Limiting of Settlement of Shallow Foundations in Normally Consolidated Clay

by

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Introduction

Settlement of foundations is computed from a knowledge of the stress distribution induced in the medium by the foundation structure and the consolidation characteristics of the soil. Formulae based on theory of elasticity are normally used in the calculations of stress distribution and the consolidation characteristics of the medium are determined from standard oedometer tests conducted on undisturbed samples. Boussinesq's equation is ordinarily used for stress distribution calculations. Whereas the Boussinesq's equation gives the stress distribution due to a load at the surface of a semi-infinite elastic half space, Mindlin's equation gives the stress distribution due to a load at the interior of the half space, a relatively more realistic condition for foundations, however, less popular because of the tedium involved in the use of Mindlin's equation. The authors in an earlier paper (Kaniraj and Ranganatham, 1974) have shown that treating the overburden or depth of embedment of footings simply as surcharge and using Boussinesq stress distribution instead of the more appropriate Mindlin stress distribution will result in conservative (large) estimate of settlement. Either of the two parameters, coefficient of volume compressibility (m_v) and compression index (C_c) borne out of oedometer test results, is frequently used to define the consolidation characteristics of the soil. Since C_c is a constant in the normally consolidated range, the computation procedure using C_c is simpler than that which makes use of m_v which varies nonlinearly with the stress level even for normally consolidated clay.

In the design of a footing, generally a tentative plan dimension, complying with the bearing capacity criterion, is chosen to start with. The settlement of such a foundation is then determined, assuming a certain level of load transfer. If the settlement is found to exceed the desirable limits it is brought within limits by either of one or by a combination of the following two methods, namely, *increasing the depth of embedment of the foundation* and increasing the bearing area of the foundation. In the first approach the level of load transfer is shifted to a deeper level which is more resistant to settlement compared to any level above this, because of stress history and stress state pertaining to respective levels. This is true in the case of all uniform deposits and further, in the case of uniform deposits of limited

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thickness the depth of compressible layer below the level of load transfer gets reduced, thus aiding the reduction of settlement. In the second approach, because of increase in the bearing area there is a decrease in the stress increase the all levels within the soil medium (at and below the level of load transfer) which consequently leads to less settlement. A third, but a little obscure method of limiting settlement would be by adopting appropriate plan dimensions. The settlement of a footing and its plan dimensions are inter-dependent, hence a change in the plan dimension of a footing keeping the other factors of the problem such as load, depth of embedment and the characteristics of the medium the same, involves a corresponding change in the settlement. Analyses to quantify the effects of depth of embedment, bearing area and that of plan dimensions on the settlement of footings have been developed and are presented in the following sections. The analyses developed for rectangular and square loaded areas are based on Boussinesq's and Mindlin's equations, separately, for stress distribution. The medium is assumed to be uniformly normally consolidated from the surface and to be a semi-infinite, homogeneous, isotropic and linearly elastic continuum for the purpose of computing stress distribution

Settlement of Footings

The compression of an element dz (Figure 1) vertically below the centre of a uniformly loaded area of $2a \times 2b$ embedded at a depth of h below the surface is given by,

$$\triangle \rho = \frac{C_c}{1+e_o} \log \frac{P_z+\sigma_z}{P_z} dz \qquad \ldots(1)$$

where, $\triangle \rho$ = vertical compression of the element

P_z = overburden pressure at z

σ_z = increase in pressure at z

and e_o = average initial void ratio.

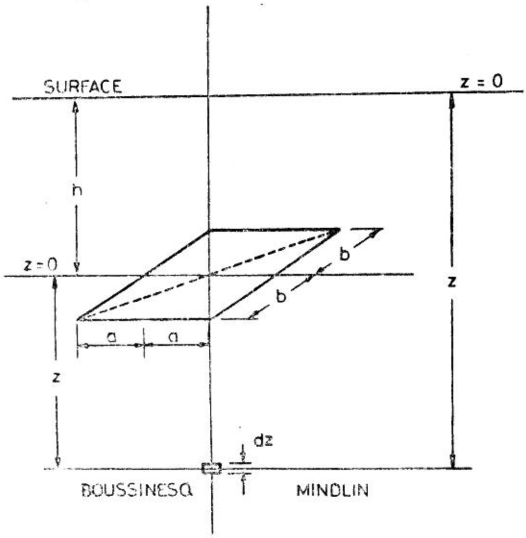

FIGURE 1. Loaded area in the interior of semi-infinite deposit

Figure 1 shows the origins of ordinate z, left half of the figure being for Boussinesq stress distribution and the right half for Mindlin stress distribution. The expression for vertical stress at depth z according to Boussinesq's equation (σ_{zB}) is given by (Harr 1966),

$$\sigma_{zB} = \frac{2q}{\pi}\left[\frac{abz\,(a^2+b^2+2z^2)}{(a^2+z^2)\,(b^2+z^2)\sqrt{a^2+b^2+z^2}} + \sin^{-1}\frac{ab}{\sqrt{a^2+z^2}\sqrt{b^2+z^2}}\right] \quad ...(2)$$

where q = intensity of loading.

