Settlement of Buried Loaded Areas in Normally Consolidated Clay Deposits of Finite Thickness

by

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Introduction

A knowledge of the stress distribution in the medium and of its consolidation characteristics are essential to calculate the settlement of a loaded area. The stress distribution is obtained from formulae based on the theory of elasticity and standard oedometer test results give the consolidation characteristics of the soil. Boussinesq's equation is ordinarily used for stress distribution calculations. Boussinesq's equation gives the stress distribution due to a load at the surface of a semi-infinite half space. However, rarely are the deposits semi-infinite in extent. Most often they are finite in thickness and are underlain by stiff stratum. The problem of stress distribution in such cases has been investigated by many investigators. Due to the presence of the rigid layer there tends to be a concentration of the stresses below the centre of the loaded area and close to the rigid stratum. Sovinc (1961) has shown that when the rigid layer is present at distances greater than 2.5 times the length (larger dimension) of the loaded area below the loaded area, the difference in the actual stress distribution and that obtained by Boussinesq's equation tends to be insignificant. It is attempted in this paper to investigate the settlement of loaded areas in normally consolidated clay deposits underlain by rigid layer. Firstly, the Boussinesq stress distribution and then the more realistic Sovinc stress distribution have been used in the analyses in order to evaluate the conventional method of using Boussinesq stress distribution.

Analysis

The compression, $\triangle\rho$ of an element dz with initial void ratio e_o, (Figure 1) vertically below the centre of a uniformly loaded rectangular area of $2a \times 2b$ embedded at a depth h below the surface is given by,

$$\triangle\rho = \frac{C_c}{1+e_o} \log \frac{p_z+\sigma_z}{p_z} \, dz \qquad \ldots(1)$$

where p_z and σ_z are respectively overburden pressure and increase in vertical stress in the element at z. The expression for vertical stress at depth z according to Boussinesq's equation (σ_{zB}) is given by (Harr, 1966),

$$\sigma_{zB} = \frac{2q}{\pi}\left[\frac{abz(a^2+b^2+2z^2)}{(a+z^2)\,(b^2+z^2)\sqrt{(a^2+b^2+z^2)}} + \sin^{-1}\frac{ab}{\sqrt{a^2+z^2}\,\sqrt{b^2+z^2}}\right] \qquad \ldots(2)$$

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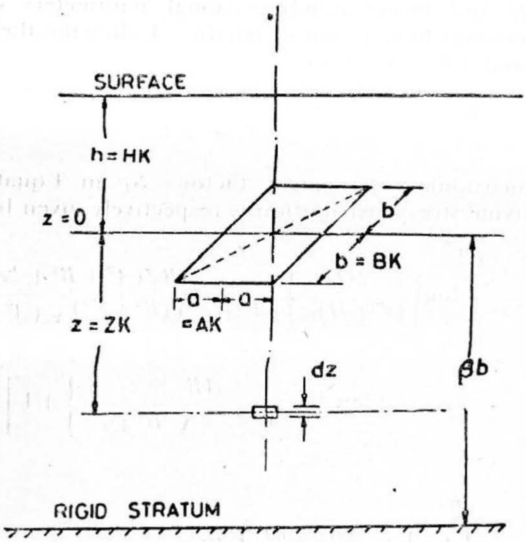

FIGURE 1. Loaded area in the interior of the compressible deposit of finite thickness

where q is the intensity of loading. The solution for vertical stress by Sovinc is expressed as a dimensionless quantity of σ_{zS}/q, the variation of which with z/b is given in the form of a chart. The chart depicts the variation for different values of b/a and β, β being the ratio of the depth to the rigid stratum below the loaded area to half the length of the loaded area. The settlement at the centre of the loaded area, ρ, is obtained by integration of Eqn. 1 substituting for σ_z value.

$$\rho = \frac{1}{C}\int_{0}^{\beta b} \log\left[1+\frac{\sigma_{zB}}{\gamma(z+h)}\right] dz \qquad \ldots(3)$$

for Boussinesq stress distribution and

$$\rho = \frac{1}{C}\int_{0}^{\beta b} \log\left[1+\frac{\sigma_{zS}/q}{\gamma(z+h)/q}\right] dz \qquad \ldots(4)$$

for Sovinc stress distribution. In Equations 3 and 4, $C[=(1+e_o)/C_c]$ is assumed a constant and γ is the bulk density of the soil. The following substitutions are made in Equations 3 and 4.

