Short Communication

Stress Distribution under Embedded Circular Footings

by

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Introduction

Stress distribution in soils under surface loads is given by Boussinesq's theory (Boussinesq, 1885) and also by Westergaard's theory (Westergaard, 1938). However, foundations are mostly embedded and as such stress distribution under subsurface loads becomes an important aspect in foundation engineering. Mindlin's theory (Mindlin, 1936) is used for such loads. The assumptions made in Boussinesq and Mindlin theories are identical. Geddes (1975) made use of Boussinesq-Mindlin theory to develop solutions for axi-symmetric sub-surface loads.

Westergaard (1938) considered not only surface point load but also point load inside a semi-infinite body. Extensive solutions making use of Westergaard's theory for surface loads are available (Babu Shanker, 1973) but not for the sub-surface loads. Westergaard-Mindlin (or subsurface Westergaard) solutions are presented in this note for circular foundations and these are compared with other similar solutions.

Analysis

The vertical stress (σ_z) at a radial distance r and at a depth Z below the surface due to a point load (P) acting at a depth D ($<Z$) below the surface (see Figure 1a) is given by Westergaard (1938) as:

$$\sigma_z = \frac{P}{4\pi}\left[\frac{C\,(Z-D)}{\{r^2+C^2\,(Z-D)^2\}^{3/2}}+\frac{C\,(Z+D)}{\{r^2+C^2\,(Z+D)^2\}^{3/2}}\right] \qquad \ldots(1)$$

where $C^2=(1-2\nu)/(2-2\nu)$, ν being the Poisson's ratio of the medium. Equation (1) with suitable integration will lead to the following solution of the circular subsurface load problem shown in Figure 1b:

$$\sigma_z = q\left[1-\tfrac{1}{2}\left\{\frac{C\,(Z-D)}{(R^2+C^2\,(Z-D)^2)^{1/2}}+\frac{C\,(Z+D)}{(R^2+C^2\,(Z+D)^2)^{1/2}}\right\}\right] \qquad \ldots(2)$$

where σ_z refers to the vertical stress at a depth Z directly under the centre of circular footing of radius R and depth D with a loading intensity of q. Using $n=R/D$ and $m=Z/D$, Equaion (2) can be written as:

$$\sigma_z = q\left[1-\tfrac{1}{2}\left\{\frac{C\,(m-1)}{\{n^2+C^2\,(m-1)^2\}^{1/2}}+\frac{C\,(m+1)}{\{n^2+C^2\,(m+1)^2\}^{1/2}}\right\}\right]$$

$$= q\,I \qquad \ldots(3)$$

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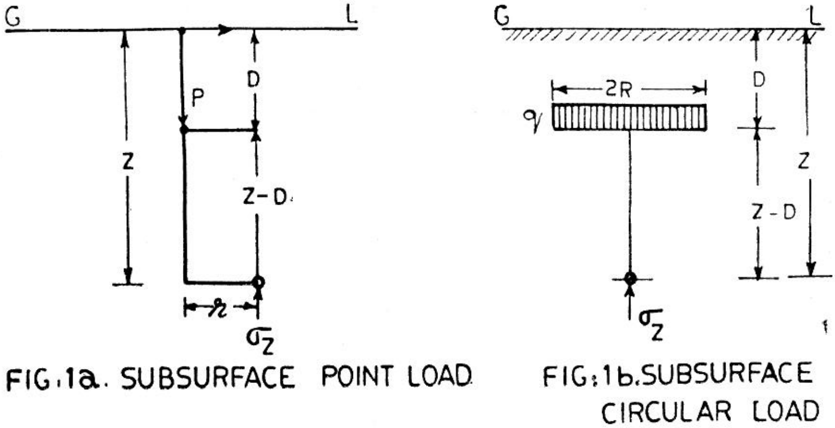

FIGURE. 1. ***a.*** **Subsurface point load**
b. **Subsurface circular load**

where I is the influence factor, which is a function of non-dimensional parameters m, n and C (or ν). For an incompressible medium with $\nu=0.5$, $I=1.0$. Equation (3) may be programmed for a computer to obtain data to enable drawing Newmark type influence charts. For example with $\nu=0$ and $m=1.5$, one obtains the following relationship between I and n:

I	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$n=R/D$	0.24	0.45	0.68	1.00	1.50	2.15	3.15	5.10	10.5

This data may be used to construct a Newmark type influence chart with a (reference) scale length showing the magnitude of D (say 1cm) on the chart.