Mindlin's equation for vertical stress (σ_{zM}) is given by (Skopek 1960),

$$\sigma_{zM} = \frac{q}{\pi\,(1\text{-}\mu)}\Bigg[(1-\mu)\left\{\text{arctg}\frac{ab}{(z-h)R_1} + \text{arctg}\frac{ab}{(z+h)R_2}\right\} + \frac{(z-h)\,aR_1}{2br_1^2}$$
$$-\frac{a\,(z-h)^3}{2br_3^2R_1} + \frac{a\,\{(3-4\mu)\,z\,(z+h) - h\,(5z-h)\}\,R_2}{2br_2^2\,(z+h)}$$
$$-\frac{\{(3-4\mu)\,z\,(z+h)^2 - h\,(z+h)\,(5z-h)\}\,a}{2br_4^2\,R_2} + \frac{2hz\,(z+h)\,aR_2^3}{b^3\,r_2^4}$$
$$+\frac{3hza\,R_2\,r_5^2}{(z+h)b^3\,r_2^2} - \frac{hz\,(z+h)^3a}{br_4^4\,R_2}\left\{\frac{2b^2-(z+h)^2}{b^2} - \frac{a^2}{R_2^2}\right\}\Bigg] \quad ...(3)$$

where, $R_1^2 = a^2+b^2+(z-h)^2$; $r_1^2 = a^2+(z-h)^2$;
$r_3^2 = b^2+(z-h)^2$; $R_2^2 = a^2+b^2+(z+h)^2$;
$r_2^2 = a^2+(z+h)^2$; $r_4^2 = b^2+(z+h)^2$;
$r_5^2 = b^2-(z+h)^2$ and μ = Poisson's ratio.

The settlement at the centre of the loaded area, ρ, is obtained by integration of Equation 1 substituting for σ_z value.

$$\rho = \frac{1}{C}\int_0^\infty \log\left(1+\frac{\sigma_{zB}}{\gamma(z+h)}\right)dz \quad ...(4)$$

for Boussinesq stress distribution and

$$\rho = \frac{1}{C}\int_0^\infty \log\left(1+\frac{\sigma_{zM}}{\gamma z}\right)dz \quad ...(5)$$

for Mindlin stress distribution. In Equations 4 and 5 C $(=(1+e_o)/C_c)$ is assumed a constant and γ is the bulk density of the soil. The following substitutions are made in Equations 4 and 5.

$$a = AK \quad ...(6a)$$
$$b = BK \quad ...(6b)$$
$$z = ZK \quad ...(6c)$$
$$h = HK \quad ...(6d)$$
$$q = QK\gamma \quad ...(6e)$$

where A, B, Z, H and Q are nondimensional parameters and K is a constant having the dimension of length. Following these substitutions, Equations 4 and 5 transform to,

$$\rho = \frac{K}{C} S_F \qquad \ldots(7)$$

where S_F is a dimensionless settlement factor. S_F in Equation 7 with Boussinesq stress distribution is given by,

$$S_F = \int_0^\infty \log \left[\frac{2Q}{(Z+H)\pi} \left\{ \frac{ABZ\,(A^2+B^2+2Z^2)}{(A^2+Z^2)\,(B^2+Z^2)\,\sqrt{A^2+B^2+Z^2}} + \sin^{-1} \frac{AB}{\sqrt{A^2+Z^2}\sqrt{B^2+Z^2}} \right\} + 1 \right] dZ \qquad \ldots(8)$$

and S_F in Equation 7 with Mindlin stress distribution is given by,

$$S_F = \int_H^\infty \log \left[\frac{Q}{\pi(1-\mu)} \left\{ \frac{(1-\mu)}{Z} \left\{ \text{arctg} \frac{AB}{(Z-H)R_{11}} + \text{arctg} \frac{AB}{(Z+H)R_{22}} \right\} + \frac{(Z-H)AR_{11}}{2BZ\,r^2_{11}} - \frac{A(Z-H)^3}{2B\,r^2_{33}\,R_{11}\,Z} + \frac{\{(3-4\mu)\,Z\,(Z+H)-(5Z-H)H\}\,AR_{22}}{2(Z+H)B\,r^2_{22}\,Z} - \frac{\{(3-4\mu)\,Z\,(Z+H)^2-H\,(Z+H)\,(5Z-H)\}A}{2B\,r^2_{44}\,R_{22}\,Z} + \frac{2H\,(Z+H)\,AR^3_{22}}{B^3\,r^4_{22}} + \frac{3HAR_{22}\,r^2_{55}}{(Z+H)B^3\,r^2_{22}} - \frac{(Z+H)^3AH}{Br^4_{44}\,R_{22}} \left\{ \frac{2B^2-(Z+H)^2}{B^2} - \frac{A^2}{R^2_{22}} \right\} \right\} + 1 \right] dZ \qquad \ldots(9)$$

where, $R^2_{11} = A^2+B^2+(Z-H)^2$

$r^2_{11} = A^2+(Z-H)^2$

$r^2_{33} = B^2+(Z-H)^2$

$R^2_{22} = A^2+B^2+(Z+H)^2$

$r^2_{22} = A^2+(Z+H)^2$

$r^2_{44} = B^2+(Z+H)^2$

and $r^2_{55} = B^2-(Z+H)^2$

The theory dealt with in the preceding paragraphs concerns with stresses and settlements due to uniform loading of an area. However, foundations tend to impose conditions rather close to uniform displacement than to uniform loading. For accurate settlement calculations it is desirable to allow for the effect of rigidity of foundations. In elastic displacement theory an approximation to the uniform displacement is obtained from the maximum and minimum displacements of a uniformly loaded area (Davis and Poulos 1968). The rigid footing displacement is known to be close to the mean displacement of the uniformly loaded area (Fox 1948). The approximate mean displacement is obtained by assuming the displaced surface under the loaded area to be parabolic, in which case the centre and corner dispacements define the mean dispacement of the rectangular footings completely. The approximation gives,

For a rectangle, mean displacement $=\frac{1}{3}\,(2\,\rho_{centre}+\rho_{corner})$ ...(10)

Adopting the above approximation for consolidation settlement also the mean settlement of the loaded area, ρ_m, is given by,

$$\rho_m = \frac{K}{C}\, S_{Fm} \qquad ...(11)$$

where, S_{Fm} = mean settlement factor.

