Stability of Slopes Under Foundation Load

by

A. Siva Reddy*
G. Mogaliah**

Introduction

Foundations are sometimes built on sloping sites or adjacent to the top edge of a slope or a cutting. In such situations the analysis should include the solution of the stability of slopes including the foundation load so as to evaluate the bearing capacity of foundations as the minimum of the bearing capacities obtained from ultimate bearing capacity point of view and stability of slopes. Gibson and Morgenstern (1962), Lo (1965), Livneh (1967) and Hunter and Schuster (1968) analysed the stability of slopes in soils whose undrained shear strength is anisotropic and nonhomogeneous. These studies do not include the effect of surcharge acting over a given width on the top of the slope. Meyerhof (1957) replaced load over the whole foundation area by a surcharge on the whole horizontal top surface of the slope and extended the solution of the slope stability obtained on the basis of dimensionless parameters by Janbu (1954). In the present investigation the analysis is made for the case of slope failure (through toe or base) under a foundation load using the friction circle method (Taylor 1937) for $c-\phi$ soils possessing anisotropy and nonhomogeneity in cohesion.

Analysis

(*a*) *Soils with* ϕ *greater than zero*

Figure 1 shows the schematic diagram for the case of slope failure under a foundation load Q_o acting over a width B_f. The edge of the load is at a distance b from the edge of the top of the slope. The slope inclined to the horizontal at an angle β_s is of height H_s. The slip circle which passes through A, makes an angle 2α at the centre O of the circle. The position of the centre O is described by two variable angles α and λ as shown in Figure 1. The cohesion of the soil is anisotropic and the cohesion on a plane corresponding to any value of ψ is :

$$c=c_H\,[1 + (k-1)\,sin^2\psi] \qquad \dots (1)$$

where ψ =inclination of major principal stress with horizontal, c_H= cohesion value for which $\psi=0°$, $k=c_V/c_H$, degree of anisotropy and c_V= cohesion value for which $\psi=90°$. The cohesion in a given direction also increases linearly with depth (Figure 1) and c_H at depth 'h' from top of the

*Professor of Civil Engineering, Indian Institute of Science, Bangalore-12.

**Lecturer, University Visvesvaraya College of Engineering, Bangalore University, Bangalore-56.

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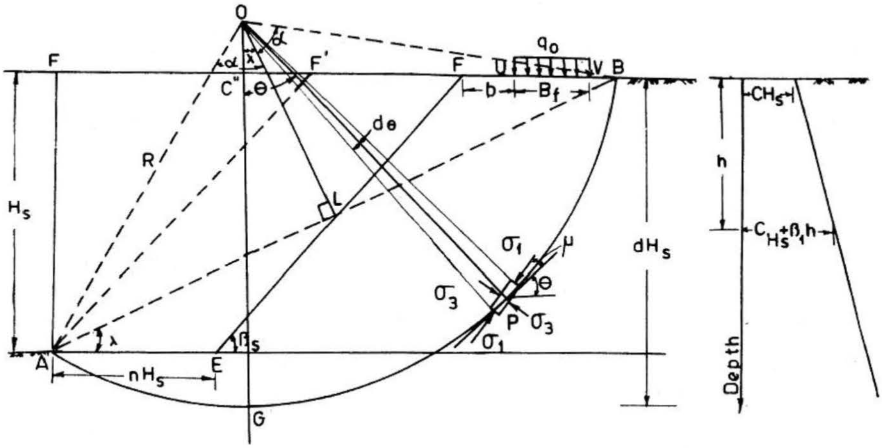

FIGURE 1. Failure surface showing various parameters for slope stability failure

slope is given by

$$c_H = c_{Hs} + \beta_1 h \qquad \cdots (2)$$

where c_{Hs}=cohesion corresponding to horizontal direction at the surface, $\beta_1 = \frac{\alpha_c}{k}$ and α_C=rate of variation of c_V with depth.

