Design Charts for Rigid Airport Runways Part II

by

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Introduction

Idealising the soil by Winkler model or elastic continuum Pickett and Ray (1951) presented influence charts for obtaining deflections and moments in concrete slabs for different positions of the loaded area. Reddy and Pranesh (1975) presented influence charts for obtaining deflections and moments for interior loads by idealising the subgrade soil by Filonenko-Borodich and Reissner models which have continuity between spring elements and at the same time the solutions can be obtained without much labour. Another foundation model for which there is continuity in the deflection is the Hetenyi model proposed by Hetenyi (1946). In this model continuity is introduced by embedding a plate in the Winkler foundation model which itself lacks continuity. In this paper analysis and influenee charts for deflections and moments in concrete slabs for interior loads are presented by idealising the subgrade soil by Hetenyi foundation.

Derivations of Equations

Figure 1 shows a concrete slab resting on Hetenyi foundation. The concrete slab is of infinite radius. The governing differential equation for the concrete slab for the axisymmetric case is

$$D \nabla_r^4 w = q - p_1, \quad \nabla_r^4 = \nabla_r^2 \cdot \nabla_r^2 = \left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) \nabla_r^2 \qquad \ldots(1)$$

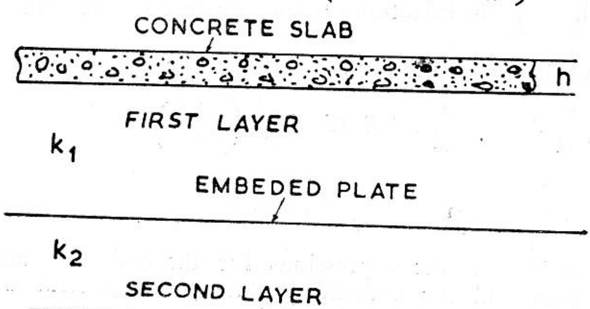

FIGURE 1. Concrete slab on Hetenyi Foundations

in which p_1^* is $k_1 (w - w_1)$ and denotes the pressure at the bottom of concrete slab, w and w_1 are the deflections of the concrete slab and the

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embedded plate respectively, k_1 is the modulus of subgrade reaction of the top layer of the Hetenyi foundation, q is uniformly distributed load and $D=Eh^3/12(1-\mu^2)$ i.e. flexural rigidity of the concrete slab, E=Young's modulus of concrete, h=thickness of concrete slab and μ=Poisson's ratio for concrete.

Introducing non-dimensional parameters $W=w/l$, $W_1=w_1/l$ and $R=r/l$ where $l=\sqrt[4]{D/k_1}$, Equation (1) takes the form

$$\nabla_R^4 W=\frac{ql^3}{D}-W+W_1 \qquad \text{...(2)}$$

Therefore,

$$W_1=\nabla_R^4 W+W-\frac{ql^3}{D} \qquad \text{...(3)}$$

The differential equation of the embedded plate is

$$D_1 \nabla_r^4 w_1=-(p_2-p_1) \qquad \text{...(4)}$$

where D_1 is the flexural rigidity of the embedded plate, p_2 is k_2w_1 and denotes the reaction on the embedded plate, k_2 is modulus of subgrade reaction of bottom layer. Equation (4) can be written as

$$D_1\nabla_r^4 w_1=k_1(w-w_1)-k_2w_1=k_1w-(k_1+k_2)\,w_1 \qquad \text{...(5)}$$

which in nondimensional form is

$$\nabla_R^4 W_1=\frac{k_1 W^4}{D_1}-\frac{(k_1+k_2)\,W_1l^4}{D_1} \qquad \text{...(6)}$$

Introducing the parameters $F=D_1/D$ and $S=k_2/k_1$, Equation (6) reduces to

$$\nabla_r^4 W_1=\frac{W}{F}-\frac{1+S}{F}W_1 \qquad \text{...(7)}$$

Operation with ∇_R^4 on Equation (3) and substituting the value of $\nabla_R^4 W_1$

from Equation (7) results in the following equation

$$\nabla_R^8 W+A_1\nabla_R^4 W+B_1 W=\frac{ql^3}{D}\left(\frac{1+S}{F}\right) \qquad \text{...(8)}$$

where, $\nabla_R^8=\nabla_R^4\nabla_R^4$, $A_1=\frac{1+S+F}{F}$, and $B_1=\frac{S}{F}$

Equations (8) and (3) are the expressions for the deflection coefficients of the concrete slab and the embedded plate. The general solution for Equation (8) is (Reddy and Pranesh, 1972)

$$W=\frac{ql^3}{D}\left\{\frac{1+S}{F}\right\}+M_1Z_1(m_1R)+M_2Z_2(m_1R)+M_3Z_3(m_1R)+M_4Z_4(m_1R)$$
$$+M_5Z_1(m_2R)+M_6Z_2(m_2R)+M_7Z_3(m_2R)+M_8Z_4(m_2R) \qquad \text{...(9)}$$

in which M_1 to M_8=constants, $Z_1(m_1R)$=ber (m_1R), $Z_2(m_1R)$ =−bei (m_1R)

$$Z_3(m_1R)=-\frac{2}{\pi}\text{ kei }(m_1R),\ Z_4(m_1R)=-\frac{2}{\pi}\text{ ker }(m_1R),$$

$$m_1=\sqrt[4]{\alpha_1-\beta_1},\ m_2=\sqrt[4]{\alpha_1+\beta_1},\ \alpha_1=\frac{A_1}{2}\text{ and }\beta_1=\sqrt{\frac{A_1^2}{4}-B_1}$$