For a rectangle, $S_{Fm} = \frac{1}{3}\,(2S_{F\text{-}centre}+S_{F-corner})$

Equations 8 and 9 give expressions for $S_{F-centre}$. Similar expressions for $S_{F\text{-}corner}$ can be found for the two approaches for stress distribution using which the mean settlement factor can be finally computed.

The above is the analysis for settlement of a rectangular loaded area in general. A procedure is now developed using which the settlement of different loaded areas embedded in the same medium could be compared in term of comparable settlement factors. The procedure helps to quantitatively assess the effects of depth of embedment, change in the bearing area and that of plan dimensioning on the settlement of footings.

Let X and Y be two footings or loaded areas as shown in Figure 2 embedded in the same medium. Let a_x, b_x be the half sides of the loaded

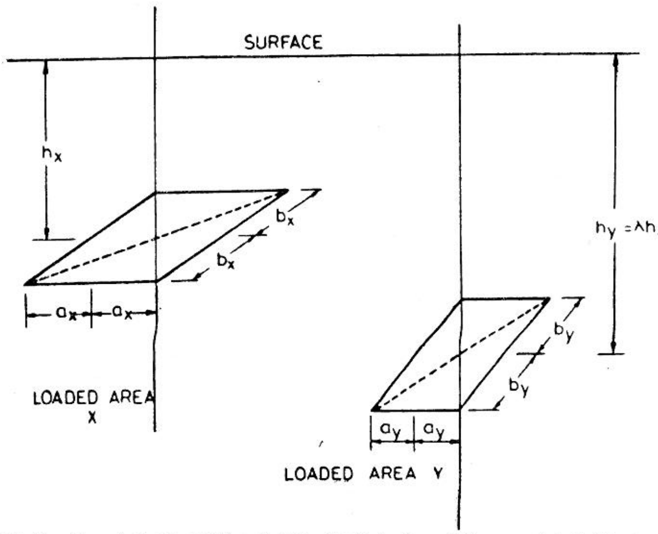

FIGURE 2. Loaded areas X and Y in the interior of the semi infinite deposit

area X. Let q_x be the load intensity on X and h_x be its depth of embedment. Then for X,

$$a_x = A_x K_x \qquad ...(12a)$$

$$b_x = B_x K_x \qquad ...(12b)$$

$$z = Z_x K_x \quad ...(12c)$$

$$q_x = Q_x K_x \gamma \quad ...(12d)$$

$$h_x = H_x K_x \quad ...(12e)$$

and

$$\rho_{mx} = \frac{K_x}{C} S_{Fmx} \quad ...(13)$$

where ρ_{mx} is the mean settlement of X. Similarly let a_y and b_y be the half sides of the loaded area Y whose area is $(1+p)$ times that of X, where p is given by,

$$p = (\text{area of } Y - \text{area of } X)/\text{area of } X \quad ...(14)$$

and p can be either positive or negative. Let q_y be the load intensity on Y and h_y be its depth of embedment. Then for Y,

$$a_y = A_y K_y \quad ...(15a)$$

$$b_y = B_y K_y \quad ...(15b)$$

$$z = Z_y K_y \quad ...(15c)$$

$$q_y = Q_y \gamma K_y \quad ...(15d)$$

$$h_y = H_y K_y \quad ...(15e)$$

The mean settlement of Y, ρ_{my} is given by the expression,

$$\rho_{my} = \frac{K_y}{C} S_{Fmy} \quad ...(16)$$

The S_{Fm} value of X and Y according to Equations 13 and 16 can be directly compared to know the relative settlement of X and Y only when K_x is equal to K_y. Equation 16 can be expressed as,

$$\rho_{my} = \frac{K_x}{C} S'_{Fmy} \quad ...(17)$$

whereby S'_{Fmy} is the mean settlement factor for Y which is directly comparable to S_{Fmx} in Equation 13. Equation 17 is derived in the following manner. From the bearing areas of X and Y,

$$a_y b_y = a_x b_x (1+p) \quad ...(18)$$

Substituting for a_x, a_y, b_x and b_y in Equation 18 from Equations 12 and 15,

$$A_y B_y K_y^2 = A_x B_x K_x^2 (1+p) \quad ...(19)$$

which gives,

$$K_y = K_x \sqrt{\frac{A_x B_x}{A_y B_y} (1+p)} \quad ...(20)$$

Substituting for K_y in Equation 16,

$$\rho_{my} = \frac{K_x}{C} \sqrt{\frac{A_x B_x}{A_y B_y} (1+p)} \quad S_{Fmy} \quad ...(21)$$

Denoting,

$$S'_{Fmy} = \sqrt{\frac{A_x B_x}{A_y B_y} (1+p)} \quad S_{Fmy} \quad (22)$$

which is the settlement factor as in Equation 17 which is directly comparable to S_{Fmx}. Similarly, the equivalence between the non-dimensionalised parameters Q_x and Q_y and between H_x and H_y can be established in the following manner.

