

# Bearing Capacity of Strip Footings on Partly Saturated Soils

by

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## Introduction

A fairly common problem encountered in foundation engineering in semi-arid and arid regions is the determination of ultimate bearing capacity of footings on partly saturated soils. The behaviour of partly saturated soils does not conform with that of saturated soils and also the presence of air-water interfaces produces negative pore pressures causing additional intergranular pressures, which vary appreciably with changes in water content. Though it was recognised long time back that the behaviour of partly saturated soils differs considerably from that of saturated soils and the degree of saturation in soils has considerable influence on bearing capacity of soils, only a few attempts have been made to evaluate this influence. Bishop (1954, 1955) described how Skempton's pore pressure parameters (Skempton, 1954) could be applied to the problem of determining the effective stresses in earth dam during construction and during rapid drawn down, and to the analysis of the stability of slopes. To study effect of degree of saturation on the ultimate bearing capacity of flexible pavements, Broms (1964) expressed the apparent shear strength parameters in terms of effective shear strength parameters, pore pressure parameters and initial pore pressures using the Mohr's stress circles with respect to effective stresses and total stresses at failure. Siva Reddy and Mogaliah (1970) in their brief technical note presented an analysis for determining the ultimate bearing capacity of partly saturated soils using Skempton's pore pressure parameters and the method of characteristics. Herein, detailed analysis and results are presented to bring out clearly the influence of initial pore pressure and degree of saturation on ultimate bearing capacity of these soils.

## Assumptions

1. The soil is rigid plastic at failure.
2. The effective stress concept for saturated soils holds good for partly saturated soils. The effective normal stress with respect to shear strength is given by (Bishop et al 1960)\*

$$\begin{aligned}\tau &= c + [\bar{\sigma} - u_a + \lambda(u_a - u_w)] \tan \phi \\ &= c + (\bar{\sigma} - u) \tan \phi\end{aligned}\quad \dots(1)$$

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\* In this paper stresses with bars are total stresses.

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- where  $\tau$  = maximum shearing resistance on a given plane,  
 $c$  and  $\phi$  = cohesion and angle of internal friction, in terms of effective stresses  
 $\bar{\sigma}$  = total normal stress,  
 $u_a$  = pore air pressure,  
 $u_w$  = pore water pressure,  
 $\chi$  = a parameter which depends upon degree of saturation and soil type, and  
 $u$  = equivalent pore pressure =  $[\chi u_w + (1 - \chi)u_a]$

3. The pore pressure change,  $\Delta u$ , under conditions of no drainage is given by (Skempton, 1954)

$$\Delta u = B[\Delta \bar{\sigma}_3 + A(\Delta \bar{\sigma}_1 - \Delta \bar{\sigma}_3)] \quad \dots(2)$$

where  $\Delta \bar{\sigma}_1, \Delta \bar{\sigma}_3$  = changes in total stresses in major and minor principal stress directions, and

$A, B$  = pore pressure parameters

These pore pressure parameters are stress dependent. However, in the present investigation these are assumed to be constant.

4. At the instant of failure, a wedge of soil  $00' L$  which is in elastic state (Figure 1) is formed beneath the foundation.

### Analysis

The analysis presented herein follows that of Siva Reddy and Mogaliah (1970) and Mogaliah (1974).

Consider the state of stress at a point  $P$  (Figure 2) in a soil mass which is in plastic equilibrium. The slip lines make equal angles  $\pm\mu$  with the direction of major principal stress, where  $\mu = \frac{\pi}{4} - \frac{\phi}{2}$ .

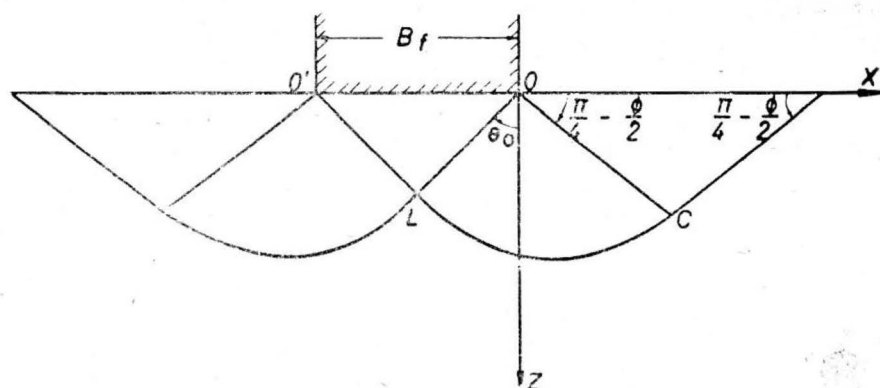


FIGURE 1. Assumed wedge below footing and coordinate axes

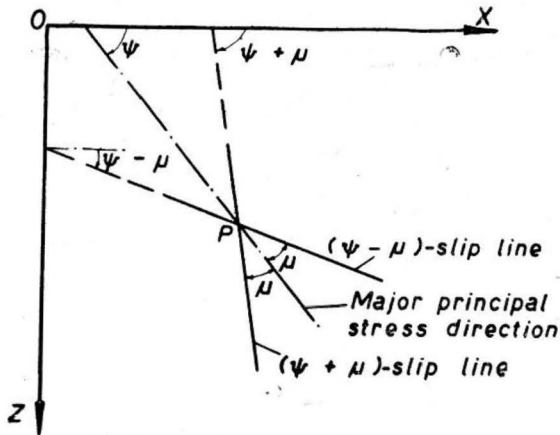


FIGURE 2. Orientation of slip lines at a point

For two-dimensional problems, the stress condition in a soil mass which is in plastic equilibrium is given by Mohr-Coulomb criterion as

$$\frac{1}{4}(\sigma_x - \sigma_z)^2 + \tau_{xz}^2 = \frac{\sin^2 \phi}{4} (\sigma_x + \sigma_z + 2H)^2 \quad \dots(3)$$

where,

$\sigma_x, \sigma_z, \tau_{xz}$  = effective stress components, and

$$H = c \cot \phi$$

If  $\bar{\sigma}_{1i}, \bar{\sigma}_{3i}$  and  $u_o$  are the initial total major principal stress, initial minor principal stress and initial pore pressure respectively, at a point  $P$  which is at a depth  $z$  from the surface, they can be expressed as

$$\bar{\sigma}_{1i} = \gamma_b z \quad \dots(4)$$

$$\bar{\sigma}_{3i} = K_o(\gamma_b z - u_o) + u_o \quad \dots(5)$$

where,

$\gamma_b$  = total unit weight of the soil mass, and

$K_o$  = coefficient of earth pressure at rest

The changes in total major principal stress,  $\Delta \bar{\sigma}_1$ , total minor principal stress,  $\Delta \bar{\sigma}_3$ , and pore pressure  $\Delta u$  can be written as

$$\left. \begin{aligned} \Delta \bar{\sigma}_1 &= \bar{\sigma}_1 - \bar{\sigma}_{1i} \\ \Delta \bar{\sigma}_3 &= \bar{\sigma}_3 - \bar{\sigma}_{3i} \\ \Delta u &= u - u_o \end{aligned} \right\} \quad \dots(6)$$

where,

$\bar{\sigma}_1$  = final total major principal stress,

$\bar{\sigma}_3$  = final total minor principal stress, and

$u$  = final pore pressure

Writing  $\bar{\sigma}_1$  and  $\bar{\sigma}_3$  in terms of stress components,

$$\frac{\bar{\sigma}_1}{\bar{\sigma}_3} = \frac{1}{2}(\sigma_x + \sigma_z + 2u) \pm \left[ \frac{1}{4}(\sigma_x - \sigma_z)^2 + \tau_{xz}^2 \right]^{1/2} \quad \dots(7)$$

On substituting for  $\bar{\sigma}_1$ ,  $\bar{\sigma}_3$ ,  $\bar{\sigma}_{1i}$  and  $\bar{\sigma}_{3i}$  from Equations (7), (4) and (5) into Equations (6) the expressions for  $\Delta \bar{\sigma}_1$ ,  $\Delta \bar{\sigma}_3$  and  $\Delta u$  are obtained in terms of effective stress components and pore pressure. Then substituting into Equation (2) and simplifying, yields the following :

$$u = \frac{B}{1-B} \left[ \frac{1}{2}(\sigma_x + \sigma_z) + (2A-1) \left\{ \frac{1}{4}(\sigma_x - \sigma_z)^2 + \tau_{xz}^2 \right\}^{1/2} \right. \\ \left. + \gamma_b z(AK_o - A - K_o) + u_o(K_o - 1)(1-A) + \frac{u_o}{B} \right] \quad \dots(8)$$