The general solution for Equation (7) is given by (Reddy and Pranesh, 1972)

$$W_1=\frac{ql^3}{D}\left(\frac{1}{S}\right)+(1-m_1^4)\ [M_1Z_1\ (m_1R)+M_2Z_2\ (m_1R)+M_3Z_3\ (m_1R)$$
$$+M_4Z_4\ (m_1R)]+(1-m_2^4)\ [M_1Z_1\ (m_2R)+M_2Z_2\ (m_2R)+M_3Z_3\ (m_2R)$$
$$+M_4Z_4\ (m_4R)] \qquad ...(10)$$

For a concentrated force P acting at the origin of an infinite plate, the constants M_1 to M_8 are determined using the following conditions

(i) $W(\infty)=0$, (ii) $W'(\infty)=0$, (iii) $W_1(\infty)=0$, (iv) $W'_1(\infty)=0$, (v) $W'(0)=0$, (vi) $W'_1(0)=0$, (vii) $\underset{R\to 0}{\text{Lt}}\ 2\pi R\ \ Q_R+(Pl/D)=0$ in the concrete slab and (viii) $\underset{R\to 0}{\text{Lt}}\ 2\pi R\ \ Q_R=0$ in the embedded plate in which

$$W'=\frac{dW}{dR}\text{ and }Q_R=-\left(\frac{d^3W}{dR^3}+\frac{1}{R}\frac{d^2W}{dR^2}-\frac{1}{R^2}\frac{dW}{dR}\right)$$

From conditions i to iv, $M_1=M_2=M_5=M_6=0$. From conditions v and vi, $M_4=M_8=0$. Using conditions vii and $viii$,

$$M_3=-\frac{1}{m_1^2}\ \frac{(1-m_2^4)}{(m_2^4-m_1^4)}\ \frac{Pl}{4D}$$

$$M_7=\frac{(1-m_1^4)}{m_2^2\,(m_2^4-m_1^4)}\ \frac{Pl}{4D}$$

Therefore,

$$W=\frac{ql^3}{D}\left(\frac{1+S}{F}\right)+\frac{Pl}{4D\,(m_2^2-m_1^4)}\left[\frac{-(1-m_2^4)}{m_1^2}\ Z_3\ (m_1R)\right.$$
$$\left.+\frac{(1-m_1^4)}{m_2^2}Z_3\,(m_2R)\right] \qquad ...(11)$$

$$W_1=\frac{ql^3}{DS}-\frac{(1-m_1^4)}{m_1^2}\ \frac{(1-m_2^4)}{(m_2^4-m_1^4)}\ \frac{Pl}{4D}Z_3\,(m_1R)$$
$$+\frac{(1-m_2^4)\,(1-m_1^4)}{m_2^2\,(m_2^4-m_1^4)}\ \frac{Pl}{4D}Z_3\,(m_2R) \qquad ...(12)$$

Deflections from Distributed Loads

From Maxewell's reciprocal theorm Equation (11) gives the deflection at the origin due to a concentrated load at a variable co-ordinate point. Therefore, replacing P by $q\ r\ d\theta\ dr$, non-dimensional deflection of concrete slab for load acting on an area bounded by $R=0$ and R_1 and $\theta=\theta_2$ and θ_1 is given by

$$W=\frac{W}{l}=\int_0^{R_1}\int_{\theta_1}^{\theta_2}\frac{qrl^2}{4D}\left[\frac{-(1-m_2^4)}{m_1^2\,(m_2^4\, m_1^4)}Z_3\,(m_1R)\right.$$

$$\left.+\frac{(1-m_1^4)}{m_2^2\,(m_2^4-m_1^4)}Z_3\,(m_2R)\right]dr d\theta \qquad \ldots(13)$$

This can be written in the final form as

$$W=\frac{0.0005\,ql^4}{D}\,N \qquad \ldots(14)$$

where,

$$N=C_3\frac{(-m_1R)}{m_1^2}\left\{\left[\log\frac{2}{m_1R}-\gamma\right]\sum_{n=0}^{\infty}(-1)^{n+1}\frac{\left(\frac{m_1R}{2}\right)^{3+4n}}{(2n+1)!\,(2n+2)!}\right.$$

$$-\frac{1}{m_1R}\sum_{n=0}^{\infty}(-1)^n\frac{\left[\left(\frac{m_1R}{2}\right)\right]^{4n}}{(2n!)^2}+\frac{\pi}{4}\left[\sum_{n=0}^{\infty}(-1)^n\frac{\left(\frac{m_1R}{2}\right)^{1+4n}}{(2n!)\,[(2n+1)!]}\right]$$