Since the major principal stress direction varies along the slip surface, the value of ψ for the portions GB and GA of the failure surface from

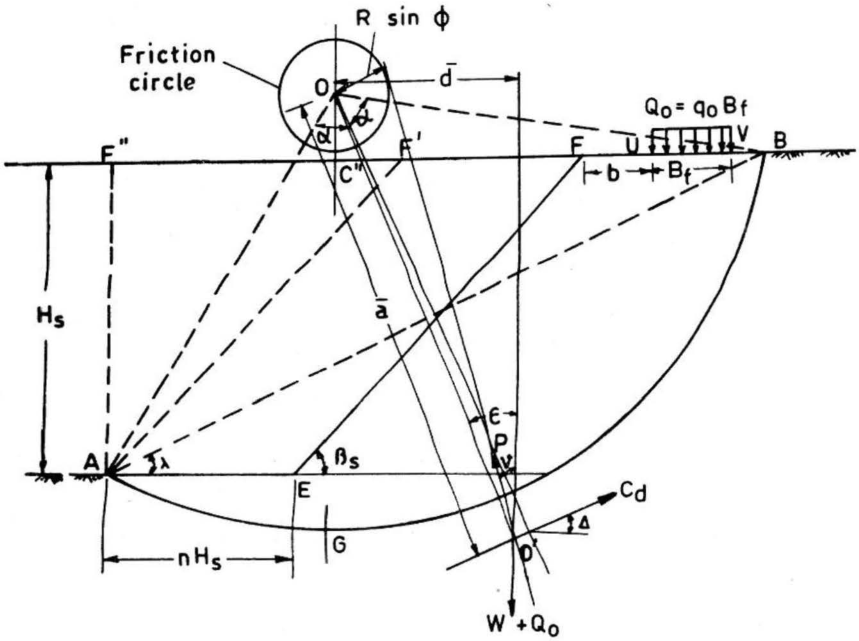

FIGURE 2. Force on soil mass above failure surface

Figure 1 is given by

$$\psi = \theta \pm \mu \qquad \ldots (3)$$

where θ=angle made by the radius rector OP with OG and μ=the angle between the major principal stress and the failure plane.

The forces acting on the soil mass above the failure surface are shown in Figure 2. These are the force W due to weight of the soil, the total load Q_o applied to the foundation on top of the slope, the resultant cohesion C_d required for equilibrium and P, the resultant normal force across the failure surface AB.

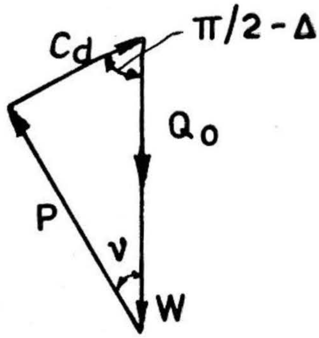

FIGURE 3. Force Polygon

Figure 3 shows the force polygon. The intersection of $(W+Q_o)$ and C_d is O' (Figure 2). Since the three forces $(W+Q_o)$, C_d and P must be concurrent, P must also pass through this intersection. With $(W+Q_o)$ known in magnitude and direction, and direction of C_d also known, the force polygon may be constructed if a second point is obtained on P to determine its direction. This is accomplished by the basic assumption of the ϕ—circle method. The assumption introduced is that P is tangent to the ϕ—circle.

From Figure 4,

$$OO' = \bar{d}\ cosec\epsilon = \bar{a}\ sec\ (\triangle - \epsilon) = R\ sin\phi\ cosec\ (\epsilon - V) \qquad \ldots(4)$$

From Figure 3 it is seen that

$$\frac{W+Q_o}{C} = \frac{cos\ (\triangle - V)}{sin\ V} = cos\ \triangle\ cot\ V + sin\ \triangle \qquad \ldots (5)$$

By substituting the expressions for W, Q_o and C_d in the above equation and after simplification, the expression for factor of safety F_s is obtained as

$$F_s = \frac{c_{Vs}}{\gamma H_s} N_s \qquad \ldots (6)$$

where γ = unit weight of soil and N_s is called stability number, given by

$$N_s = \frac{(\cos\triangle\ \cot V + \sin\triangle)\ T_2}{T_1} \qquad \ldots (7)$$

in which

$$\triangle = \tan^{-1} \frac{F_{cV}}{F_{cH}}$$

$$T_2 = \left[\frac{F^2_{cV} + F^2_{cH}}{(H_s c_{Vs/2})^2}\right]^{\frac{1}{2}}$$