Two new variables  $\sigma$  defined by

$$\sigma = \frac{1}{2}(\sigma_x - \sigma_z) + c \cot \phi = \frac{1}{2}(\sigma_x - \sigma_z) + H \quad \dots(9)$$

and  $\psi$  (the angle between the major principal stress direction and the  $x$ -axis measured positive in the clock-wise direction) are introduced. The effective stress components  $\sigma_x$ ,  $\sigma_z$ ,  $\tau_{xz}$  and  $u$  of Equations (3) and (8), can be expressed in terms of these two variables as

$$\sigma_x = \sigma(1 + \sin \phi \cos 2\psi) - H \quad \dots(10)$$

$$\sigma_z = \sigma(1 - \sin \phi \cos 2\psi) - H \quad \dots(11)$$

$$\tau_{xz} = \sigma \sin \phi \sin 2\psi \quad \dots(12)$$

$$u = \frac{B}{1-B} \left[ \sigma(1 + (2A-1) \sin \phi) - H \right. \\ \left. + \gamma_b z(AK_o - A - K_o) + u_o(K_o - 1)(1-A) + \frac{u_o}{B} \right] \quad \dots(13)$$

Herein,  $u_o$  is taken to be independent of depth  $z$ . Approximate procedures for finding  $K_o$  and  $u_o$  in partly saturated soils are given by Bishop and Henkel (1962). A study of Equation (13) shows that as  $B$  approaches one this expression becomes indeterminate.

The above Equations (10) through (13) are substituted into the following equations of equilibrium

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial u}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0 \quad \dots(14)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} + \frac{\partial u}{\partial z} = \gamma_b \quad \dots(15)$$

where  $\gamma_b$  = bulk density of the soil. Then two equations are obtained in terms of the partial derivatives of  $\sigma$  and  $\psi$ . Multiplying the first of these equations by  $\sin(\psi \pm \mu)$  and the second by  $-\cos(\psi \pm \mu)$  and after some simplification the following equations are obtained :

$$(1+M) \frac{\partial \sigma}{\partial x} \pm M \tan \phi \tan(\psi \pm \mu) \frac{\partial \sigma}{\partial x}$$

$$\mp 2\sigma \tan \phi \frac{\partial \psi}{\partial x} \pm \frac{N \cos (\psi \pm \mu)}{\cos \phi \cos (\psi \mp \mu)}$$

$$+ \left[ (1+M) \frac{\partial \sigma}{\partial z} \mp M \tan \phi \cot (\psi \mp \mu) \frac{\partial \sigma}{\partial z} \right.$$

$$\left. \mp 2\sigma \tan \phi \frac{\partial \psi}{\partial z} \right] \tan (\psi \mp \mu) = 0$$

where,

$$M = \frac{B}{1-B} [1 + (2A-1) \sin \phi]$$

$$N = \gamma_b \left[ 1 - \frac{B}{1-B} (AK_o - A - K_o) \right]$$

Using the following dimensionless variables

$$\sigma' = \frac{\sigma}{c},$$

$$x' = \frac{x}{l},$$

$$z' = \frac{z}{l},$$

where  $l$  = a characteristic length =  $\frac{c}{\gamma_b}$ , from Equations (16) the following relationships are obtained along the characteristics.

The first family is determined by

$$\frac{dz'}{dx'} = \tan (\psi - \mu) \quad \dots(17)$$

$$[1 + M - M \tan \phi \cot (\psi - \mu)] d\sigma' - 2\sigma' \tan \phi d\psi$$

$$+ M \tan \phi [\tan (\psi - \mu) + \cot (\psi - \mu)] \frac{\partial \sigma'}{\partial x'} dx'$$

$$= - \frac{N' \cos (\psi + \mu)}{\cos \phi \cos (\psi - \mu)} dx' \quad \dots(18)$$

and the second by

$$\frac{dz'}{dx'} = \tan (\psi + \mu) \quad \dots(19)$$

$$[1 + M + M \tan \phi \cot (\psi + \mu)] d\sigma' + 2\sigma' \tan \phi d\psi$$

$$- M \tan \phi [\tan (\psi + \mu) + \cot (\psi + \mu)] \frac{\partial \sigma'}{\partial x'} dx'$$

$$= \frac{N' \cos (\psi - \mu)}{\cos \phi \cos (\psi + \mu)} dx' \quad \dots(20)$$

where,  $N' = \frac{\gamma_b l}{c} \left[ 1 - \frac{B}{1-B} (AK_o - A - K_o) \right]$ .

Substitution of  $B = 0$  and  $A = 0$  into Equations (18) and (20) reduce

them to well known equations of Sokolovsky (1965) along characteristics as

$$d\sigma' \mp 2\sigma' \tan \phi d\psi = \frac{\gamma_b l}{c \cos \phi} (\cos \phi dz' \mp \sin \phi dx').$$

The above four equations may be used to find the values of  $x'$ ,  $z'$ ,  $\sigma'$  and  $\psi$  at the points of intersection of the characteristics, starting from known boundaries. After determining these quantities, pore pressure,  $u$ , may be determined by using Equation (13). On writing Equations (17) through (20) in finite difference form and after simplification, the quantities  $x'$ ,  $z'$ ,  $\sigma'$  and  $\psi$  at a point of intersection of the characteristics are obtained as

$$x' = \frac{x_A' \tan(\psi_A - \mu) - z_A' - x_B' \tan(\psi_B + \mu) + z_B'}{\tan(\psi_A - \mu) - \tan(\psi_B + \mu)} \quad \dots(21)$$

$$z' = z_B' + (x' - x_B') \tan(\psi_B + \mu) \quad \dots(22)$$

$$\sigma' = \frac{1}{D_1} \left( D_2 \sigma_B' + D_3 M \tan \phi \frac{\partial \sigma'}{\partial x'} + D_4 N' \right) \quad \dots(23)$$

$$\psi = \frac{1}{2 \tan \phi} \left[ 2\psi_A \tan \phi + D_5 \frac{(\sigma' - \sigma_A')}{\sigma_A'} + D_6 M \tan \phi \frac{\partial \sigma'}{\partial x'} + D_4 N' \right] \quad \dots(24)$$

where,

$$D_1 = 1 + M + M \tan \phi \cot(\psi_B + \mu)$$

$$+ \frac{\sigma_B'}{\sigma_A'} [1 + M - M \tan \phi \cot(\psi_A - \mu)]$$

$$D_2 = 2(1 + M) + M \tan \phi [\cot(\psi_B + \mu) - \cot(\psi_A - \mu)] + 2(\psi_B - \psi_A) \tan \phi$$

$$D_3 = [\tan(\psi_B + \mu) + \cot(\psi_B + \mu)] (x' - x_B') - \frac{\sigma_B'}{\sigma_A'} [\tan(\psi_A - \mu) + \cot(\psi_A - \mu)] (x' - x_A')$$

$$D_4 = \frac{\cos(\psi_B - \mu)}{\cos \phi \cos(\psi_B - \mu)} (x' - x_B'),$$

$$- \frac{\sigma_B' \cos(\psi_A + \mu)}{\sigma_A' \cos \phi \cos(\psi_A - \mu)} (x' - x_A'),$$

$$D_5 = 1 + M - M \tan \phi \cot(\psi_A - \mu),$$

$$D_6 = [\tan(\psi_A - \mu) + \cot(\psi_A - \mu)] \frac{(x' - x_A')}{\sigma_A'},$$

$$D_7 = \frac{\cos(\psi_A + \mu)}{\sigma_A' \cos \phi \cos(\psi_A - \mu)} (x' - x_A')$$

Here quantities with subscript  $A$  refer to the point  $A$  which is on a slip line of the  $(\psi - \mu)$ -family and quantities with subscript  $B$  refer to the point  $B$  which is on a slip line of the  $(\psi + \mu)$ -family.

Equations (23) and (24) contain term  $\frac{\partial \sigma'}{\partial x'}$  which is unknown. In order to arrive at value of  $\sigma'$  from Equation (23) iteration procedure may be used.