From the known loads on the two respective loaded areas,

$$q_x\, a_x\, b_x = M\, q_y\, a_y\, b_y = M\, q_y\, a_x\, b_x\, (1+p) \qquad \ldots(23)$$

where $M = P_x/P_y$, P_x and P_y being the total load on X and Y respectively. Substituting in Equation 23 for a_x, b_x, q_x and q_y from Equations 12 and 15,

$$Q_x \gamma K_x = Q_y \gamma K_y\, (1+p)\, M \qquad \ldots(24)$$

Substituting for K_y from Equation 20 and on simplification,

$$Q_y = \frac{Q_x}{M} \sqrt{\frac{A_y B_y}{A_x B_x}} \left(\frac{1}{\sqrt{1+p}}\right)^3 \qquad \ldots(25)$$

The relationship between the depths of embedment h_x and h_y is given by,

$$h_y = \lambda\, h_x \qquad \ldots(26)$$

where $\lambda > 0$. Substituting for h_x and h_y from Equations 12 and 15 and on simplification,

$$H_y = \lambda H_x \sqrt{\frac{A_y B_y}{A_x B_x\, (1+p)}} \qquad \ldots(27)$$

In order to find the relative settlements of the two loaded areas X and Y, S_{Fmx} and S'_{Fmy} according to Equations 13 and 22 respectively are calculated and the ratio of these values give the relative settlements of X and Y.

Results

Calculations for S_{Fm} values have been carried out with a view to quantify the effects of depth of embedment, the change in the bearing area and that of plan dimensioning, separately, on settlement of foundations. Mean settlement factors in each case have been calculated using the two approaches for stress distribution, namely Boussinesq and Mindlin. Settlement factor values have been calculated by numerical integration of the appropriate expressions using Simpson's one-third rule formula. The settlement factors are calculated up to a Z value after which the addition in settlement due to the next segment does not exceed 0.001. S_F values are computed keeping the non-dimensionalised width parameter A (A_x, A_y) to be unity in all the cases and μ is taken as 0.5 for solutions with Mindlin stress distribution. The results of the three different cases of the study are as presented below.

(a) Effect of Depth of Embedment: In this cases the settlements of two equally loaded areas of identical plan dimensions but embedded at different levels within the medium are considered. The relationship between two such loaded areas and their parameters are defined as follows:

$$A_x = A_y = 1;\; B_x = B_y;\; M = 1;\; p = 0;$$

$$Q_x = Q_y \text{ and } H_y = \lambda H_x$$

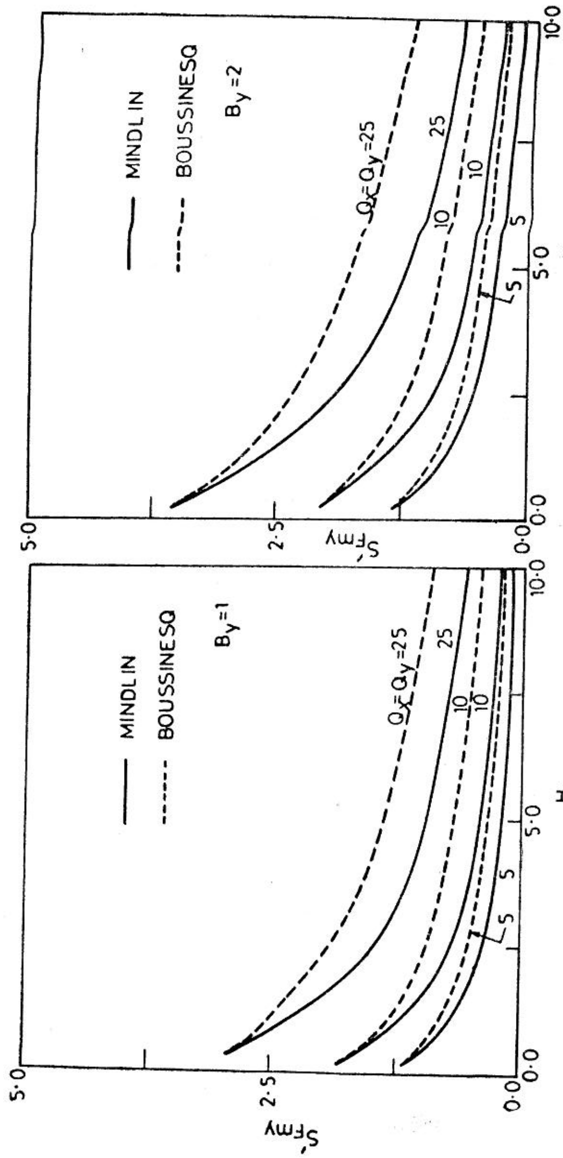

FIGURE 3 *a* & *b*. Variation of mean settlement factor, S_{Fmy} with depth of embedment parameter, H_y

From the above and from Equation 22 it follows that

$$S'_{Fmy} = S_{Fmy}$$

or in other words S_{Fmy} itself is directly comparable to S_{Fmx} in this case.

With A_x as unity, calculations for S_{Fmx} are carried out varying B_x from 1 to 10 and Q_x from 5 to 25 for a constant value of H_x equal to unity. Similarly A_y as unity calculations for S'_{Fmy} (or S_{Fmy}) have been carried out for the following range of variation of parameters: B_y from 1 to 10 and Q_y from 5 to 25. The various depths of embedment for Y relative to X could be selected by varying the value of λ, which has been varied from 0.25 to 10. Thus the loaded area X in this case has a constant depth of embedment, defined by $H_x=1$, against the settlement of which the settlement of an identical load area Y embedded at various depths, defined by $H_y=0.25$ to 10, will be compared. At $H_y=1$ the settlement of Y is equal to that of X. Figures 3*a* and 3*b* shows the variation of S'_{Fmy} with H_y for $B_y=1$ and $B_y=2$ respectively. The curves in full line in them are for Mindlin stress distribution and those in dashed lines are for Boussinesq stress distribution. This helps to bring out distinctly the influence of the approach for stress distribution on S_{Fm} values. Since the current practice is to use Boussinesq stress distribution, the use of more appropriate Mindlin stress distribution could be accounted by a reduction factor, F, defined as $S_{Fm\text{-}Mindlin}/S_{Fm\text{-}Boussinesq}$. The variation of F (the definition of F here would be $S'_{Fmy\text{-}Mind.}/S'_{Fmy\text{-}Bous.}$) with H is reported in Figures 4*a* and 4*b* each figure being for a particular value of B_y and each curve in a figure shows the variation for one particular value of Q_y. From Figures 3 and 4 it is evident that for values of H_y below 1 there is no significant difference (less than 10%) between the S_{Fm} values computed with Boussinesq stress distribution and Mindlin stress distribution, other conditions remaining the same. But beyond $H_y=1$ there is a considerable increase in the difference between S_{Fm} values computed with the two types of distribution. Hence the settlements computed based on Boussinesq's equation tend to err highly on the conservative side beyond $H_y=1$. From Figures 4 it is evident that for square footings with depth of embedment same as the side of the footing (i.e. $H_y=2$), consideration of the overburden as surcharge and the use of Boussinesq stress distribution instead of the more appropriate Mindlin stress distribution accounts for about 30% over-prediction of settlement. It is also evident from Figure 4 that there is a slight reduction in this magnitude of overprediction as the load level (Q) on the area increases and also as B_y increases.