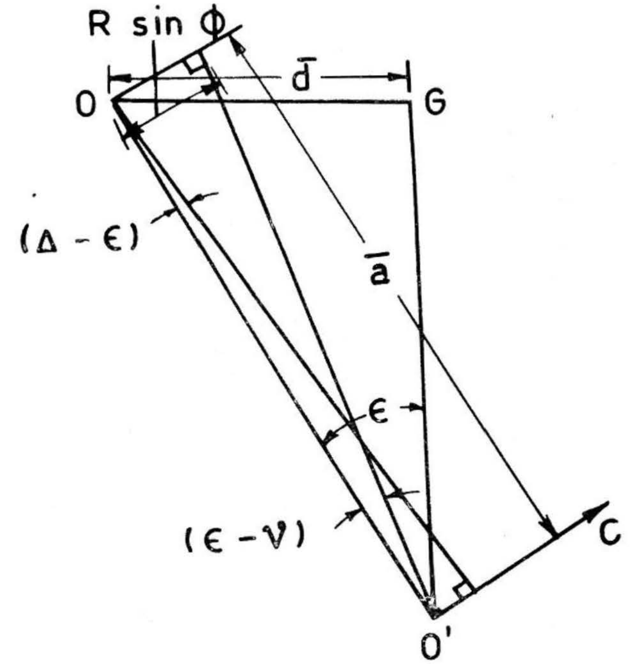

FIGURE 4. Relationship between γ, ϵ and $\triangle$

$$T_1 = \tfrac{1}{2}\operatorname{cosec}^2\lambda\,(\alpha\operatorname{cosec}^2\alpha - \cot\alpha) + \cot\lambda - \cot\beta_s - 2n + \frac{2_{qo}\,B_f}{\gamma H_s^2}$$

$$\sin(\epsilon - V) = \frac{H_s}{2\bar{d}}\sin\epsilon\operatorname{cosec}\alpha\operatorname{cosec}\lambda\sin\phi$$

$$\cot\epsilon = \frac{\bar{a}}{\bar{d}}\sec\triangle - \tan\triangle$$

$$F_{cH} = \frac{H_s}{2}\operatorname{cosec}\alpha\operatorname{cosec}\lambda\left[c_{Hs}\,I_{cH} + \frac{\beta_1 H_s}{2}\operatorname{cosec}\alpha\operatorname{cosec}\lambda\,J_{cH}\right]$$

$$F_{cV} = \frac{H_s}{2}\operatorname{cosec}\alpha\operatorname{cosec}\lambda\left[c_{Hs}\,I_{cV} + \frac{\beta_1 H_s}{2}\operatorname{cosec}\alpha\operatorname{cosec}\lambda\,J_{cV}\right]$$

$$\bar{a} = \frac{M_R}{(F^2_{cV} + F^2_{cH})^{1/2}}$$

$$\begin{aligned} M_r = {} & \left(\frac{H_s}{2}\operatorname{cosec}\alpha\operatorname{cosec}\lambda\right)^2\left[c_{Hs}\left\{(k+1)\alpha - \frac{(k-1)}{2}\sin 2\alpha\cos 2(\lambda+\mu)\right\}\right] \\ & + \left(\frac{H_s}{2}\operatorname{cosec}\alpha\operatorname{cosec}\lambda\right)^3\beta_1\Big[(k+1)\sin\alpha\cos\lambda \\ & - \alpha(k+1)\cos(\alpha+\lambda) + \frac{(k-1)}{2}\Big\{-\sin\alpha\cos(\lambda+2\mu) \\ & - \tfrac{1}{3}\sin 3\alpha\cos(3\lambda+2\mu) + \sin 2\alpha\cos(\alpha+\lambda)\cos 2(\lambda+\mu)\Big\}\Big] \end{aligned}$$

$$\bar{d} = \left[\frac{\gamma H^3_s}{12} (Y-Z) + \frac{q_o H_s B_f T}{2} \right] \div$$

$$\frac{\gamma H^2_s}{4} \left[\alpha \operatorname{cosec}^2 \alpha \operatorname{cosec}^2 \lambda - \cot\alpha \operatorname{cosec}^2\lambda \right.$$

$$\left. +2 (\cot\lambda - \cot \beta_s - 2n) + \frac{4\, q_o B_f}{\gamma H_s^2} \right]$$

$$Y = 1-2 \cot^2 \beta_s + 3 \cot\alpha \cot\lambda + 3 \cot\beta_s \cot\lambda - 3 \cot\beta_s \cot\alpha$$

$$Z = 6n\, (n + \cot\beta_s - \cot\lambda + \cot\alpha)$$

$$T = 2 \cot \beta_s - \cot\lambda + \cot\alpha + 2n + \frac{2b}{H_s} + \frac{B_f}{H_s}$$

$$I_{cH} = (k+1) \sin\alpha \cos\lambda - \frac{(k-1)}{2} \left[\sin\alpha \cos (\lambda + 2\mu) \right.$$