### Calculation of bearing capacity

The above analysis gives expressions for the required quantities at points of intersection of characteristics. The solution of the problem is accomplished by starting from boundary with known values and successive calculation throughout failure zone in the soil mass. In the present case uniform surcharge  $q$  along  $x$ -axis (Figure 1) is considered and hence the shear stress is zero. For this condition the equation along the characteristics gives  $\psi = 0$  at all the points of intersection of the characteristics in the zone above OC (see Figure 1). Therefore, this zone will be in Rankine's passive state. The passive pressure acting on OL is determined by successive numerical integration starting from the boundary of Rankine's passive zone OC. This boundary is inclined to the  $x$ -axis at angle  $\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$ , and along it

$$\psi = 0 \quad \dots(25)$$

$$\sigma' = \frac{\left(\frac{q}{c} + \cot \phi - \frac{u_0}{c}\right) (1 + M + M \tan \phi \cot \mu) + N'z'}{(1 - \sin \phi) (1 + M + M \tan \phi \cot \mu)} \quad \dots(26)$$

and

$$u' = \frac{u}{c} = \frac{B}{1-B} \left[ \sigma' \{1 + (2A-1) \sin \phi\} - H' + \frac{\gamma_0 l z'}{c} (AK_0 - A - K_0) + \frac{u_0}{c} (K_0 - 1) (1 - A) + \frac{u_0}{CB} \right] \dots(27)$$

where,  $H' = \cot \phi$

The quantities  $x'$ ,  $z'$ ,  $\sigma'$ ,  $\psi$  and  $u'$  for the points in the rupture zone between OC and OL (Figure 1) are determined using Equations (21) through (24) and (27).

For determining the points of intersection of the characteristics with OL the following two conditions are used :

$$\left. \begin{aligned} z' &= mx' = -x' \tan (90 - \theta_0) \\ \tau_{nt} &= -(\sigma_n' + H') \tan \phi \end{aligned} \right\} \quad \dots(28)$$

where,  $\theta_0 =$  the angle made by the inclined face of soil with the vertical axis,

$m =$  slope of the line OL, and

$\tau_{nt}, \sigma_n' =$  the dimensionless tangential and normal effective stresses on OL.

With these two conditions and with the equations along  $(\psi - \mu)$ -family slip line which intersects OL, the equations for determining the required quantities along OL are given by

$$\psi = \theta_0 + \frac{\pi}{4} + \frac{\phi}{2} \quad \dots(29)$$

$$x' = \frac{z_1' - x_1' \tan(\psi_1 - \mu)}{m - \tan(\psi_1 - \mu)} \quad \dots(30)$$

$$z' = mx' \quad \dots(31)$$

$$\sigma' = \frac{1}{1 + M - M \tan \phi \cot(\psi_1 - \mu)} \left[ \sigma_1' \{2(\psi - \psi_1) \tan \phi + 1 + M - M \tan \phi \cot(\psi_1 - \mu)\} - M \tan \phi \{ \tan(\psi_1 - \mu) + \cot(\psi_1 - \mu) \} (x' - x_1') \frac{\partial \sigma'}{\partial x'} - \frac{N' \cos(\psi_1 + \mu)}{\cos \phi \cos(\psi_1 - \mu)} (x' - x_1') \right] \quad \dots(32)$$

and  $u'$  may be determined by using Equation (27) where the quantities with subscript 1 refer to the last point determined on the  $(\psi - \mu)$ -family slip line.

After obtaining the quantities  $x'$ ,  $z'$ ,  $\psi$ ,  $\sigma'$  and  $u'$  at several points along the surface OL (Figure 1) of the wedge OLO', the same stresses are assumed to act on O'L and the ultimate bearing capacity of the footing,  $q_o'$ , is calculated by considering the equilibrium of wedge OLO' as

$$q_o' = \frac{2P_{pv}'}{B_f'} - \frac{1}{4} \frac{\gamma_b l}{c} B_f' \tan(90^\circ - \theta_o) \quad \dots(33)$$

where  $P_{pv}' =$  the non-dimensional vertical component of total resultant pressure on surface OL which is found by summing up  $\Delta P_{pv}'$  values,

$$\Delta P_{pv}' = \frac{\sigma_n' + H'}{\cos \phi} \Delta L' \sin(\theta_o - \phi) + \Delta L' u' \sin \theta_o, \text{ and}$$

$B_f' =$  dimensionless width of footing.

## Results and Discussion

Numerical results are obtained for different values of  $u_o/c$ ,  $B$ ,  $A$  and  $K_o$ . The results are presented in the form of bearing capacity factors  $N_c$ ,  $N_q$  and  $N_\gamma$  representing the contributions due to cohesion, surcharge and weight, respectively. The ultimate bearing capacity is expressed as

$$q_o' = N_c + q' N_q + \frac{1}{2} \frac{\gamma_b l}{c} B_f' N_\gamma \quad \dots(34)$$

In order to determine the bearing capacity factors three sets of calculations are done. In the first set of calculations the values of  $\gamma_b l/c$  and  $q'$  are assumed equal to unity and zero, respectively. The ultimate bearing capacity thus obtained, designated as  $q'_{c\gamma}$ , is due to cohesion and weight of the soil and is given by

$$q'_{c\gamma} = N_c + \frac{B_f'}{2} N_\gamma \quad \dots(35)$$



The second set of calculations are done assuming  $\gamma_b l/c$  as well as  $q'$  as zero to obtain the value of  $N_c$ . With the values of  $N_c$  thus obtained, the values of  $N_\gamma$  are determined using Equation (36). For determining the factor  $N_q$  the third set of calculations are done by assuming a certain value of  $q'$  and  $\gamma_b l/c$  equal to zero. The bearing capacity thus obtained will be due to cohesion and surcharge which is given by

$$q_{ca}' = N_c + q' N_q \quad \dots(36)$$

With the values of  $N_c$  and using Equation (36) the values of  $N_q$  are determined. All the three sets of calculations are done assuming the base angle of the wedge  $OLO'$  equal to  $\phi$ . For all the calculations a fine mesh size that gives sufficiently accurate results is used.

The bearing capacity factors are determined for  $B = 0.2, 0.5$  and  $0.7$ ,  $A = 0.2, 0.5, 1.0$  and  $1.2$ ,  $u_o/c = -0.25, -1.0, -2.0$  and  $-3.0$  and  $K_o = 0.4$ . Figure 3 shows the influence of  $B$  on the shape of slip lines for  $\phi$  equal to  $10^\circ$ . It is seen that for the same footing width, the slip lines of the  $(\psi - \mu)$ -family in the case of  $B$  equal to  $0.7$  are relatively shallower and cylindrical when compared to  $B$  equal to  $0.2$ . This indicates that, as the degree of saturation increases, the slip lines of  $(\psi - \mu)$ -family tend to become cylindrical in shape. Figure 4 shows the influence of  $A$  on the slip lines for  $\phi = 10^\circ$ . From this figure it is seen that the influence of  $A$  on the shape of slip lines though slight is similar to that of  $B$ . The influence of initial negative pore pressure  $u_o/c$  on the slip lines is shown in Figure 5 for  $\phi$  equal to  $10^\circ$ . It is observed that for the same footing width the slip lines of  $(\psi - \mu)$ -family extend to larger distances from footing in the case of  $u_o/c = -3.0$  when compared to the case of  $u_o/c = -0.25$ . This shows that the influence of initial negative pore pressure on the slip lines is in the same

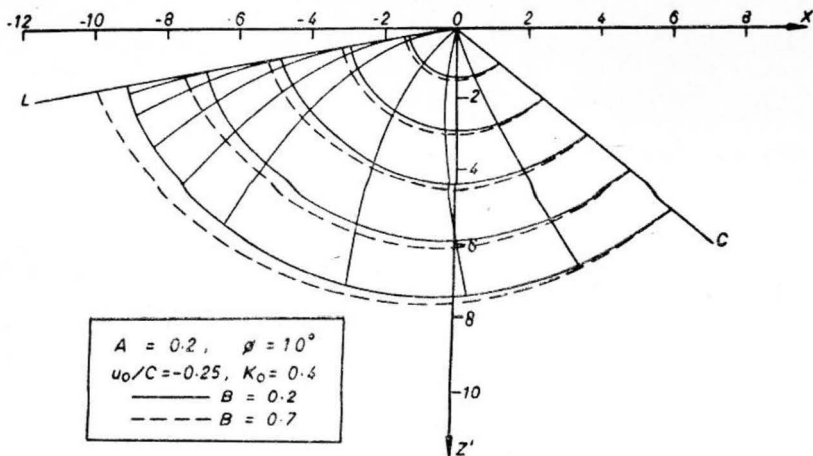


FIGURE 3. Influence of  $B$  on slip lines for  $\phi = 10^\circ$ ,  $A = 0.2$  and  $\frac{u_o}{c} = 0.25$

manner as that of  $\phi$ , since as initial negative pore pressure increases, the soil has higher shear strength due to enhanced level of effective stress. It may be summarised that the effect of decrease in  $B$  and  $A$  and increase in  $-u_0/c$  is to increase the shear strength and in turn affect the slip lines in a manner similar to the increase of  $\phi$ . It is also seen in these figures (Figures 3 through 5) that the  $(\psi + \mu)$ -family slip lines starting on  $OL$  are tangential to  $OL$ , since the critical condition (Equation 28) is assumed along  $OL$  of wedge  $OLO'$  (Figure 1).