In order to quantify the effect of depth of embedment on settlement of footings, values of ratio R defined as,

$$R = S'_{Fmy}/S_{Fmx} = S'_{Fmy(Hy)}/S'_{Fmy(Hy=1)} \qquad \ldots(28)$$

have been calculated for identical loaded areas with difference in depth of embedment. The ratio R gives the relative settlement of a loaded area Y (with H_y) with respect to an identical loaded area X with $H_x=1$ (or the same loaded area Y with $H_y=1$). The variation of R with H_y is shown in Figures 5 and 6 for Mindlin and Boussinesq stress distribution respectively. It is evident from these figures that as the depth of embedment of a loaded area is increased in a uniform deposit of normally consolidated clay its settlement decreases. This effect is so pronounced that for square footings ($B=1$) when H_y increases from, say 1 to 3 (the normal range of depth of

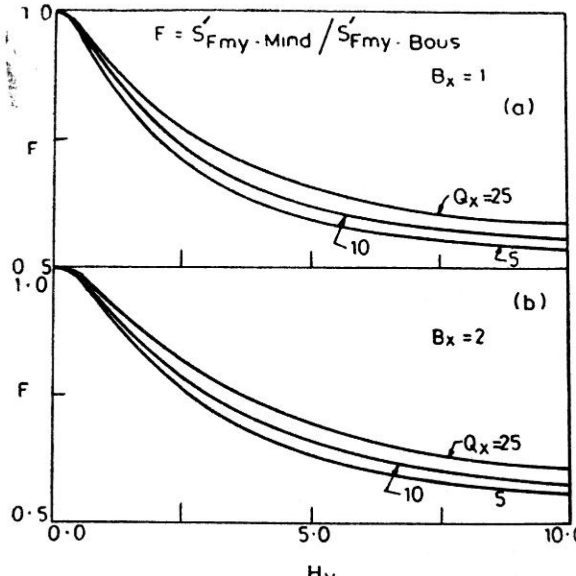

FIGURE 4*a* and *b*. Variation of F with embedment parameter, H_y

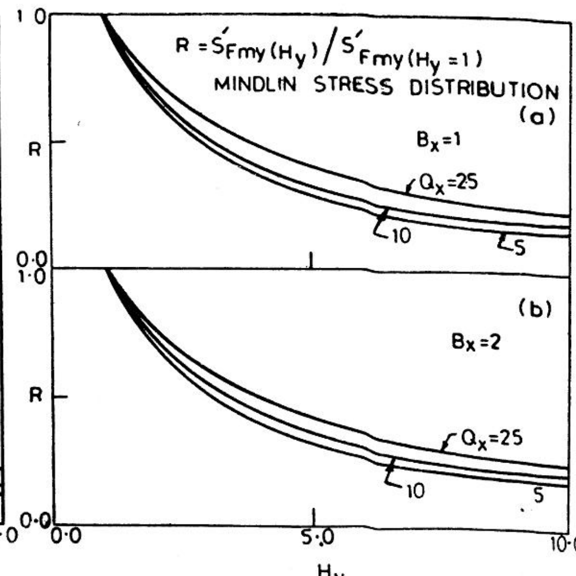

FIGURE 5*a* and *b*. Variation of R with depth of embedment parameter, H_y for Mindlin stress distribution

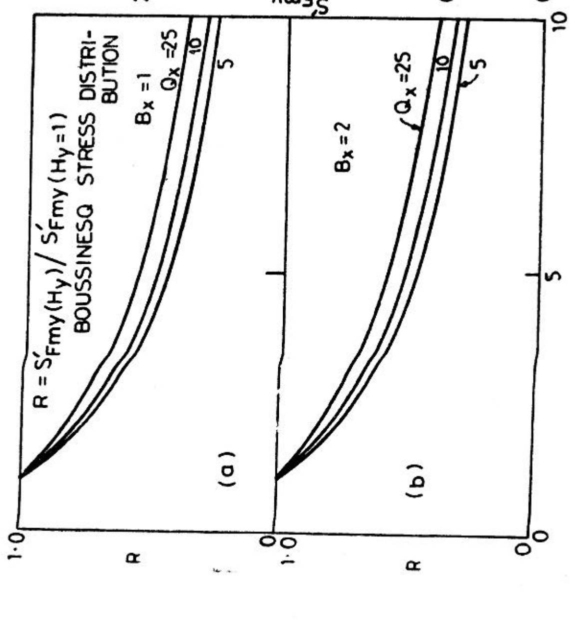

FIGURE 6*a* and *b*. Variation of *R* with depth of embedment parameter, H_y for Boussinesq stress distribution