$$\left.-\sum_{n=0}^{\infty}(-1)^n\frac{\left(\frac{m_1R}{2}\right)^{4n+3}}{(2n+1)!\,(2n+2)!}\left(1+\tfrac{1}{2}+\cdots\cdots\frac{1}{2n+2}\right)\right\}_0^R$$

$$+C_4\left\{\frac{1}{m_2^2}(-m_2R)\left\{\left[\log\frac{2}{m_2R}-\gamma\right]\sum_{n=0}^{\infty}(-1)^{n+1}\frac{\left(\frac{m_2R}{2}\right)^{3+4n}}{(2n+1)!\,(2n+2)!}\right.\right.$$

$$-\frac{1}{m_2R}\sum_{n=0}^{\infty}(-1)^n\frac{\left(\frac{m_2R}{2}\right)^{4n}}{(2n!)^2}$$

$$+\frac{\pi}{4}\left[\sum_{n=0}^{\infty}(-1)^n\frac{\left(\frac{m_2R}{2}\right)^{1+4n}}{(2n!)\,(2n+1)!}\right]-\sum_{n=0}^{\infty}(-1)^n\frac{\left(\frac{m_2R}{2}\right)^{4n+3}}{(2n+1)!\,(2n+2)!}$$

$$\left.\left.\left(1+\tfrac{1}{2}+\cdots\frac{1}{2n+2}\right)\right\}\right\}_0^R \qquad \ldots(15)$$

$$C_3=-\frac{2}{\pi}C_1\,,\ C_4=-\frac{2}{\pi}C_2$$

$$C_1=500\ (\theta_2-\theta_1)\left\{\frac{-(1-m_2^4)}{m_1^2(m_2^2-m_1^4)}\right\},\ C_2=500\ (\theta_2-\theta_1)\left\{\frac{1-m_1^4}{m_2^2(m_2^4-m_1^4)}\right\}$$

and $\gamma=0.57721567$.

Moments from Distributed Loads

The moment at point (r, θ) in the direction of the reference axis due to a load at the origin is given by

$$M = -\frac{D}{2}\left[(1+\mu)\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)+(1-\mu)\left(\frac{d^2w}{dr^2}-\frac{1}{r}\frac{dw}{dr}\right)\cos 2\theta\right] \quad \ldots(16)$$

Due to the reciprocal theorm, Equation (16) also gives the moment at the origin in the direction of the reference axis due to the load at the point (r, θ). Substituting for w from Equation (13) in Equation (16) and integrating, the moment due to a load on area bounded by $R=0$ and R and $\theta=\theta_1$ and θ_2 is obtained as

$$M=\frac{ql^2N}{10000} \quad \ldots(17)$$

where,

$$N=C_5\left\{B_1m_1R\,Z'_{3,\,m_1R}\,(m_1R)+B_2m_2R\,Z'_{3,\,m_2R}\,(m_2R)\right\}_0^R$$
$$+C_6\left\{B_1\left[m_1R\,Z'_{3,\,m_1R}\,(m_1R)-2Z_3\,(m_1R)\right]_0^R\right.$$
$$\left.+B_2\left[m_2R\,Z'_{3,\,m_2R}\,(m_2R)-2Z_3\,(m_2R)\right]_0^R\right\} \quad \ldots(18)$$

$$B_1=\frac{-(1-m_2^4)}{m_1^2\,(m_2^4-m_1^4)},\quad B_2=\frac{1-m_1^4}{m_2^2\,(m_2^4-m_1^4)}$$

$$C_5=-\frac{10,000}{8}\,(\theta_2-\theta_1)\,(1+\mu)$$

$$C_6=-\frac{10,000}{16}\,(\sin 2\theta_2-\sin 2\theta_1)\,(1-\mu)$$

$$Z_3\,(mR)=-\frac{2}{\pi}\left[(\log 2-\gamma-\log mR)\left\{\left(\frac{mR}{2}\right)^2\right.\right.$$
$$\left.+\sum_{n=1}^{\infty}(-1)^n\left(\frac{mR}{2}\right)^{4n+2}\frac{1}{\{(2n+1)!\}^2}\right\}-\frac{\pi}{4}\left\{1+\sum_{n=1}^{\infty}(-1)^n.\left(\frac{mR}{2}\right)^{4n}\right.$$
$$\left.\left.\frac{1}{\{(2n)!\}^2}\right\}+\left(\frac{mR}{2}\right)^2+\sum_{n=1}^{\infty}(-1)^n\frac{\left(\frac{mR}{2}\right)^{4n+2}}{\{(2n+1)!\}^2}\left(1+\tfrac{1}{2}+\ldots\ldots+\frac{1}{2n+1}\right)\right]$$

$$Z'_{3,\,mR}\,(mR)=-\frac{2}{\pi}\left[(\log 2-\gamma)\right.$$

$$\left\{\frac{(2mR)}{2^2}+\sum_{n=1}^{\infty}(-1)^n\frac{(4n+2)}{2^{4n+2}}\frac{(mR)^{4n+1}}{\{(2n+1)!\}^2}\right\}$$