The values of  $N_c$  are presented as a function of  $\phi$  in Figures 6 through 17 for different values of  $B$ ,  $A$  and  $u_0/c$  keeping  $K_0$  equal to 0.4. It is seen that the curves obtained are similar to those of the case where the effect of pore pressures and degree of saturation in soils are not considered (Terzaghi

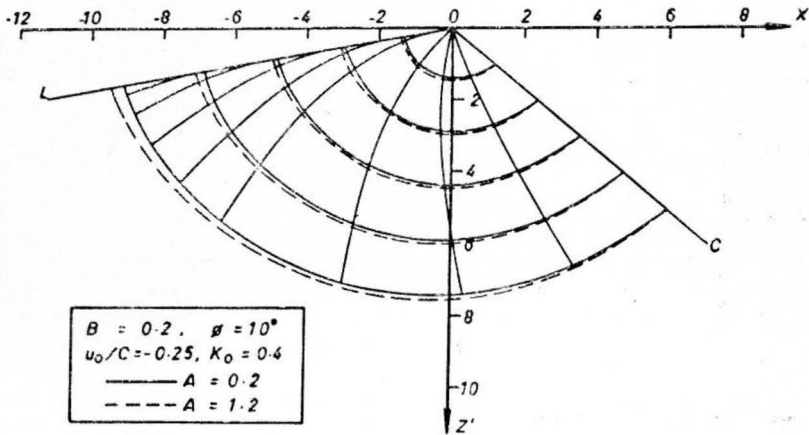


FIGURE 4. Influence of  $A$  on slip lines for  $\phi = 10^\circ$ ,  $B = 0.2$  and  $\frac{u_0}{c} = 0.25$

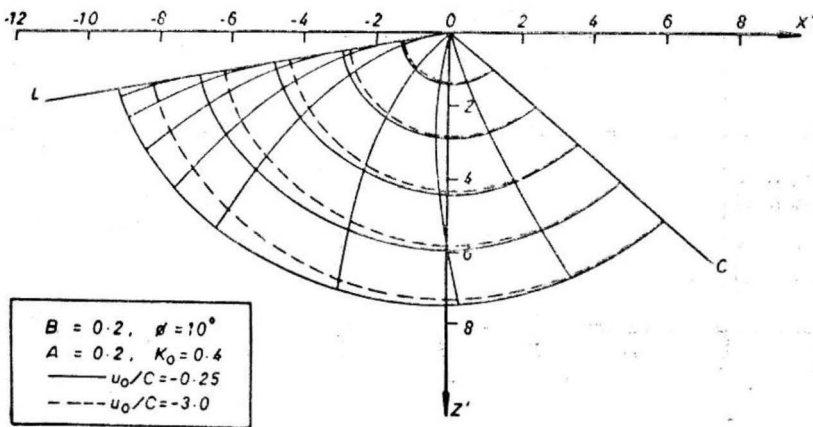


FIGURE 5. Influence of  $\frac{u_0}{c}$  on slip lines for  $\phi = 10^\circ$ ,  $B = 0.2$  and  $A = 0.2$