$$a = AK \qquad \ldots(5a)$$

$$b = BK \qquad \ldots(5b)$$

$$z = ZK \qquad \ldots(5c)$$

$$h = HK \qquad \ldots(5d)$$

$$q = Q\gamma K \qquad \ldots(5e)$$

where A, B, Z, H and Q are nondimensional parameters obtained by introducing K, a constant having unit of length. Following these substitutions, Equation 3 and 4 transform to,

$$\rho = \frac{K}{C} S_F \qquad \ldots(6)$$

where S_F is a dimensionless settlement factor. S_F in Equation 6 with Boussinesq and Sovinc stress distribution is respectively given by,

$$S_F = \int_0^{\beta B} \log\left[\frac{2Q}{(Z+H)\pi}\left\{\frac{ABZ(A^2+B^2+2Z^2)}{(A^2+Z^2)(B^2+Z^2)\sqrt{(A^2+B^2+Z^2)}} + \sin^{-1}\frac{AB}{\sqrt{A^2+Z^2}\sqrt{B^2+Z^2}}\right\}+1\right] dZ \qquad \ldots(7)$$

and

$$S_F = \int_0^{\beta B} \log\left[1+\frac{Q(\sigma_{zs}/q)}{Z+H}\right] dZ \qquad \ldots(8)$$

Effect of Rigidity of Foundation Structures

The theory dealt with in the preceding paragraphs concerns with stresses and settlements due to uniform loading of an area. However, foundations in practice tend to impose conditions closer to uniform displacement than to uniform loading. For accurate settlement predictions it is desirable to allow for rigidity of foundations. In elastic displacement theory an approximation to the uniform displacement is obtained from the maximum and minimum displacements of a uniformly loaded area (Davis and Poulos, 1968). This approximation is obtained in two stages. First, the rigid footing displacement is known to be close to mean displacement of the uniformly loaded area (Fox, 1948). Second the approximate mean displacement is obtained by assuming the displaced surface under the loaded area to be parabolic, in which case the centre and corner displacements define the mean displacement completely. This approximation gives :

For a rectangle, mean displacement $\simeq 1/3\ (2\rho_{-centre} + \rho_{-corner})$ ...(9)

Adopting the above approximation for consolidation settlement also the mean settlement, ρ_m of the loaded area is given by,

$$\rho_m = \frac{K}{C} S_{Fm} \qquad \ldots(10)$$

where S_{Fm} is the mean settlement factor.

For a rectangle, $S_{Fm} \simeq 1/3\ (2S_{F\text{-}centre} + S_{F\text{-}corner})$

Equations 7 and 8 give expressions for $S_{F\text{-}centre}$. Similar expressions for $S_{F\text{-}corner}$ can be found for the two types of approaches for stress distribution using which the mean settlement factor can be finally computed.

Results

Calculations for mean settlement factor for rectangular loaded areas are made for the two types of stress distribution. S_F values are evaluated by numerical integration of the expressions for S_F using Simpson's one-third rule formula. S_F values are calculated keeping A to be unity. B is varied from 1 to 10, Q from 2 to 25, H from 0.25 to 10 and β from 1 to 10. $B=1$ represents the case of a square loaded area. The values of the ratio of compression in a depth z below the loaded area (S_{Fm-Z}) to that of the total compression (S_{Fm}) have also been calculated.

Figure 2 shows the variation of S_{Fm}, the mean settlement factor with β for a square loaded area ($B = 1$) and $Q = 5$ for Boussinesq stress distribution, each curve in it being the variation for one value of H. Figure 3 shows the respective results for a square loaded area when Sovinc stress distribution is applied. In Figure 4 and Figure 5 $S_{Fm,Z}/S_{Fm}$ versus Z plot have been obtained for square loaded area for $H = 1$ and $Q = 5$ for Boussinesq stress distribution and Sovinc stress distribution respectively. In these figures each curve is for one value of β. These figures help to comprehend the contribution by the medium above a given depth to the total settlement of the medium. In order to compare the two approaches for stress distribution, ratios of $S_{Fm-Sovinc}$ to $S_{Fm-Boussinesq}$ have been calculated, the variation of which with β for different combination of parameters is shown in Figure 6.