$$-\frac{1}{mR}\left\{\left(\frac{mR}{2}\right)^2+\sum_{n=1}^{\infty}(-1)^n\left(\frac{mR}{2}\right)^{4n+2}\frac{1}{\{(2n+1)!\}^2}\right\}$$

$$-\log\ mR\left\{\frac{2(mR)}{2^2}+\sum_{n=1}^{\infty}(-1)^n\,\frac{(4n+2)}{2^{4n+2}}\,\frac{(mR)^{4n+1}}{\{(2n+1)!\}^2}\right\}$$

$$-\frac{\pi}{4}\left\{\sum_{n=1}^{\infty}(-1)^n\,\frac{4n}{2^{4n}}\,(mR)^{4n-1}\,\frac{1}{\{(2n)!\}^2}\right\}$$

$$+\frac{2(mR)}{2^2}+\sum_{n=1}^{\infty}\frac{(-1)^n\,(4n+2)\,(mR)^{4n+1}}{2^{4n+2}\,\{(2n+1)!\}^2}\left(1+\tfrac{1}{2}+\ldots\ldots+\frac{1}{2n+1}\right)\Bigg]$$

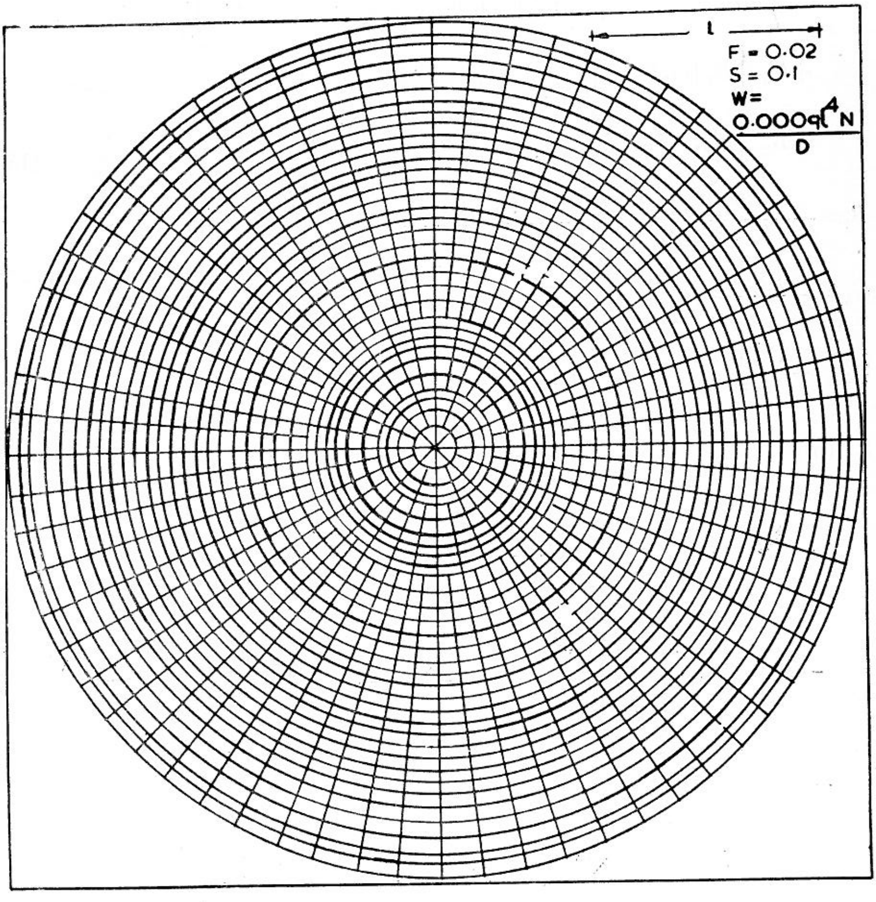

FIGURE 2. Influence chart for deflection for $F=0.02$ and $S=0.1$

Results and Analysis

The procedure for determining S is given by Reddy and Pranesh (1972). As for $S=\infty$, Hetenyi model becomes Winkler model, values of $S=0.1$ and 4.0 are used for presenting charts. Based on laboratory investigation Barden (1962) found a value of $EI_2/EI_1=1/50$ for high edge pressures beneath a beam. In view of this, in presenting the charts, values of $F=0.02$ and 0.002 are used. Poisson's ratio, μ, for concrete is taken as 0.15 for all the charts. Figures 2 and 3 are the influence charts for deflection. The deflection is given by

$$w=Wl=\frac{0.0005\ ql^4N}{D}$$

where N=number of blocks enclosed. For variation of F from 0.02 to 0.002 there is practically no change in the value of N obtained. Therefore, for a given value of S, the influence charts for deflection for both $F=0.02$ and 0.002 are practically the same. In view of this, the influence charts for deflection are presented only for $F=0.02$.