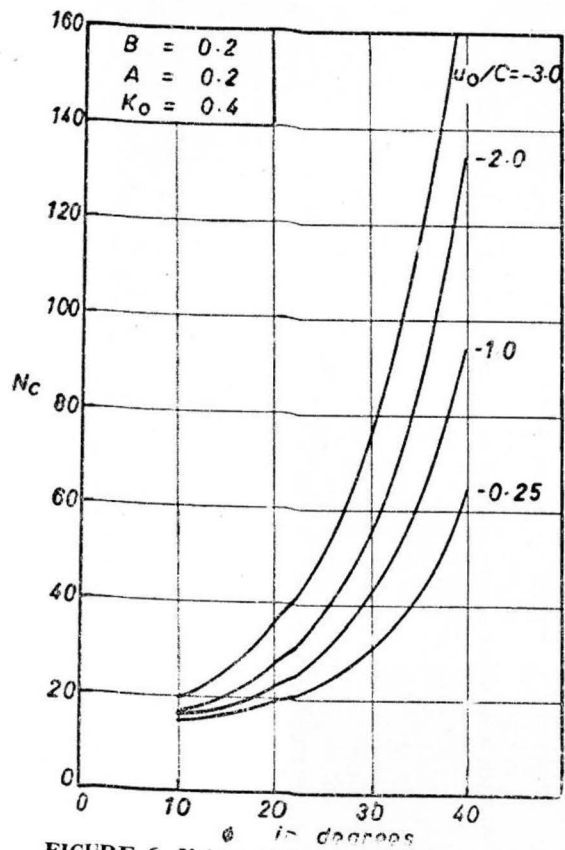


FIGURE 6. Values of  $N_c$  for  $B=0.2$  and  $A=0.2$

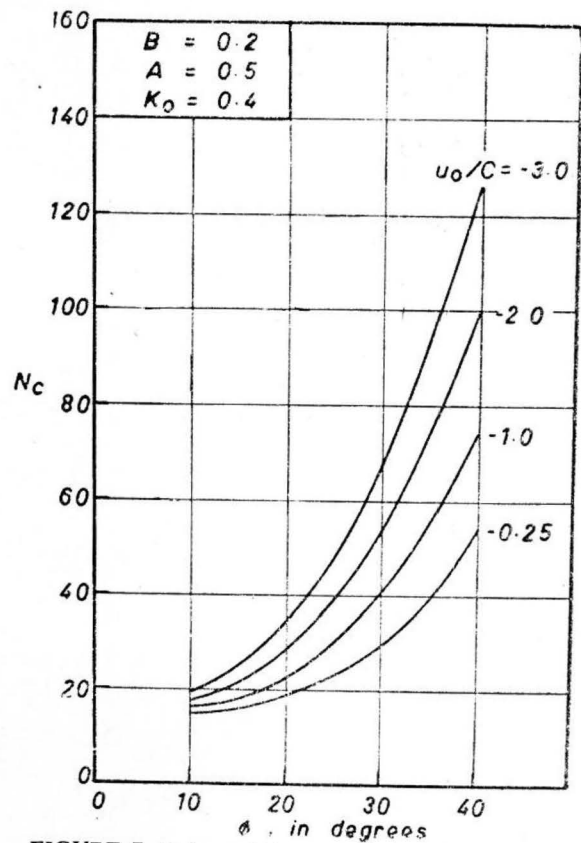
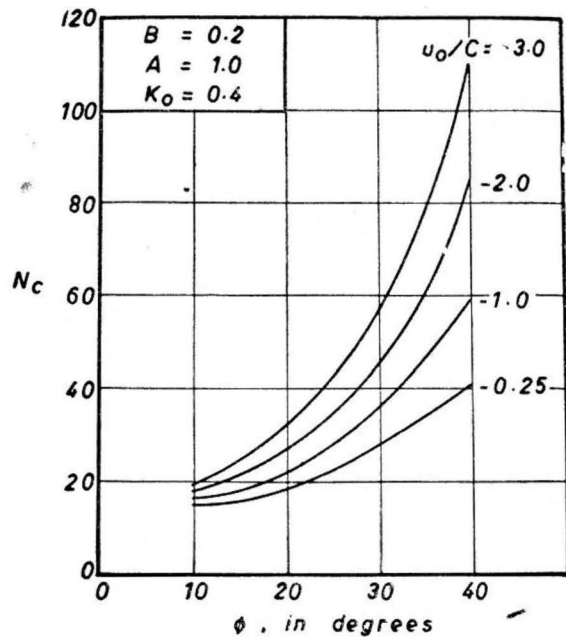
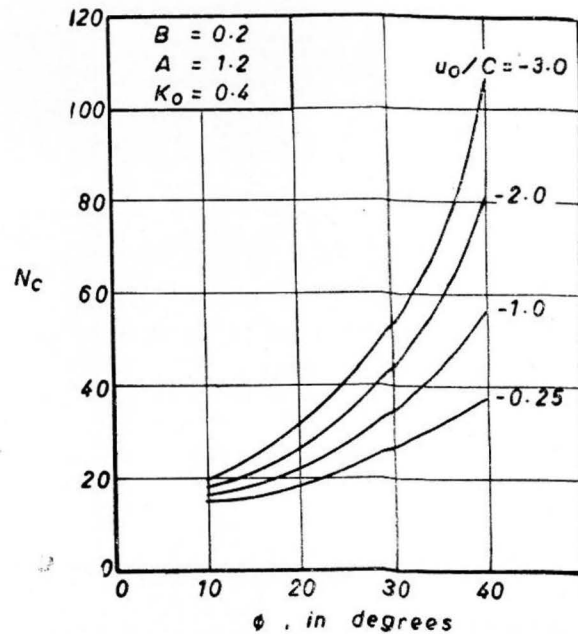
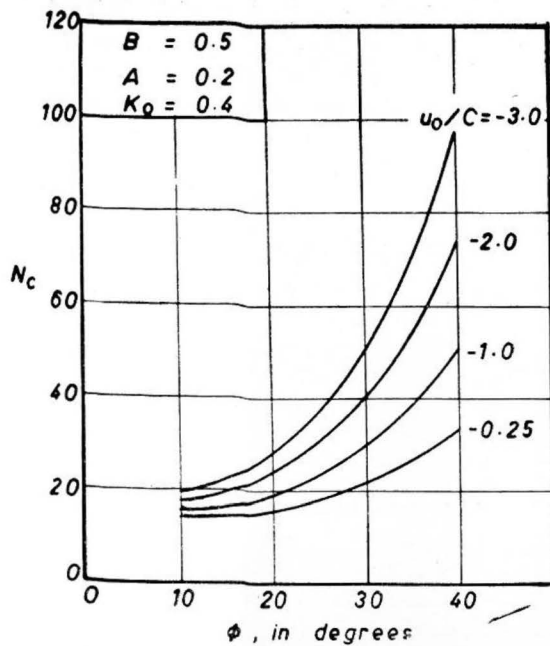
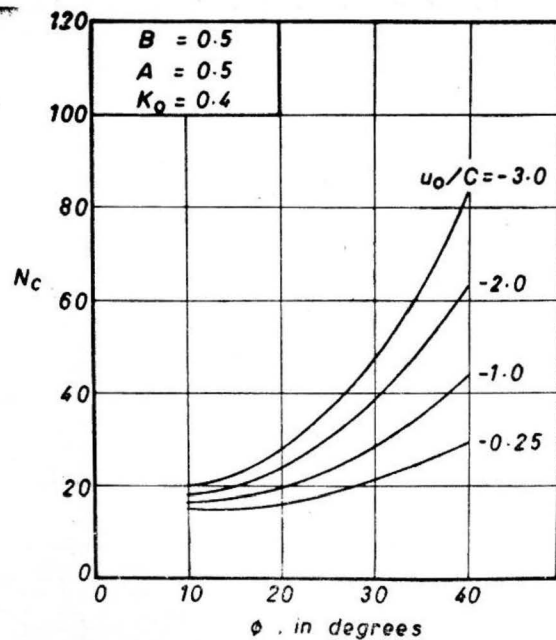
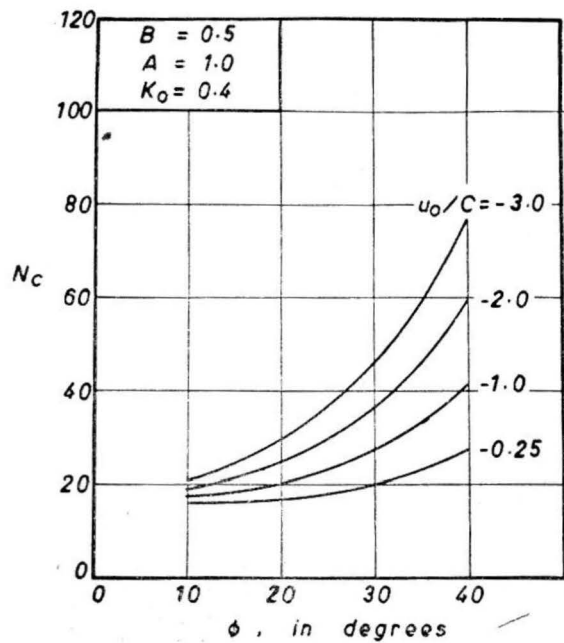
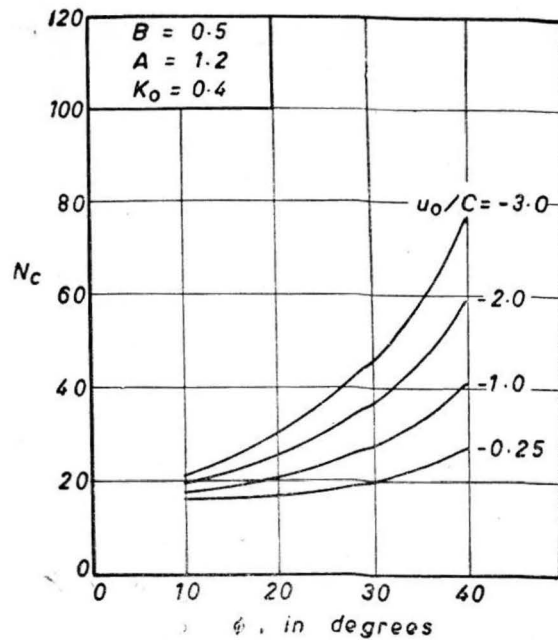


FIGURE 7. Values of  $N_c$  for  $B = 0.2$  and  $A = 0.5$

FIGURE 8. Values of  $N_c$  for  $B = 0.2$  and  $A = 1.0$ FIGURE 9. Values of  $N_c$  for  $B = 0.2$  and  $A = 1.2$

FIGURE 10. Values of  $N_c$  for  $B = 0.5$  and  $A = 0.2$ FIGURE 11. Values of  $N_c$  for  $B = 0.5$  and  $A = 0.5$

FIGURE 12. Values of  $N_c$  for  $B = 0.5$  and  $A = 1.0$ FIGURE 13. Values of  $N_c$  for  $B = 0.5$  and  $A = 1.2$

