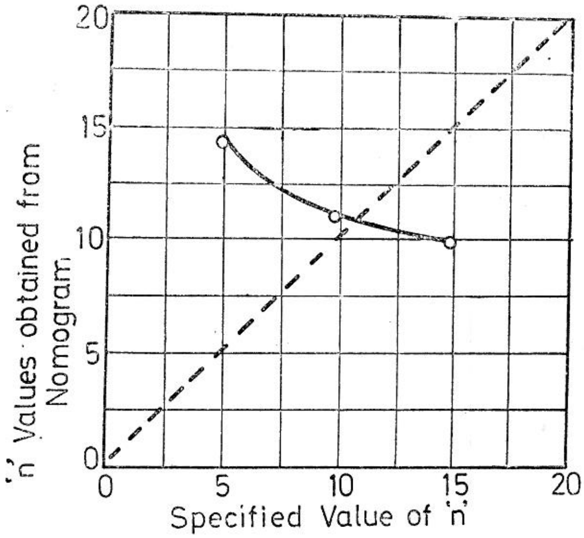

FIGURE 3. Plot between specified and obtained values of '*n*'

For square pattern, the spacing '*S*' for the sand drains comes to 2.925*m*. and for triangular pattern the spacing '*S*' is 3.143*m*.

Effect of Time and Thickness of Clay Layer on Sand Drains Spacing

The effect of time and thickness of clay layer on sand drain spacing is studied briefly in this investigation.

With the help of the nomograph, the sand drain spacing was obtained for different timings ranging from 30 days to 110 days. The solution was sought for $C_{vv}=3.726\times10^{-2}$ cm²/sec, $H=1.5m$. and for 90 percent consolidation. It is observed from Figure 4 that the relation between the time and sand drain spacing is linear.

The effect of clay thickness on sand drain spacing was obtained similarly for $t=180$ days; $C_{vv}=3.726\times10^{-4}$ cm²/sec and for different thickness of clay strata ranging from 1.5*m*. to 20 *cm*. It is observed from Figure 5 that the relation between thickness of clay strata to the sand drain spacing is non-linear.

In both the above cases double drainage was assumed.

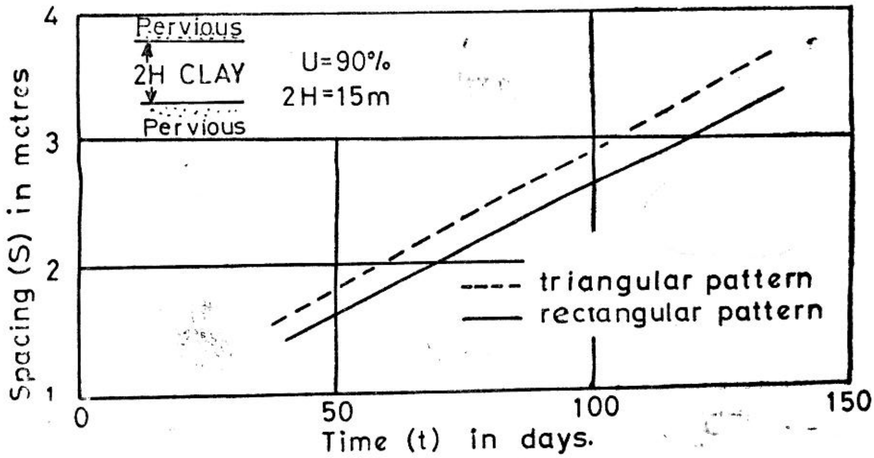

FIGURE 4. Graph between time and spacing

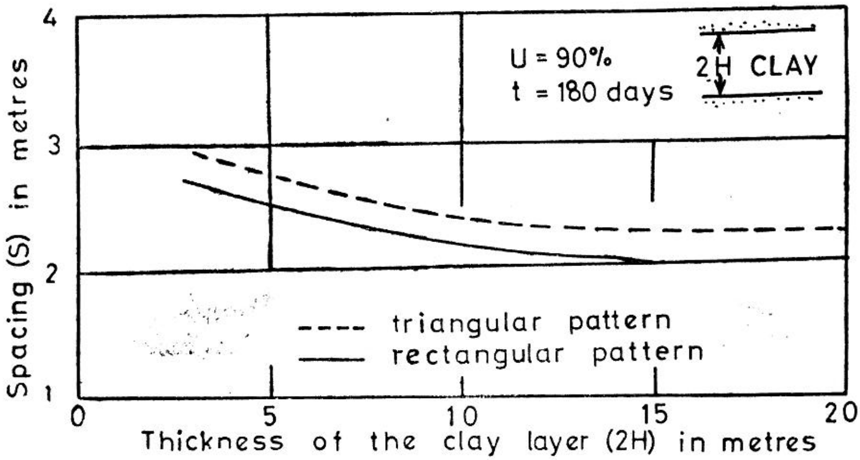

FIGURE 5. Graph between thickness and spacing

Conclusions

1. An effective method of design of sand drain installations has been presented for a constant load, constant permeability and no smear case by using alignment chart. The chief advantage of this method is that one nomograph is sufficient for obtaining the design parameters for various values of percentage of consolidation. Knowing the soil properties and time to achieve the required average degree of consolidation, the drain configuration can be obtained efficiently by entering into the nomograph.

2. The relation between the time and sand drain spacing is found to be linear when other parameters of the sand drain and soil strata are kept constant.

3. Non-linear relationship is observed between the thickness of clay layer and sand drain spacing when other parameters of the sand drain and soil strata are kept constant.

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