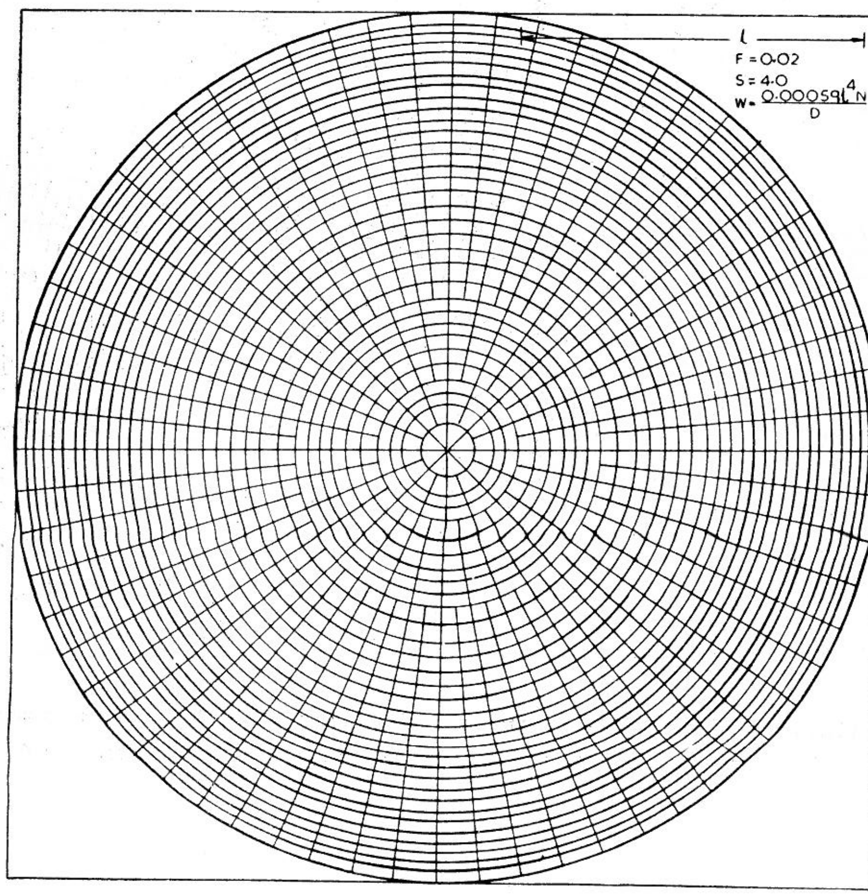

FIGURE 3. Influence chart for deflection for F=0.02 and S=4.0

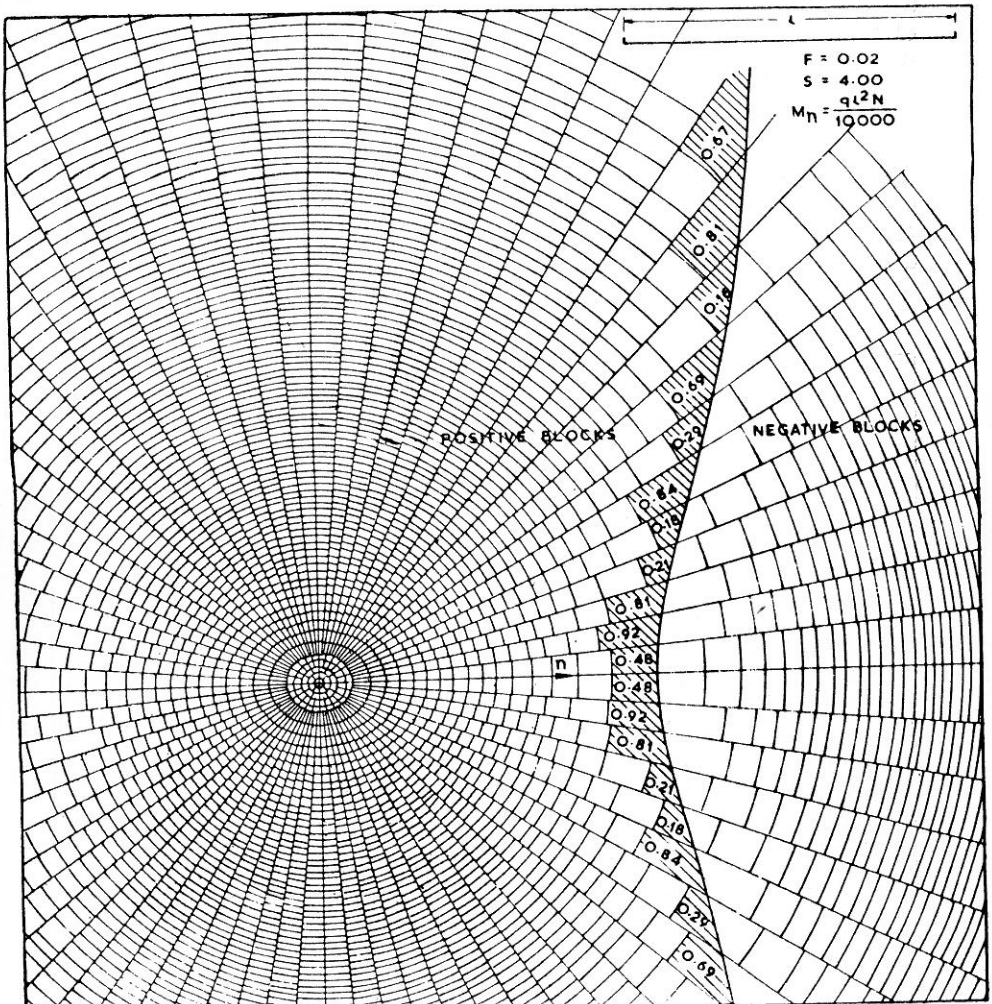

FIGURE 4. Influence chart for moment, $M_n = \frac{ql^2 N}{10,000}$, at point '$O$' ($N$=number of positive blocks enclosed minus number of negative blocks enclosed) for F=0.02 and S=4.0

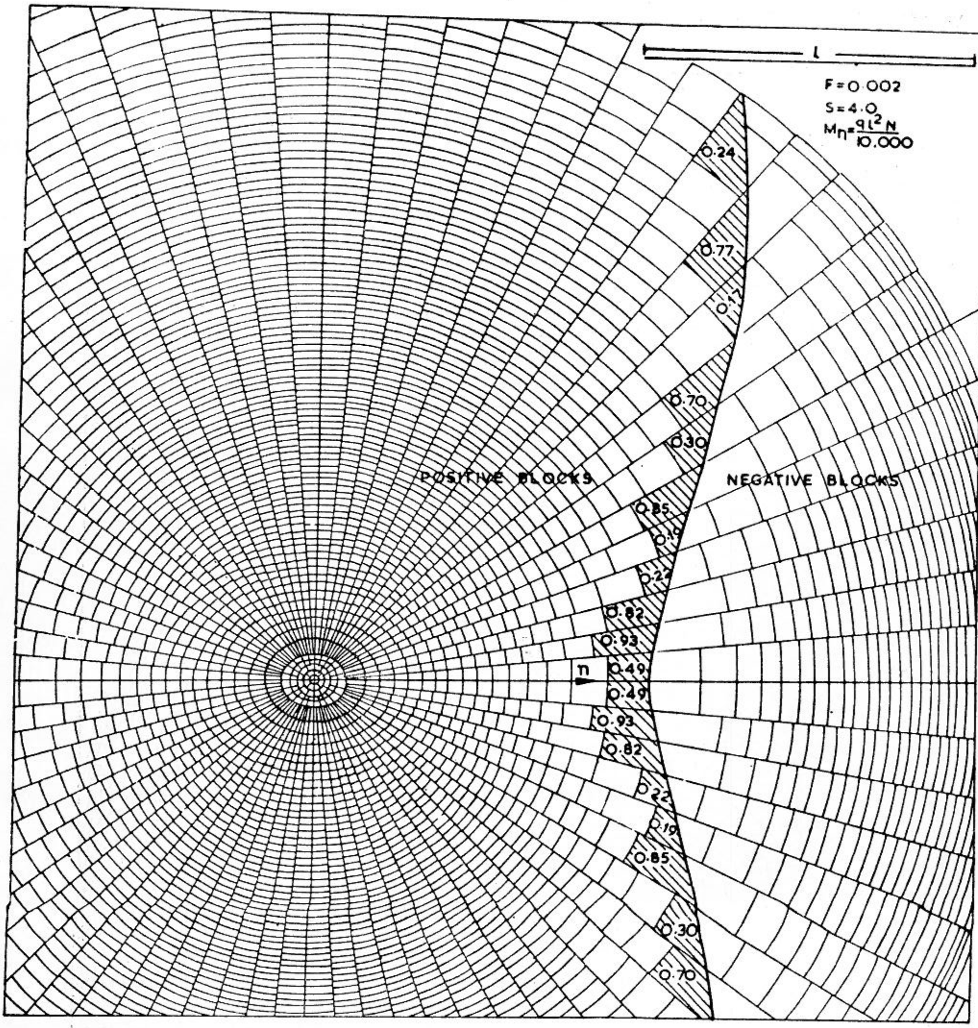

FIGURE 5. Influence chart for moment $M_n = \frac{ql^2N}{10,000}$, at point 'O' ($N$=number of positive blocks enclosed minus number of negative blocks enclosed) for F=0.002 and S=4

Figures 4 to 7 are in the influence charts for moments. For Figures 4 and 5, the moment M_n at the origin in the n-direction is given by

$$M_n = \frac{ql^2N}{10,000}$$

whereas for Figures 6 and 7 it is given by

$$M_n = \frac{ql^2N}{2000}$$

since in Figures 6 and 7 every fifth member of block as given by Equation (17) is plotted. The procedure for using the influence charts is the same as that for charts presented by Pickett and Ray (1951) and Reddy and Pranesh (1975). A numerical example is worked out to show the use of the charts.