Discussions

It is observed from Figure 2 that if the depth of the clay layer is increased beyond $\beta = 2.5$ tne resulting increase in settlement is very small in the conventional analysis of using Boussinesq stress distribution. From Figure 4 it is evident that for $B = 1$, $Q = 5$, $H = 1$ and $\beta = 5$ the contribution to settlement from the soil deposit lying between $Z = 2.5$ and 5 is only 13 per cent. These observations demonstrate that the large portion of settlement

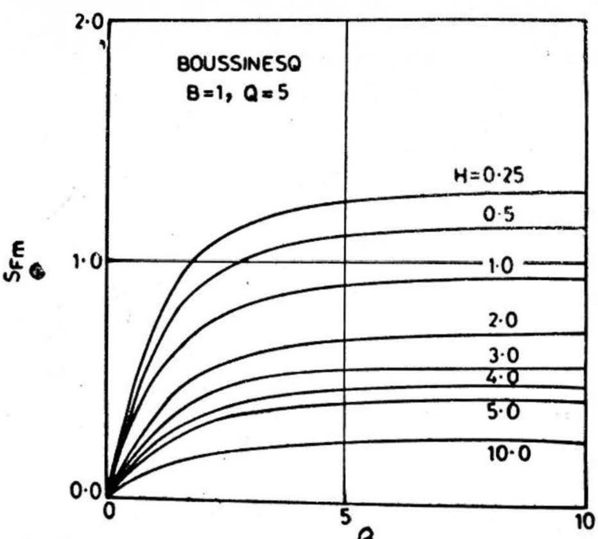

FIGURE 2. S_{Fm} **vs** β **for square loaded areas using Boussinesq stress distribution**

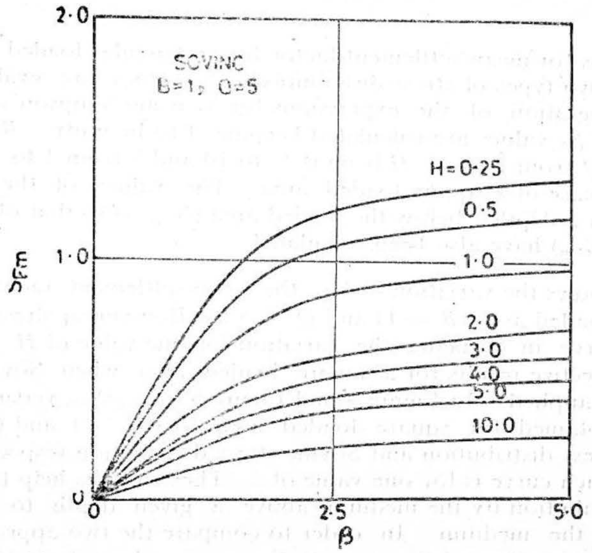

FIGURE 3. S_{Fm} **vs β for square loaded areas using Sovinc stress distributions**

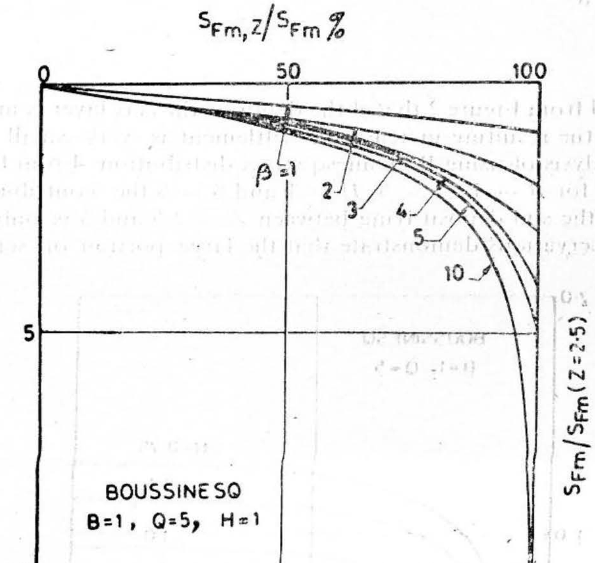

FIGURE 4. $S_{Fm,\,Z}$ **vs Z for square loaded areas using Boussinesq stress distribution**

takes place within a very shallow depth (1.25 times the length) below the footing. A similar trend is revealed when the more appropriate Sovinc distribution is used (Figures 3 and 5). However, as could be logically expected, for the same geometric and load intensity data (i.e. $B = 1$, $Q = 5$, $H = 1$ and $\beta = 5$) the contribution to settlement from the soil between $Z = 2.5$ and 5 is slightly more, being 19 per cent. This increase is as a esult of the tendency for the vertical stresses to concentrate near the rigid srtratum.