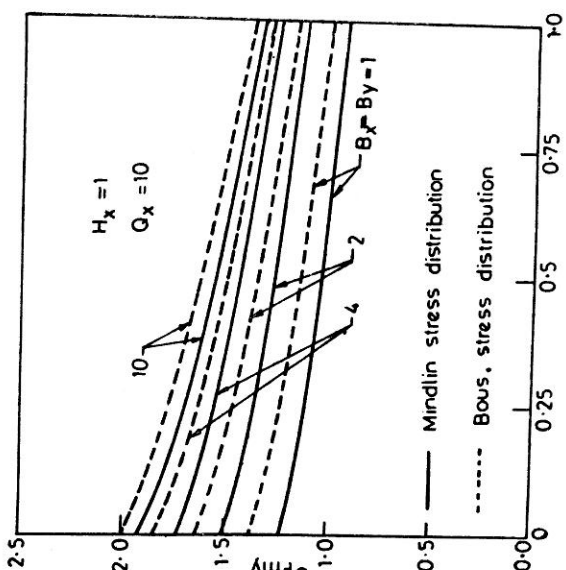

FIGURE 7*a* and *b*. Variation of mean settlement factor, S'_{Fmy} with *p*

embedment of shallow foundations) the reduction in settlement is found to be of the order of 40 to 50% for Mindlin stress distribution and 30 to 40% for Boussinesq stress distribution. A widely prevalent practice whenever the settlement limits are not complied with, is to increase the depth of embedment of shallow foundations. However, such a quantitative improvement in terms of settlement response does not seem to be that well appreciated. It could be seen from Figures 5 and 6 that the improvement in settlement response slightly decreases with increase in the load level and also as the value of B_y increases.

(*b*) *Effect of Change in the Bearing Area:* The relative settlement of two equally loaded areas with one of them having an area of $(1+p)$ times that of the other but the same sides ratio and depth of embedment is considered here. Let X and Y be the two loaded areas, Y having an area of $(1+p)$ times that of X. The relationships between the parameters of X and Y will then be defined as follows:

$$A_x = A_y = 1 \qquad \ldots(29a)$$

$$B_x = B_y \qquad \ldots(29b)$$

$$M = 1 \qquad \ldots(29c)$$

$$\lambda = 1 \qquad \ldots(29d)$$

$$Q_y = Q_x\left(\frac{1}{\sqrt{1+p}}\right)^3 \qquad \ldots(29e)$$

and $$H_y = H_x/\sqrt{1+p} \qquad \ldots(29f)$$

S'_{Fmy} is given by,

$$S'_{Fmy} = \sqrt{1+p}\;\; S_{Fmy} \qquad \ldots(30)$$

Keeping the value of A_x as unity, calculations for S_{Fmx} are carried out varying B_x from 1 to 10, H_x from 1 to 3 and Q_x from 5 to 25. Different bearing areas for loaded areas Y could be chosen by varying the value of p and p has been varied from 0 to 1. The settlement of Y is compared to that of X and at $p=0$, X and Y are identical in all respects and their settlements are equal. For other values of p, the equivalent parameters (Q_y and H_y) for the calculation of S_{Fmy} are given by the relationship according to Equation 29 and S'_{Fmy} is obtained from Equation 30. Figure 7 shows the variation of S'_{Fmy} with p for $H_x=1$ and $Q_x=10$. The curves in full line in the figure are for Mindlin stress distribution and those in broken lines are for Boussinesq stress distribution. Each curve in the figure represents the variation for one particular value of B_y $(=B_x)$. In order to quantify the effect of change in bearing area on settlement of foundations, value of ratio R' defined as

$$R' = S'_{Fmy}/S_{Fmx} = S'_{Fmy\,(p)}/S'_{Fmy\,(p=0)} \qquad \ldots(31)$$

for corresponding values of H_x and H_y and Q_x and Q_y are computed. This ratio gives the relative settlement of Y with respect to X as a result of change in the area of X. The variation of R' with p is shown in Figure 8. It is observed that all the values of R' (for the chosen range of H_x, Q_x and B_x) lie within the limits indicated by dashed lines in the figure (for both Boussinesq and Mindlin stress distribution). The average variation of R' with p is indicated by the solid line in the figure. The values of R' along the limiting dashed lines deviate from the average line by plus or minus 10 per cent. Thus the average curve represents the variation mostly with an accuracy of greater than 90 per cent. It is evident from this curve that settlement is reduced by 25 per cent when the load bearing area is doubled.

(c) *Effect of Plan Dimensioning:* The study here involves the limiting of settlement by change in only the plan shape (accordingly the dimensions of the footing). The scope of the study is limited to rectangular and square plan dimensions only. The relationship between the parameters of X and Y are as follows.