Illustrative Example

A dual-tandom landing gear of a plane has a total load of 68040 kg with fore and aft and dual spacings equal to 157.5 cm and 80.0 cm,

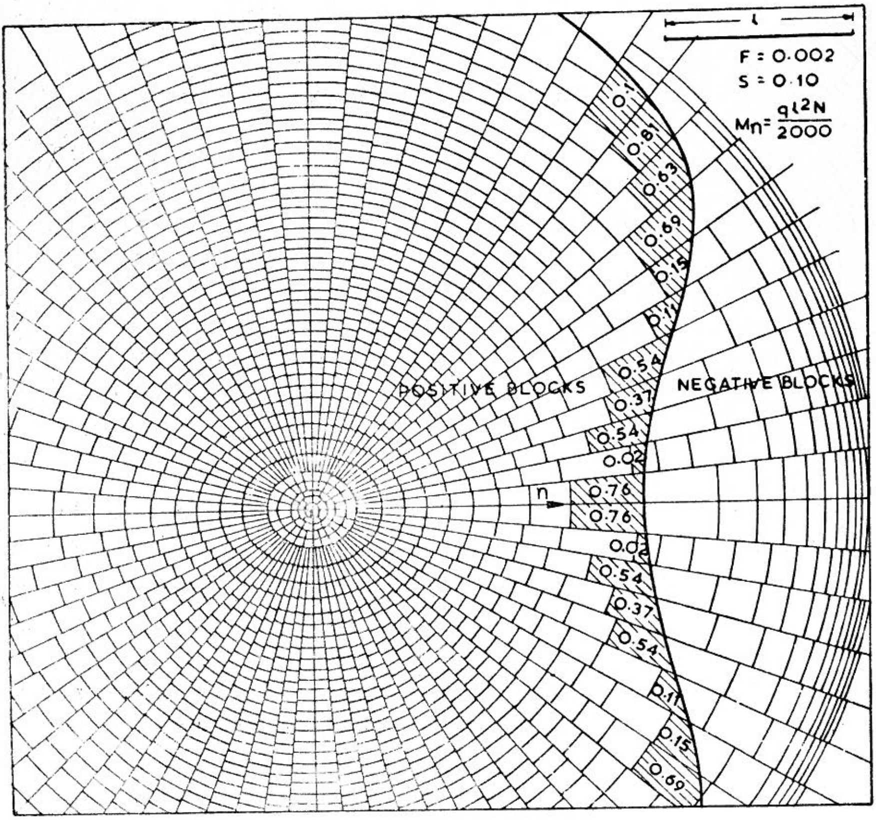

FIGURE 6. Influence chart for moment $M_n = \frac{ql^2N}{2,000}$, at point 'O' ($N$=number of positive blocks enclosed minus number of negative blocks enclosed) for F=0.002 and S=0.1