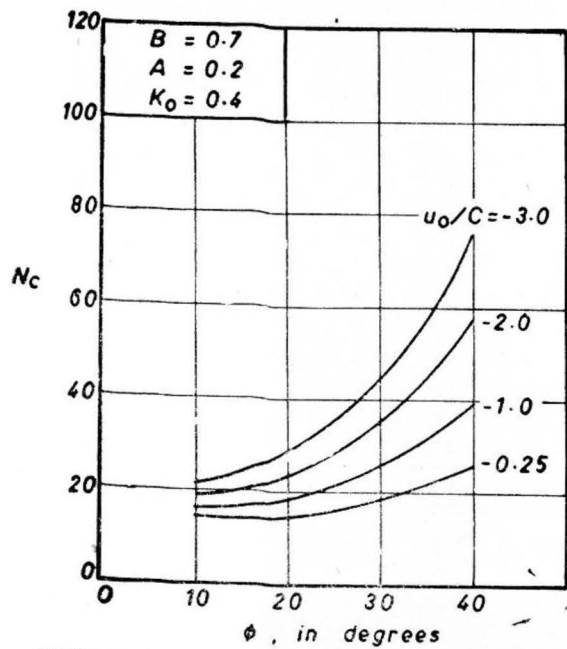


FIGURE 14. Values of  $N_c$  for  $B = 0.7$  and  $A = 0.2$

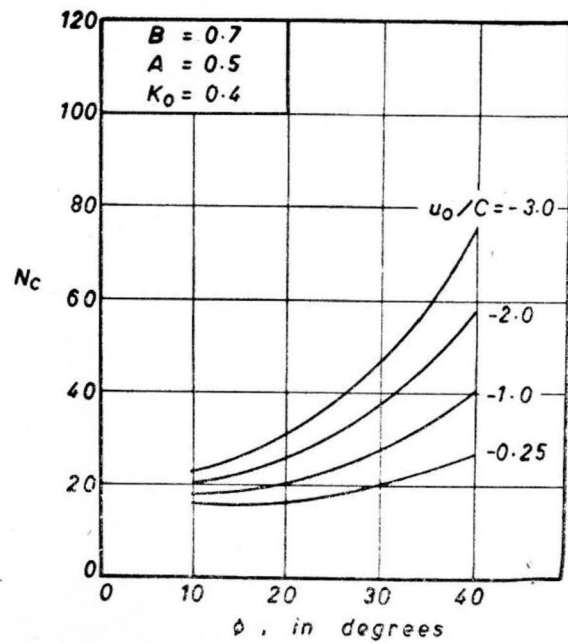


FIGURE 15. Values of  $N_c$  for  $B = 0.7$  and  $A = 0.5$

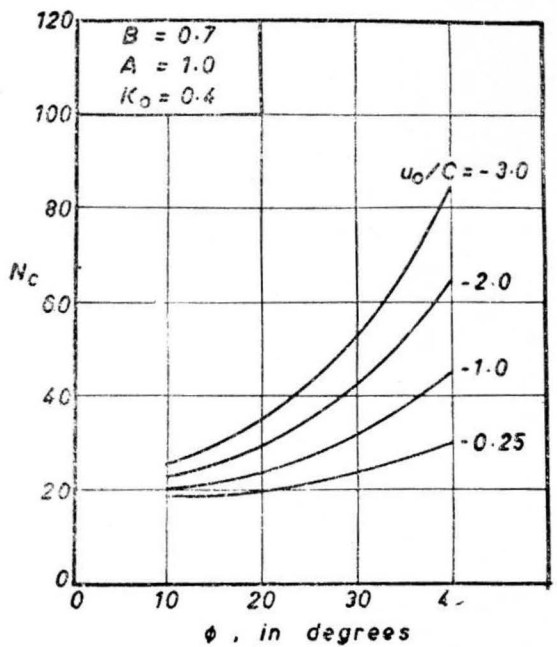


FIGURE 16. Values of  $N_c$  for  $B = 0.7$  and  $A = 1.0$

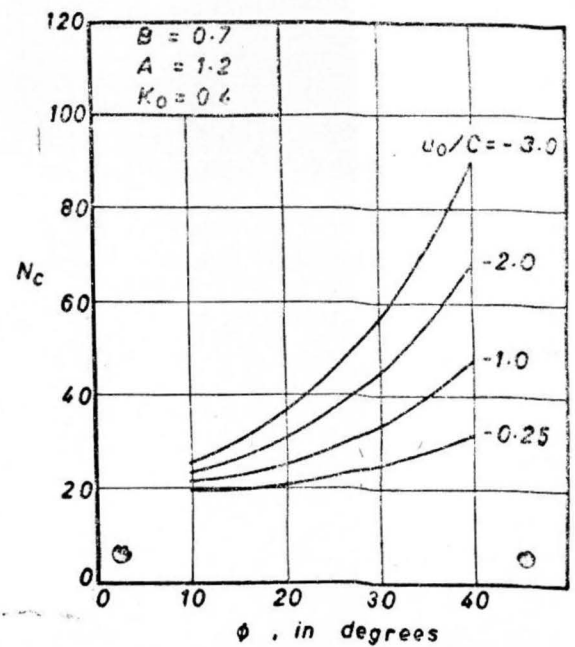


FIGURE 17. Values of  $N_c$  for  $B = 0.7$  and  $A = 1.2$



1943). It is observed from these figures that as initial negative pore pressure increases the value of  $N_c$  increases for given values of  $B$ ,  $A$  and  $\phi$ .

From these figures it is seen that when the other parameters defining the state of pore pressure ( $A$  and  $u_o/c$ ) are not insignificantly low, the effect of pore pressure parameter is to reduce  $N_c$ -value with increase of  $B$ . Increase of  $B$  from 0.2 to 0.7 decreases  $N_c$  by 56.5 per cent i.e., from 174.2 to 76.0 ( $\phi = 40^\circ$ ,  $A = 0.2$ ,  $u_o/c = -3.0$ ). For  $A = 0.2$  combinations of  $\phi = 30^\circ$  and  $u_o/c = -3.0$  and  $\phi = 40^\circ$  and  $u_o/c = -0.25$  give  $N_c$ -values of 75 and 65 for  $B = 0.2$  whereas for the same set of values at  $B = 0.7$  the corresponding  $N_c$ -values are 45 and 27. This indicates that the decrease of  $N_c$  with increase of  $B$  is more pronounced at higher values of  $\phi$ . From these figures it is also seen that when  $u_o/c$  changes from  $-0.25$  to  $-3.0$  the values of  $N_c$  increase by about 28 per cent for  $\phi$  equal to  $10^\circ$ , and by about 170 per cent for  $\phi$  equal to  $40^\circ$ , when  $B = 0.2$  and  $A = 0.2$ . For  $A = 1.2$ , keeping  $B$  constant at 0.2, the corresponding increase in  $N_c$  is about 183 per cent for  $\phi$  equal to  $40^\circ$ . The change in the value of  $N_c$  due to changes in the value of  $B$ ,  $A$  and  $u_o/c$  is thus quite appreciable and not be disregarded.

The values of  $N_\gamma$  and  $N_q$  are presented only for extreme values of initial pore pressures in Figures 18 through 29 as the bearing capacity factors  $N_\gamma$  and  $N_q$  are not significantly affected by the values of initial negative pore pressures. It is however seen that as initial negative pore pressure increases both  $N_\gamma$  and  $N_q$  increase though slightly when compared to the increase in  $N_c$ . It may also be noted that the rate of increase of  $N_\gamma$  with decrease of  $B$  is much more (for  $\phi = 40^\circ$ ,  $A = 0.2$  and  $u_o/c = -0.25$  decrease of  $B$  from 0.7 to 0.5 increases  $N_c$  by 139 per cent,  $N_q$  by 155 per cent and  $N_\gamma$  by 583 per cent). It is seen from these figures that when  $B$  changes from 0.2 to 0.7,  $N_\gamma$  reduces by about 28 per cent for  $\phi$  equal to  $10^\circ$  and about 85 per cent for  $\phi$  equal to  $40^\circ$ , when  $A = 0.2$ . When  $A = 1.2$ , the corresponding decreases in  $N_\gamma$  are, about 60 per cent for  $\phi$  equal to  $10^\circ$  and about 87 per cent for  $\phi$  equal to  $40^\circ$ , whereas  $N_q$  decreases by about 11 per cent for  $\phi$  equal to  $10^\circ$  and by about 50 per cent, when  $B$  changes from 0.2 to 0.5 and  $A = 0.2$ . Thus it is seen that the influence of  $B$  on  $N_\gamma$  is very considerable and also  $N_q$  is very much affected.

The bearing capacity factors are determined for  $\phi = 30^\circ$  when  $B$ ,  $A$  and  $u_o/c$  are equal to zero. These values are compared with the bearing capacity factors of Terzaghi (1943) in Table I. It is seen from this table that the values obtained in the present investigation agree with Terzaghi's values.

The values of  $N_c$ ,  $N_\gamma$  and  $N_q$  are presented against  $K_o$  for  $A = 0.1, 0.2$  and 1.2 and for  $B = 0.5$ ,  $\phi = 30^\circ$  and  $u_o/c = -2.0$  in Figures 30 and 31 to show the influence of  $K_o$  on the ultimate bearing capacity. It is observed from these figures that the influence of  $K_o$  on  $N_c$  is too insignificant and hence its effect may be taken to be negligible on the ultimate bearing capacity.