$$\left. + \frac{1}{3} \sin 3\alpha \cos (3\lambda + 2\mu) \right]$$

$$I_{cV} = (k+1) \sin\alpha \sin\lambda + \frac{(k-1)}{2} \left[\sin\alpha \sin (\lambda + 2\mu) \right.$$

$$\left. - \frac{1}{3} \sin 3\alpha \sin (3\lambda + 2\mu) \right]$$

$$J_{cH} = \frac{(k+1)\alpha}{2} - \tfrac{1}{2} \sin 2\lambda \cos 2\alpha - \sin (\alpha - \lambda) \cos (\alpha + \lambda)$$

$$+ \frac{(k-1)}{2} \left[\tfrac{1}{2} \sin 2\alpha \cos 2\lambda \right.$$

$$- \frac{\cos^2 (\alpha + \lambda) \sin 2 (\alpha + \lambda + \mu)}{4}$$

$$- \frac{\cos^2 (\alpha - \lambda) \sin 2 (\alpha - \lambda - \mu)}{4}$$

$$- 2\alpha \cos 2\mu - \frac{\sin 2\alpha \cos 2 (\lambda + \mu)}{4}$$

$$- \cos (\alpha + \lambda) \left\{ 2 \sin\alpha \sin\lambda - \sin\alpha \cos (\lambda + 2\mu) \right.$$

$$\left.\left. - \frac{1}{3} \sin 3\alpha \cos (3\lambda + 2\mu) \right\} \right]$$

$$J_{cV} = \frac{(k+1)}{4} \sin 2\alpha \sin 2\lambda + \cos^2 (\alpha + \lambda)$$

$$- \cos (\alpha - \lambda) \cos (\alpha + \lambda) + \frac{(k-1)}{2} \left[\frac{\alpha \sin 2\mu}{2} \right.$$

$$- \frac{\sin 4\alpha \sin (4\lambda + 2\mu)}{8} - \cos (\alpha + \lambda)$$

$$\left\{ 2 \sin\alpha \sin\lambda + \sin\alpha \sin (\lambda + 2\mu) \right.$$

$$\left.\left. - \frac{1}{3} \sin 3\alpha \sin (3\lambda + 2\mu) \right\} \right]$$

For minimum factor of safety, from Equation (6) it is seen that N_s should be minimum. Therefore, the minimum N_s is obtained by minimizing the right hand side expression [Equation (7)] with respect to α and λ such that

$$\left.\begin{aligned}\frac{\partial N_s}{\partial \alpha} &= 0\\ \frac{\partial N_s}{\partial \lambda} &= 0\end{aligned}\right\} \qquad \ldots (8)$$

(*b*) *Soils with* $\phi = 0$

For the case of slopes in soils with $\phi = 0$, the following expression for N_s is obtained after substituting $\phi = 0$ in the above analysis and simplifying :

$$\begin{aligned}N_s = & \frac{3}{k\sin^2\alpha \sin^2 \lambda\left(Y-Z+\frac{6q_oB_fT}{\gamma H_s^2}\right)}\Big[(k+1)\,\alpha \\ & -\frac{(k-1)}{2}\sin 2\alpha \cos 2\,(\lambda+\mu) \\ & +\frac{\beta_1 H_s}{2c_{Hs}}\operatorname{cosec}\alpha \operatorname{cosec}\lambda\Big\{(k+1)\sin\alpha\cos\lambda \\ & -\alpha\,(k+1)\cos\,(\alpha+\lambda)\Big\}+\frac{\beta_1 H_s\,(k-1)\operatorname{cosec}\alpha\operatorname{cosec}\lambda}{4c_{Hs}} \\ & \Big\{-\sin\,\alpha\,\cos\,(\lambda+2\mu)-\frac{1}{3}\sin 3\alpha\cos\,(3\lambda+2\mu) \\ & +\sin 2\alpha\cos\,(\alpha+\lambda)\,\cos\,(\lambda+\mu)\Big\}\Big] \qquad \ldots (9)\end{aligned}$$

When $\beta_1 = 0$ and $Q_o = 0$, the above equation reduces to

$$N_s = \frac{3\left[(k+1)\alpha - \frac{(k-1)}{2}\sin 2\alpha\ \cos(\lambda+2\mu)\right]}{k\,(Y-Z)\sin^2\alpha\,\sin^2\lambda}$$

This expression is the same as given by Lo (1965) when $k = \frac{1}{k}$ and $\mu = \frac{\pi}{2} - f$, the right hand side terms being those defined by Lo.