$$A_x = A_y = 1 \quad \ldots(32a)$$

$$B_x \neq B_y \quad \ldots(32b)$$

$$M = 1 \quad \ldots(32c)$$

$$\lambda = 1 \quad \ldots(32d)$$

$$p = 0 \quad \ldots(32e)$$

$$Q_y = Q_x\sqrt{B_y/B_x} \quad \ldots(32f)$$

$$H_y = H_x\sqrt{B_y/B_x} \quad \ldots(32g)$$

and S'_{Fmy} is given by,

$$S'_{Fmy} = \sqrt{B_x/B_y}\ S_{Fmy} \quad \ldots(33)$$

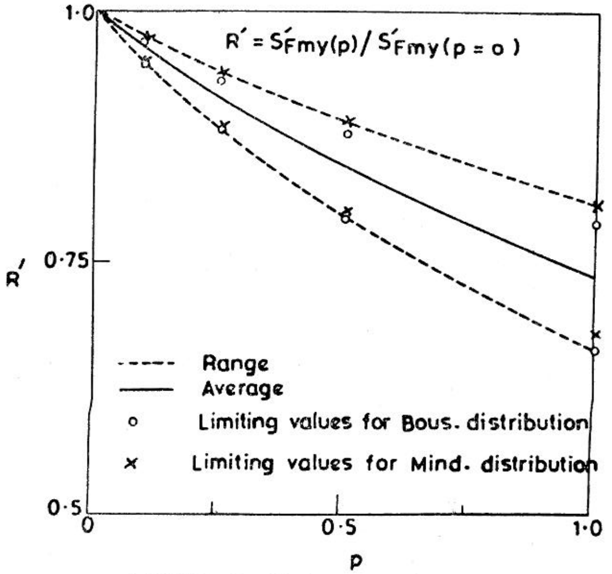

FIGURE 8. Variation of R' with p

S_{Fmx} values dave been calculated keeping A_x and B_x as unity (ie. X is square). The range of variation chosen for other non-dimensionalised parameters are: H_x from 1 to 3 and Q_x from 5 to 25. Similarly in the calculations of S_{Fmy}, A_y is kept as unity and B_y is varied from 1 to 10. The values of H_y and Q_y corresponding to H_x and Q_x are determined by Equation 32. Thus a square loaded area (X) is transformed into rectangles (Y) with length to breadth ratio varying up to 10 : 1 and the settlements of these

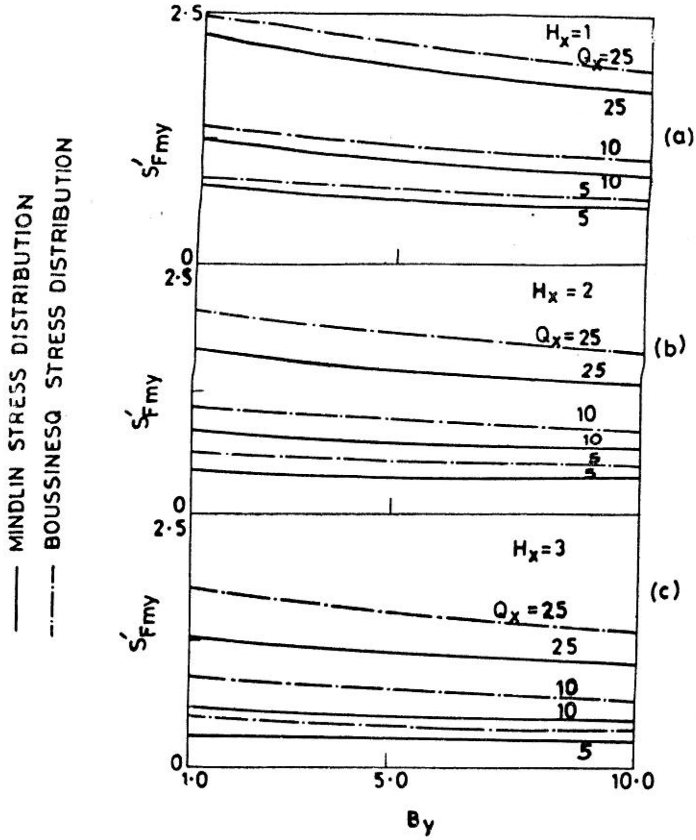

FIGURE. 9*a*, *b*, and *c*. **Variation of mean settlement factor, S'_{Fmy} with breadth paramater B_y**

rectangles are compared against that of the square. The settlement of Y with side ratio 1: 1 ($B_y=1$) is equal to the settlement of X. Figures 9*a*, 9*b* and 9*c* show the variation of S'_{Fmy} with B_y, each figure being for one value of H_x with each curve in a figure showing the variation for one particular value of Q_x. The curves in full line in the figures are for Mindlin stress distribution and those in broken lines are for Boussinesq stress distribution. In order to quantify the effect of plan dimensioning on settlement, values of ratio R'' defined as

$$R''=S'_{Fmy}/S_{Fmx}=S'_{Fmy\ (By)}/S'_{Fmy\ (By=1)} \qquad \text{...(34)}$$

have been calculated. This ratio brings out the influence that plan dimensioning has on settlement. The variation of R'' with B_y is shown in Figure 10. It has been observed that R'' values calculated for all the chosen range of parameters lie within the limits as indicated by the dashed lines in Figure 10 for both Boussinesq and Mindlin stress distributions. The average variation is shown by the solid line in the figure. For the same reasons as explained for Figure 8, the average curve represents the variation with an accuracy of greater than 90 per cent. It is clear from Figures 9 and 10 that the settlement of a loaded area decreases as it is made more and more oblong. It is seen from Figure 10 that the settlement of a loaded area decreases on the

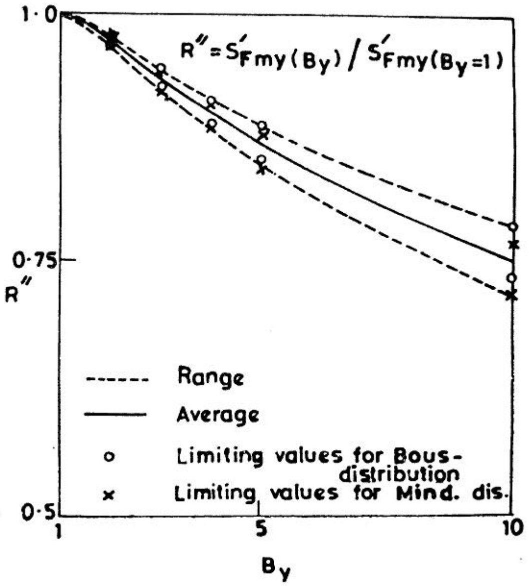

FIGURE 10. Variation of R'' with breadth parameter, B_y

average by 25 percent when it is changed from a square into a rectangle with sides ratio of 1:10. However, if a square loaded area is changed into a rectangular area with sides ratio of 1:3 (the usually adopted maximum ratio) the average reduction in settlement is found to be very small and amounts to 6 per cent.