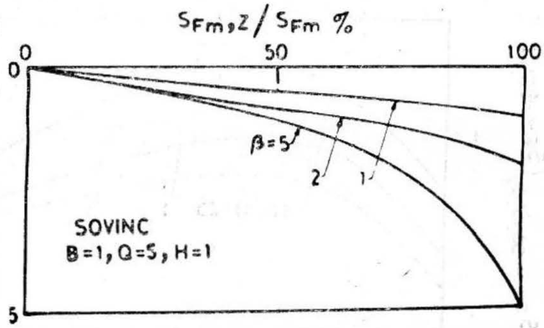

FIGURE 5. S_{Fm}, **vs** Z **for square loaded areas using Sovinc stress distribution**

From Figure 6 it is evident that the conventional approach of using Boussinesq stress distribution underestimates consolidation settlement values compared to when the more appropriate Sovinc stress distribution is used. From Figure *6a* it is seen that with the geometric parameters other than that of the depth of embedment and the load intensity parameter remaining the same, the difference between the two approaches increases as the depth of embedment decreases. Figure *6b* shows that the difference between the two approaches decreases when the load intensity is increased (keeping the geometric parameters the same) fo β nearly up to 4 and this tendency reverses for β values greater than 4. The comparison in Figure *6c* of settlements of a square and a rectangular loaded area keeping the other geometric and loading parameters the same, shows that the difference between the two approaches for stress distribution tends to be more as the loaded area becomes more oblique. However, a perusal of the Figure 6 reveals that the difference between the two approaches is not significant and the maximum increase in settlement in each case because of the use of Sovinc stress distribution is nearly 10 per cent only. This small difference also tends to decrease as the depth to the rigid stratum below the footing increases. It may also be pointed out that the two stress distributions considered in the investigation are due to load at the surface of the medium. The actual stress distribution is modified because of embedment of the loaded area. It has been found (Kaniraj and Ranganatham, 1974) that in the case of buried loaded area in a semi-infinite medium the use of more appropriate Mindlin stress distribution gives less settlement values than when the conventional Boussinesq distribution is used. Hence, a more appropriate stress distribution than Sovinc stress distribution (to account for the depth of embedment in the case of compressible medium underlain by a rigid stratum), may give less settlement values. This observation together with the small difference seen in the settlement values for the two different approaches viz. Boussinesq and Sovinc indicates that Boussinesq stress distribution may reasonably be used to calculate settlement when the compressible stratum lies below the loaded area up to a depth of 2.5 times the length of footing ($\beta = 5$). Beyond $\beta = 5$ the thickness of the compressible medium contributing to settlement increases and also the difference between Boussinesq stress distribution and Sovinc stress distribution are negligible, which amounts to saying that for stress distribution calculations the depth of compressible medium below the loaded area can be regarded as approaching infinity. For the case of semi-infinite medium as already stated above (Kaniraj and Ranganatham, 1974) use of Boussinesq's equation overpredicts settlement values. Hence for beyond $\beta = 5$ the use of Mindlin's equation may be appropriate.