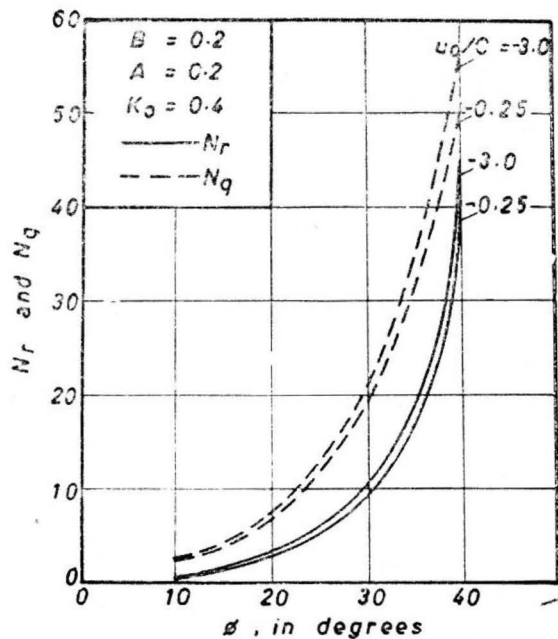


FIGURE 18. Values of  $N_r$  and  $N_q$  for  $B = 0.2$   
 and  $A = 0.2$

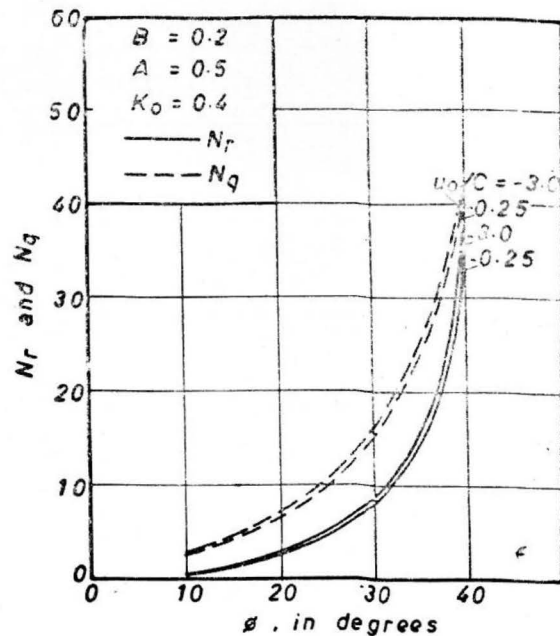


FIGURE 19. Values of  $N_r$  and  $N_q$  for  $B = 0.2$   
 and  $A = 0.5$

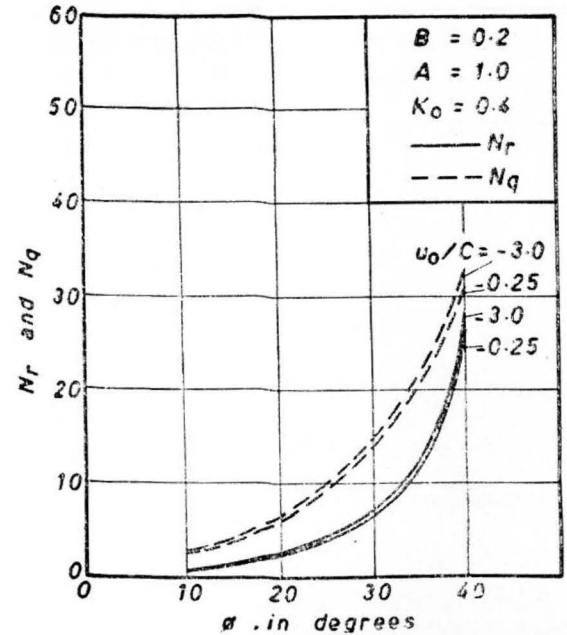


FIGURE 20. Values of  $N_r$  and  $N_q$  for  $B = 0.2$  and  $A = 1.0$

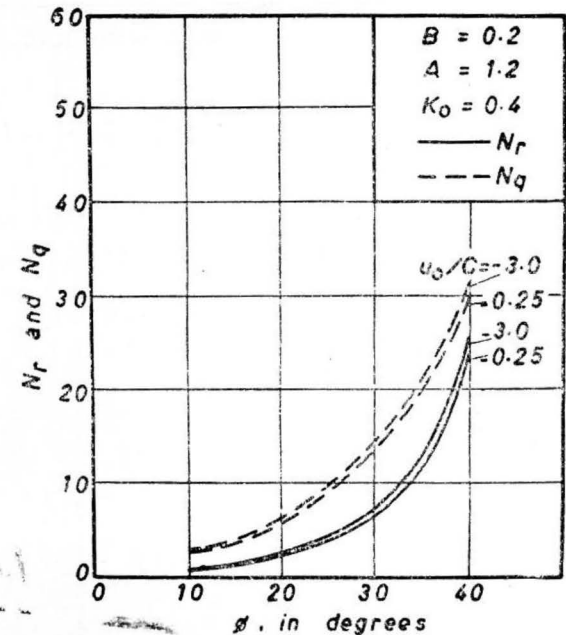


FIGURE 21. Values of  $N_r$  and  $N_q$  for  $B = 0.2$  and  $A = 1.2$

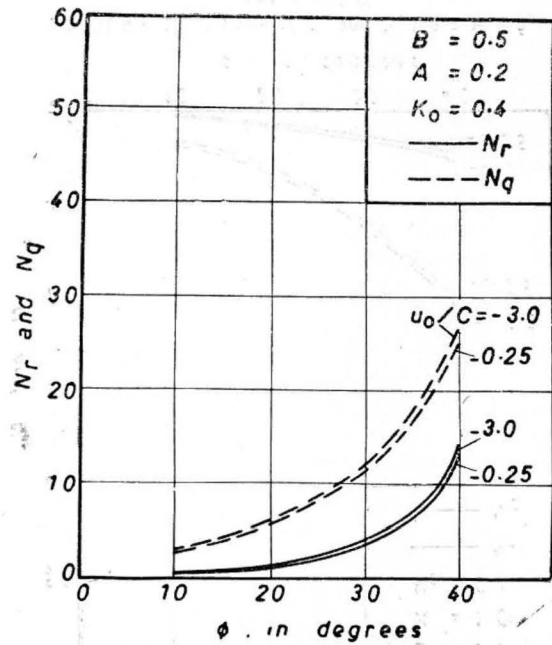


FIGURE 22. Values of  $N_r$  and  $N_q$  for  $B = 0.5$  and  $A = 0.2$

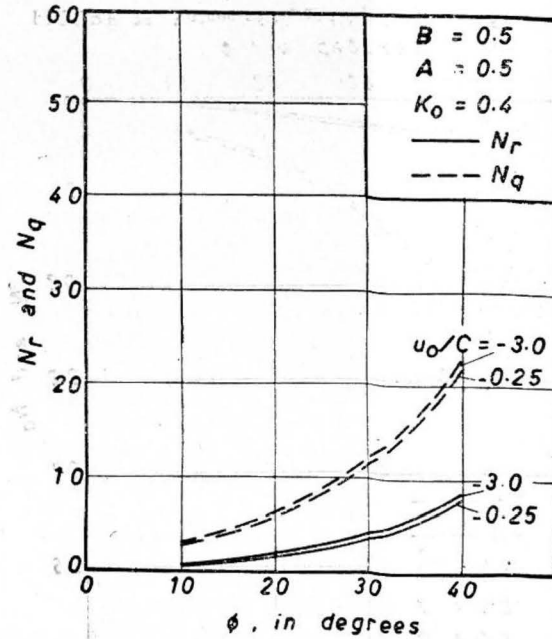


FIGURE 23. Values of  $N_r$  and  $N_q$  for  $B = 0.5$  and  $A = 0.5$

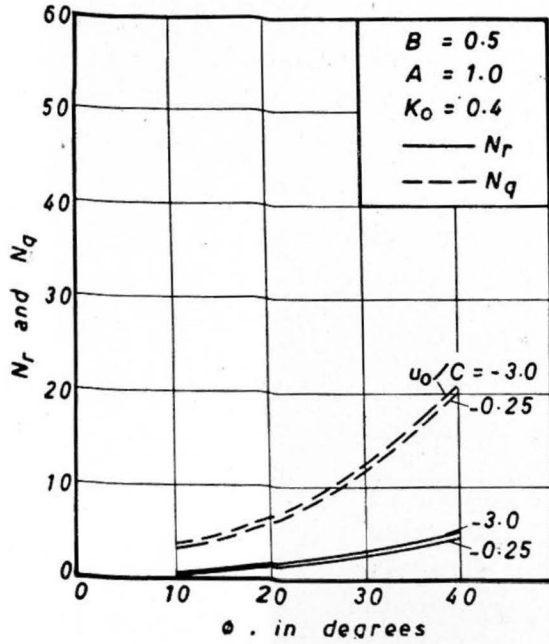


FIGURE 24. Values of  $N_r$  and  $N_q$  for  $B = 0.5$  and  $A = 1.0$

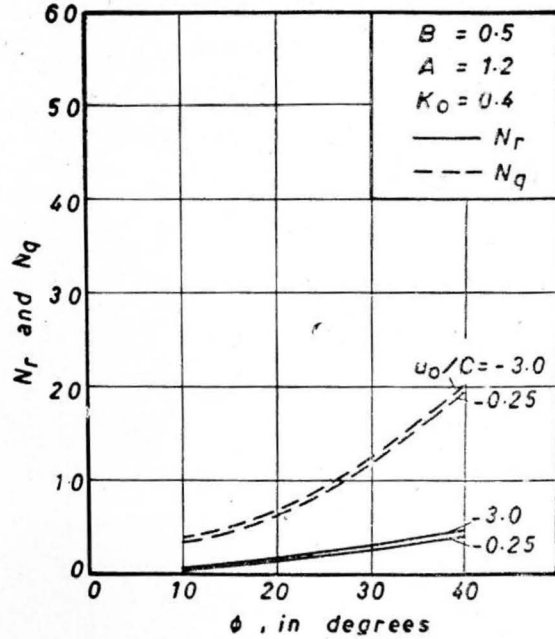


FIGURE 25. Values of  $N_r$  and  $N_q$  for  $B = 0.5$  and  $A = 1.2$

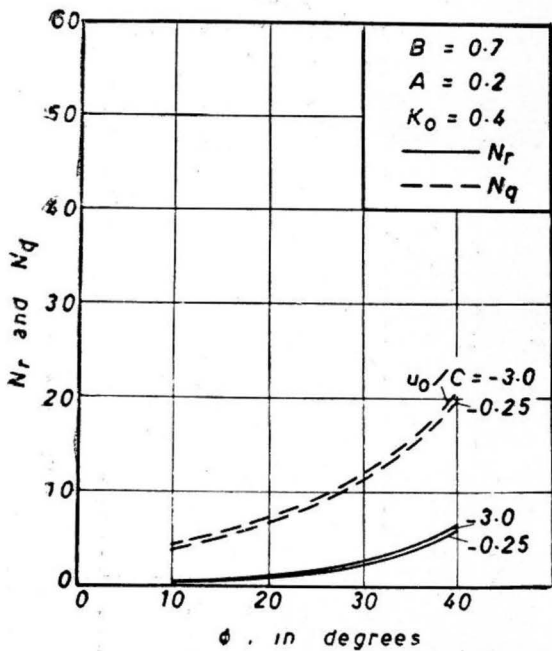


FIGURE 26. Values of  $N_r$  and  $N_q$  for  $B = 0.7$  and  $A = 0.2$

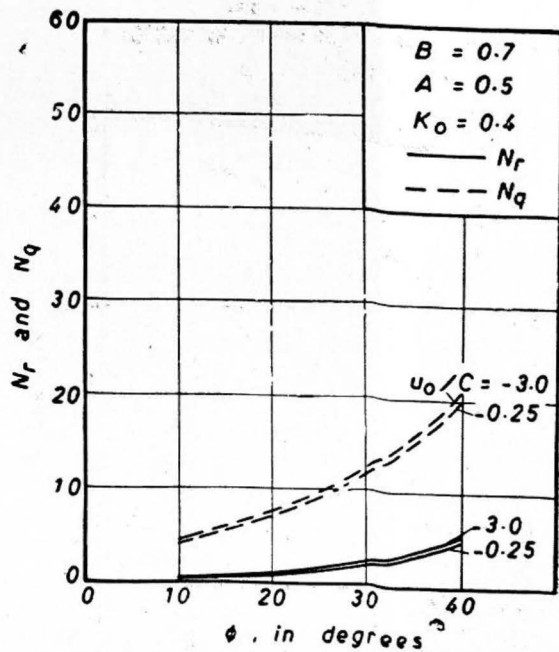


FIGURE 27. Values of  $N_r$  and  $N_q$  for  $B = 0.7$  and  $A = 0.5$

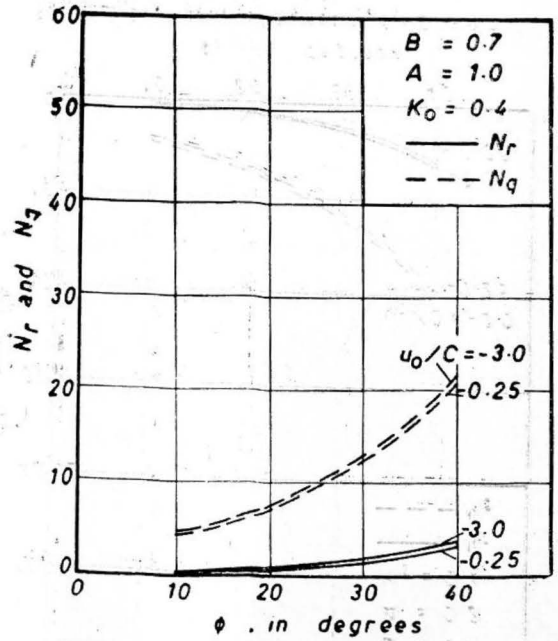


FIGURE 28. Values of  $N_r$  and  $N_q$  for  $B = 0.7$  and  $A = 1.0$

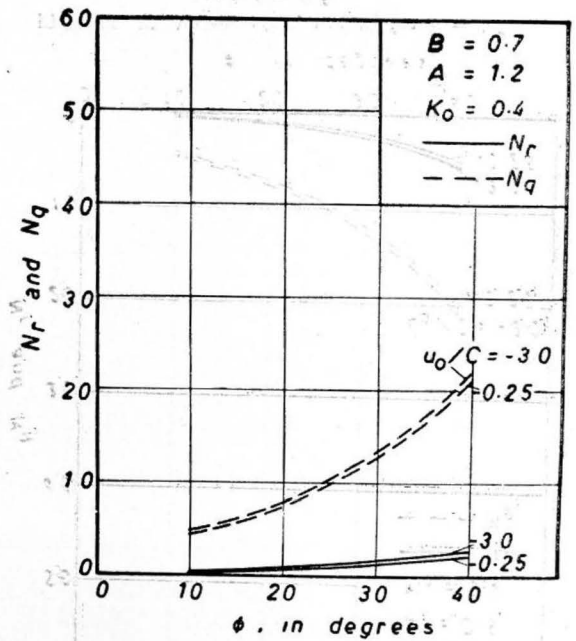


FIGURE 29. Values of  $N_r$  and  $N_q$  for  $B = 0.7$  and  $A = 1.2$

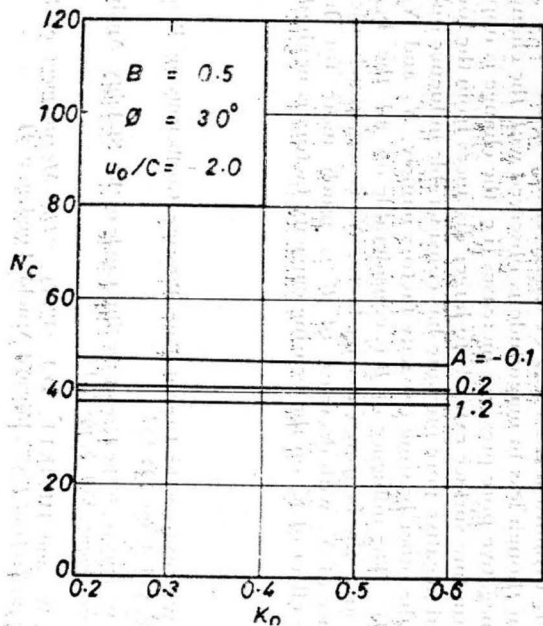


FIGURE 30. Influence of  $K_o$  on  $N_c$  for  $\phi = 20^\circ$ ,  $B = 0.5$  and  $\frac{u_o}{c} = -2.0$

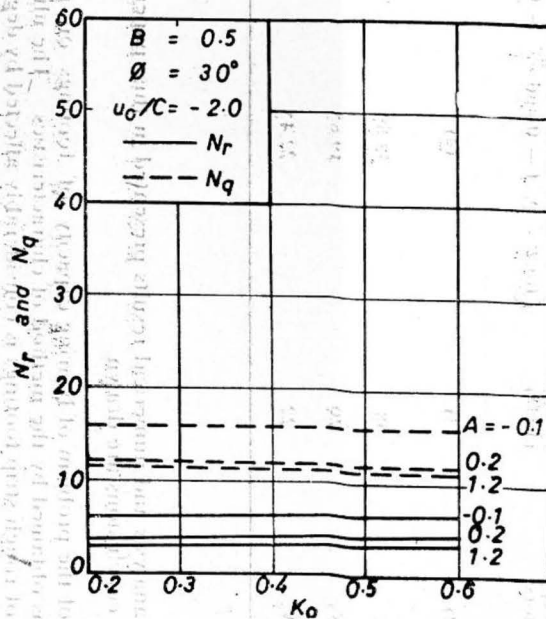


FIGURE 31. Influence of  $K_o$  on  $N_\gamma$  and  $N_q$  for  $\phi = 30^\circ$ ,  $B = 0.5$  and  $\frac{u_o}{c} = -2.0$