Discussion

From the results of the three cases presented above, it is clear that settlement of a loaded area could be reduced by one or a combination of the following methods of increasing the depth of embedment, increasing the bearing area and making the plan dimensions more oblong. The following offers an explanation of how such a reduction is brought about in the case of the last method, the explanation for the first two methods having been made earlier in the paper. The mean settlement of a loaded area is defined by the settlement at its centre and corner. When the plan shape is made oblong, the load intensity acting on the area is moved away from these controlling points. (This is true also in the case of increasing the bearing area whereby the plan dimensions become large). As a results the induced stresses at all levels below these centre and corner points decrease which consequently leads to less settlement.

Though it is said that settlement could be controlled by adopting the three presented methods, they are not without economic implications. The increase in depth of embedment requires additional earthwork excavation, materials for extended columns and might also involve other likely operations such as dewatering etc. The increase in bearing area also requires additional earthwork excavation because of the increase in the size of the footing. Further a structural redesign of the foundation should also be done. It is not proven how the quantities of materials (steel and concrete) after the redesign will differ from the original quantities. In the case of altering the plan dimensions to reduce settlement also, structural redesign of the footing is involved and hereagain it is not clear how the redesign will affect the quantities of materials in the footing. Hence, it is concluded that an engineering solution to the problem of settlement control is by a suitable combination of the above three methods when other measures such as soil stabilization are not considered. It might be also stated that qualitatively similar trend of settlement response of loaded areas can be expected in other types of uniform deposits such as under consolidated clay, over-consolidated clay and sands also, since the mechanism governing the phenomenon is the same, irrespective of the type of the deposits. However, the response might vary quantitatively.

Conclusions

Analyses in order to quantify the effects of depth of embedment, the change in the bearing area and that of plan dimensioning on the settlement of shallow foundations in normally consolidated clays have been developed. The settlement of a loaded area is expressed in terms of non-dimensional settlement factors and the settlement of two loaded areas are compared by computing the comparable settlement factors. Boussinesq and Mindlin stress distributions have been used, separately, in the calculation of settlement factors. The effect of each, namely, depth of embedment, change in bearing area and change in plan dimensions (shape), on settlement has been investigated separately. It is concluded from the study that the use of Boussinesq stress distribution instead of more appropriate Mindlin stress distribution overpredicts the settlement values. The settlement of a loaded area decreases as its depth of embedment increases, as the bearing area increases and as the loaded area is made more and more oblong. It is believed that an engineering solution to the problem of limiting of settlement lies in a suitable combination of the above three methods.

References

DAVIS, E.H. and POULOS, H.G, (1968). The Use of Elastic Theory for Settlement under Three Dimensional Conditions, *Geotechnique*, Vol. 18, No. 1, pp. 67-91.

FOX, E.N. (1948), The Mean Elastic Settlement of Uniformly Loaded Area at a Depth below the Ground Surface, *Proceedings of the 2nd International Conference on Soil Mechanics and Foundation Engineering*, Rotterdam, Vol. 1, p. 129.

HARR, M.E. (1966), *Foundations of Theoretical Soil Mechanics*, McGraw Hill Inc.

KANIRAJ, S.R. and RANGANATHAM, B.V. (1974). Settlement of Buried Loaded Areas in Normally Consolidated Clay, *Soils and Foundations*, Vol. 14, No. 2, pp. 95-103.

SKOPEK, J. (1960), Uniformly Distributed Area Load in the Interior of the Semi-Infinite Solid, *Acta Technica*, No. 5, pp. 459-475, Ceskoslovenska Akademie Ved.

Notations

a = half the breadth of loaded area

a_x, a_y = half the breadth of loaded areas X and Y respectively

A = non-dimensionalised breadth parameter

A_x, A_y = non-dimensionalised breadth parameters for X and Y

b = half the length of loaded area

b_x, b_y = half the length of loaded areas X and Y
B = non-dimensionalised length parameter
B_x, B_y = non-dimensionalised length parameters for X and Y
C = $C_c/(1+e_o)$
C_c = compression index
e_o = average initial void ratio
h = depth of embedment of loaded area
h_x, h_y = depth of embedment of loaded areas X and Y
H = non-dimensionalised depth of embedment parameter
H_x, H_y = non-dimensionalised depth of embedment parameters for X and Y
K, K_x, K_y = constants having dimension of length
log = common logarithm
m_v = coefficient of volume compressibility
M = P_x/P_y
p = (area of $Y-$ area of X)/area of X
P_x, P_y = total load acting on loaded areas X and Y respectively
q = intensity of load acting on the loaded area
q_x, q_y = intensity of load acting on loaded areas X and Y
Q = non-dimensionalised load intensity parameter
Q_x, Q_y = non-dimensionalised load intensity parameters for X and Y
S_F = dimensionless settlement factor
S_{Fmx}, S_{Fmy} = dimensionless mean settlement factors for X and Y
S'_{Fmy} = mean settlement factor of Y directly comparable to that of X
X, Y = two loaded areas
γ = bulk density of soil
μ = Poisson's ratio
ρ = settlement
σ_z = vertical stress increase at z
σ_{zB} = vertical stress increase at z due to Boussinesq's equation
σ_{zM} = vertical stress increase at z due to Mindlin's equation
λ = h_y/h_x