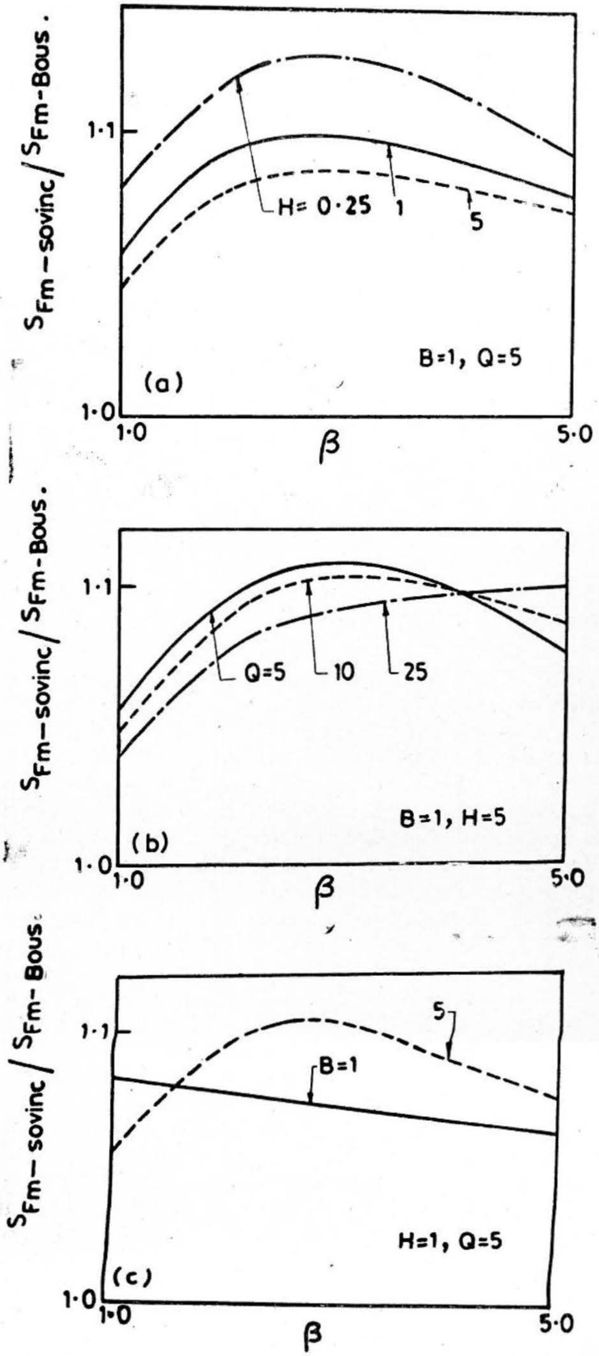

FIGURE 6. $S_{Fm\text{-}Sovinc}/S_{Fm\text{-}Bous.}$ vs β

(*a*) for variation in depth of embedment

(*b*) for variation in load intensity

(*c*) for variation in side ratio of rectangular areas

It is observed from Figure 2 and 4 that the major portion of settlement takes place within a shallow depth below the footing viz. $\beta = 2.5$. In order to investigate this trend over a large combination of parameters the ratios of mean settlement factor to the contribution to the mean settlement factor from the deposit lying between $Z = 0$ and $Z = 2.5$ have been calculated and is reported in Figure 7. If the curve for $B = 1$, $Q = 5$ and $H = 1$ is considered as the basis for comparison (for which the contribution to settlement from soil lying between $Z = 2.5$ and $Z = 10$, when $\beta = 10$, is only 18 per cent) it will be seen that as the load on the loaded area is increased (Q increased to 25) the contribution from the deep layers of soils tends to increase. This is due to the increase in the vertical stress in those layers and also because of their additional compression being relatively more than that of the soil layers at shallow depths (Equation 1). It is also seen from Figure 7 that as the depth of embedment increases again the contribution from the deep layers of soils tends to increase, but it however, tends to decrease when the loaded area becomes more oblique. The manner of calculating K and settlement of a buried loaded area is explained in Kaniraj and Ranganatham, 1974.

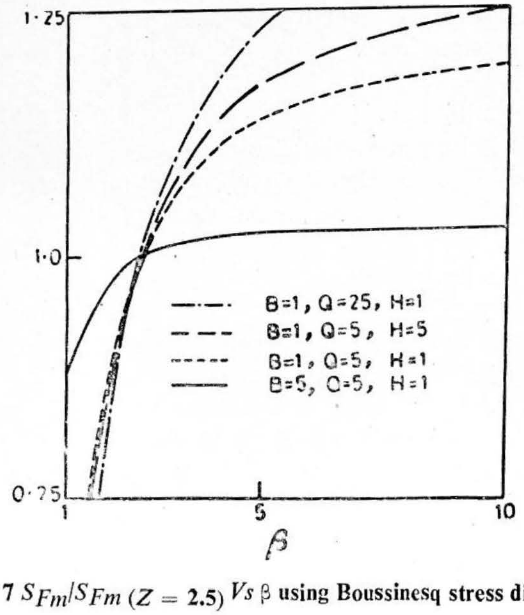

Figure 7 $S_{Fm}/S_{Fm}\,(Z = 2.5)$ *Vs* β **using Boussinesq stress distribution**

Conclusions

From the investigations carried out to evalute the conventional method of using Boussinesq stress distribution to calculate settlement when the normally consolidated clay deposit is underlain by rigid stratum it is concluded that the Boussinesq stress distribution may be used to calculate settlement when the depth to rigid stratum below the loaded area is up to 2.5 times the length of footing ($\beta = 5$). Beyond $\beta = 5$ the use of Mindlin stress distribution may be appropriate. It is seen that a large portion of the total settlement takes place within a shallow depth below the footing.

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Notation

a = half the breadth of the loaded area
A = non-dimensional breadth parameter (= 1)
b = half the length of the loaded area
B = non-dimensional length parameter
C_c = compression index
$C = (1+e_o)/C_c$
e_o = average initial void ratio
h = depth of embedment of loaded area
H = non-dimensional depth of embedment parameter
K = dimensional parameter having unit of length
m_v = coefficient of volume compressibility
p_z = overburden pressure at z
q = intensity of loading
Q = non-dimensional load intensity parameter
S_F = non-dimensional settlement factor
z = depth co-ordinate
Z = non-dimensional depth parameter
β = ratio of depto to the rigid stratum below the loaded area to half the length of the loaded area
P = settlement
P_m = mean settlement
σ_Z = increase in vertical stress at depth z
σ_{ZB} = increase in vertical stress at depth z according to Boussinesq's equation
σ_{ZS} = increase in vertical stress at depth z according to Sovinc stress distribution
γ = bulk density of soil