TABLE I

Comparison of bearing capacity factors

$\phi$	Bearing capacity Factors	Terzaghi (1943)	Present Analysis (for $B = 0$ , $A = 0$ and $\frac{u_o}{c} = 0$ )
(1)	(2)	(3)	(4)
30°	$N_c$	38	38.89
	$N_\gamma$	20	19.65
	$N_q$	22	22.45

### Conclusions

Based on the analysis and numerical results presented in this paper the following general conclusions are drawn.

The solution of the problem of bearing capacity of footings on partly saturated soils is obtained by the method of characteristics. The ultimate bearing capacity of rough strip footings is considerably affected by degree of saturation and initial pore pressures. This analysis also gives the influence of  $B$ ,  $A$  and  $u_o/c$  on the shape of failure surfaces. As  $B$  increases (i.e. with the increase of induced positive pore pressure) the rupture surface is shallower and more cylindrical than that of relatively dry soils. The same trend of behaviour though less in magnitude is observed with the change of  $A$ . When initial negative pore pressure increases (i.e. the change is opposite to that of  $B$ ) the rupture surface extends to larger distances from the footing width. The initial negative pore pressure has considerable influence only on values of  $N_c$  (i.e. the other bearing capacity factors namely,  $N_\gamma$  and  $N_q$  are not much affected). The influence of  $B$  is considerable on all the bearing capacity factors and the bearing capacity factor  $N_\gamma$  is very much affected. The rate of increase of  $N_\gamma$  with decrease of  $B$  is much more for higher values of  $\phi$ . The effect of  $K_o$  on the bearing capacity factors is negligible.

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