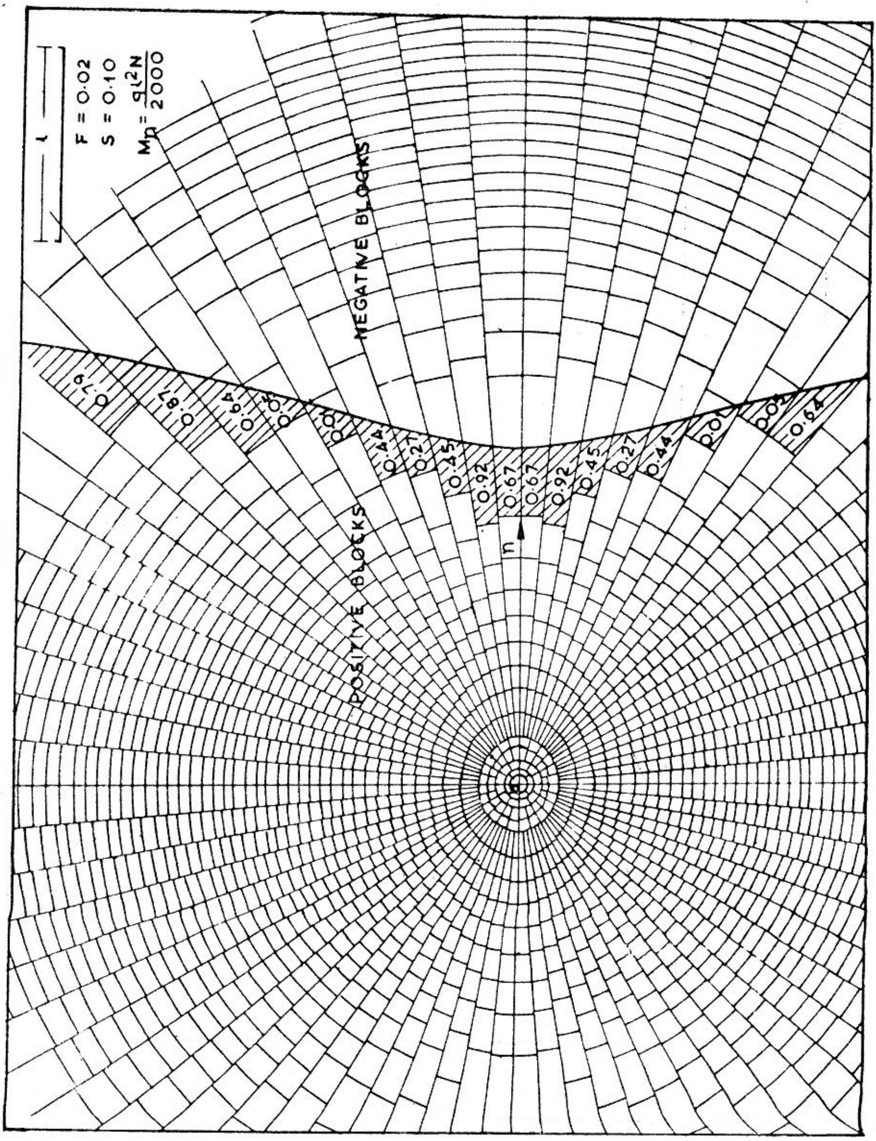

FIGURE 7. Influence chart for moment $M_n = \frac{ql^2N}{2,000}$, at point '*O*' ($N$ = number of positive blocks enclosed minus number of negative blocks enclosed) for $F = 0.02$ and $S = 0.1$

respectively. The other data are: Tire pressure=11.1 kg/cm²; thickness of concrete slab=40.7 cm; E=281200 kg/cm²; μ=0.15, F=0.002; k_1=38.6 kg/cc; and S=0.1. The orientation of the gear with the 'on' axis in Figure 6 is as shown in Figure 8. Find the stress in the slab at point 'O'.

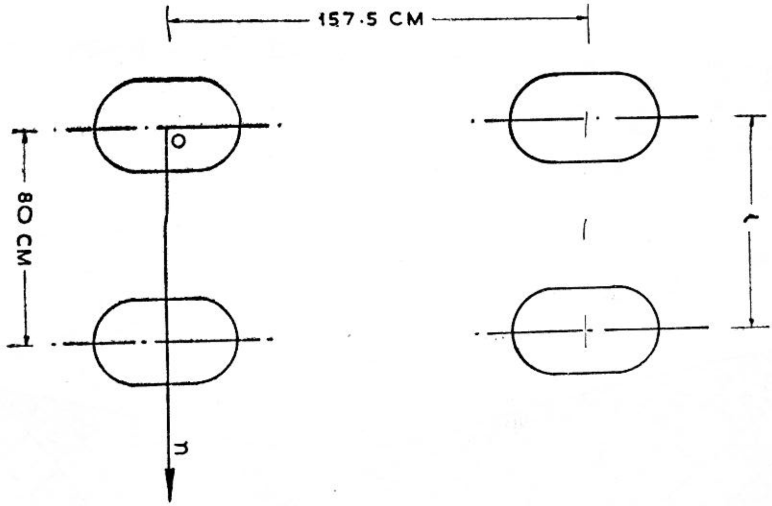

FIGURE 8. Tire imprints and their orientation for the numerical example

Taking the shape of the contact area as a rectangle with semicircular ends with width W_o equal to six-tenth of the length L and the contact pressure equal to tire pressure, contact area=0.5227 L^2.

Hence, $0.5227\ L^2 = \frac{17010}{11.1} = 1531$ sq. cm

$$\text{or } L = \sqrt{\frac{1531}{0.5227}} = 54.1 \text{ cm}$$

$$W_o = 0.6 \times 54.1 = 32.46 \text{ cm}$$

$$l = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)\ k_1}} = \sqrt[4]{\frac{281200 \times 40.7^3}{12(1-0.0225)38.6}}$$

$$= 80.4 \text{ cm.}$$

The tracing dimension (L or W_o) of the tire imprint is obtained from

$$\frac{\text{Tracing dimensiom } (L \text{ or } W_o)}{\text{Actual contact dimension}} = \frac{\text{length } l \text{ on chart}}{l \text{ of pavement}}$$

Hence, tracing $L = \frac{54.1 \times 5.0^*}{80.4} = 3.365$ cm

$$W_o = 0.6\ L = 2.02 \text{ cm}$$

Lateral spacing of tires on tracing $= \frac{80 \times 5.0}{80.5} = 4.97$ cm

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*Graphical l for large scale chart used.

Fore and aft spacing on tracing $=\frac{157.5\times 5.0}{80.4}=9.8$ cm

Using these dimensions the outline of the tire imprints is drawn. For the orientation of the tire imprints shown in Figure 8, N, the number of blocks enclosed $=165$. Therefore

$$M_n=\frac{11.1\times(80.4)^2\ 165}{2000}=5920 \text{ kg}$$

Hence, the stress in the slab at

$$\text{point } `O' =\frac{6M_n}{h^2}=\frac{6\times 5920}{(40.7)^2}=21.45 \text{ kg/cm}^2$$

Conclusion

Influence charts for deflections and moments due to interior loads on concrete slabs on Hetenyi foundation are presented. The charts can be used for the design of rigid airport runways.

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