Illustration and Comparison

Use of Equations (2) and (3) developed above is depicted in Figure 2 which shows the variation of vertical stress with depth in a medium with $\nu=0$ below an embedded circular footing. This solution of Westergaard-Mindlin is compared with other solutions like Boussinesq-Mindlin, Boussinesq and Westergaard. The corresponding analytical solution for vertical stress under embedded circular footing using equivalent Boussinesq theory (where depth of embeddment is neglected) as suggested by Geddes (1975) is:

$$\sigma_z = q\left[1 - \frac{(m-1)^3}{\{n^2+(m-1)^2\}^{3/2}}\right] \quad \ldots(4)$$

On the other hand, Boussinesq-Mindlin solution (Geddes, 1975) for the problem yields:

$$\sigma_z = q\left[1 - \frac{1}{4(1-\nu)}\left\{\frac{(1-2\nu(m-1)}{G} - \frac{(1-2\nu)(m-1)}{H} + \frac{(m-1)^3}{G^3} + \frac{(3-4\nu)\,m\,(m+1)^2-(m+1)(5m-1)}{H^3} + \frac{6m\,(m+1)^3}{H^5}\right\}\right] \quad \ldots(5)$$

where $G^2 = n^2 + (m-1)^2$ and $H^2 = n^2 + (m+1)^2$.

The corresponding solution due to equivalent Westergaard (Babu Shanker, 1973) is:

$$\sigma_z = q \left[1 - \frac{C(m-1)}{\{n^2+C^2\,(m-1)^2\}^{1/2}} \right] \qquad \ldots(6)$$

It is seen from Figure 2 that embedded solutions yield lower values for the stress as compared to the equivalent (surface) solutions neglecting the depth of embeddment. It is also seen that for $\nu=0$, Westergaard's solutions (whether surface or subsurface) yield lower values than those due to Boussinesq (1). Thus the Westergaard-Mindlin solution yields the lowest values for the vertical stress. Similar results have been shown in respect of embedded square footings by *Kul Bhushan and Baley (1976).*

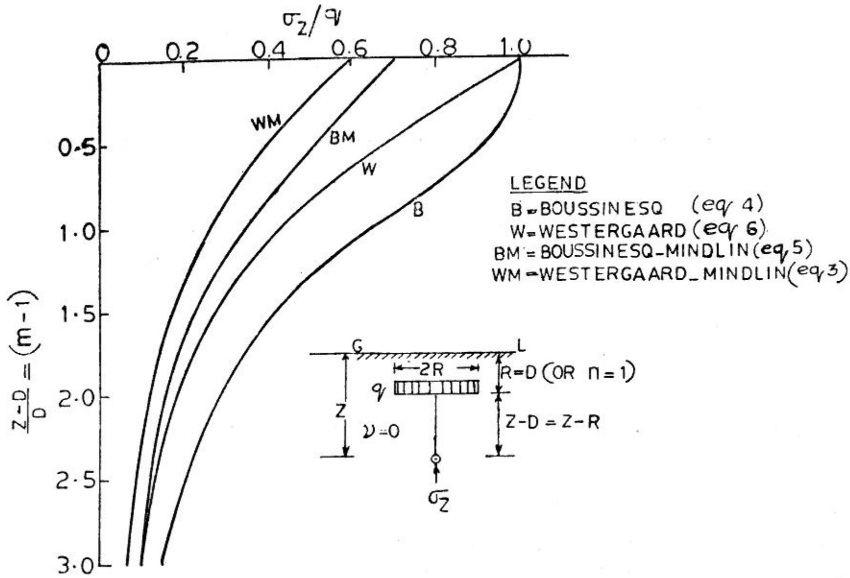

FIGURE. 2. Stress distribution under center of circular footing

Conclusions

Equations are presented for estimating vertical stresses under the centre of the embedded circular footing in a Westergaard material. Equivalent surface footing solutions overestimate the stresses. Westergaard's solutions (with $\nu=0$) yields lower values than the corresponding Boussinesq solutions.

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