Results and Discussion

The values of N_s are obtained for different values of

$$\beta_s,\ k,\ \frac{\beta_1 H_s}{2c_{Hs}},\ \frac{\beta_f}{H_s},\ \frac{b}{H_s} \text{ and } \frac{q_o}{\gamma H_s}.$$

The values of β_s and k are varied from 15° to 75° and 0.8 to 2, respectively. $\frac{\beta_1 H_s}{2cH_s}$ is varied from 0 to 0.4 and $\frac{q_o}{\gamma H_s}$ from 0 to 1.0. Since the bearing capacity of foundations on top of clay slopes is frequently governed by overall slope failure (Meyerhof 1957), the numerical results are presented for $\phi = 0°$ and 10° in Tables 1 to 4. As the computer time involved in obtaining the numerical results is very much and also the number of parameters are more, in this paper the numerical results are presented for the

TABLE 1

Values of N_s for $\phi = 0°$ and $q_o/\gamma H_s = 0$

β_s, in degrees	$\frac{\beta_1 H_s}{2c_{Hs}}$	Values of N_s				
		k=0.8	k=1.0	k=1.3	k=1.6	k=2.0
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15	0	6.2141	5.5316	4.9015	4.5077	4.1663
	0.2	11.4862	10.4546	9.2846	8.5396	7.8815
	0.4	15.8906	13.6378	12.1829	11.2764	10.7109
30	0	6.2038	5.5246	4.8934	4.5054	4.1626
	0.2	9.4985	8.7972	8.0736	7.5316	7.0576
	0.4	11.9531	11.0810	10.2489	9.7125	9.2353
45	0	6.1942	5.5243	4.8902	4.5035	4.1583
	0.2	8.0222	7.6114	7.2015	6.9267	6.6741
	0.4	9.8030	9.3138	8.8268	8.5009	8.2022
60	0	5.4221	5.2474	4.8524	4.4923	4.1541
	0.2	6.8030	6.5993	6.3854	6.2349	6.0911
	0.4	8 16132	7.9108	7.6872	7.5166	7.3542
75	0	4.6348	4.5645	4.4854	4.4264	4.1502
	0.2	5.8150	5.6340	5.4413	5.4301	5.4254
	0.4	6.7555	6.6797	6.5902	6.5255	6·4676

TABLE 2

Values of N_s for $\phi = 10°$ and $q_o/\gamma H_s = 0$

β_s, in degrees	$\frac{\beta_1 H_s}{2c_{Hs}}$	Values of N_s		
		$k=0.8$	$k=1.0$	$k=2.0$
(1)	(2)	(3)	(4)	(5)
15	0	45.175	41.132	33.014
	0.2	61.782	56.027	44.467
	0.4	78.146	70.706	55.763
30	0	13.850	12.870	10.835
	0.2	15.286	16.957	14.200
	0.4	22.659	20.986	17.516
45	0	9.558	9 088	8.048
	0.2	12.211	11.604	10.271
	0.4	14.835	14.093	12.469
60	0	7.393	7.180	6.669
	0.2	9.231	8.969	8.351
	0.4	11.054	10.743	10.016
75	0	5.856	5.779	5.575
	0.2	7.193	7.103	6.875
	0.4	8.522	8.417	8.162

TABLE 3

Values of N_s for $\phi = 0°$ and $b/H_s = 0$

β_s, in degrees	$\frac{B_f}{H_s}$	$\frac{\beta_1 H_s}{2c_{H_s}}$	$\frac{q_o}{\gamma H_s}$	Values of N_s $k=0.8$	$k=1.0$	$k=2.0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15	0.23	0	0.1	6.2131	5.5307	4.1657
			0.2	6.2119	5.5297	4.1649
			0.3	6.2109	5.5288	4.1643
			0.4	6.2098	5.5278	4.1635
			0.5	6.2086	5.5268	4.1628
			0.6	6.2075	5.5258	4.1621
			0.7	6.2063	5.5248	4.1613
			0.8	6.2051	5.5238	4.1605
			0.9	6.2039	5.5226	4.1598
			1.0	6.2027	5.5216	4.1589
		0.2	0.1	11.382	10.251	7.856
			0.2	11.241	10.126	7.779
			0.3	11.096	9.996	7.678
			0.4	10.946	9.863	7.574
			0.5	10.792	9.725	7.467
			0.6	10.632	9.583	7.356
			0.7	10.467	9.437	7.242
			0.8	10.297	9.285	7.123
			0.9	10.122	9.129	7.004
			1.0	9.947	8.973	6 886
		0.4	0.1	15.659	13.517	10.678
			0.2	15.446	13.310	10.527
			0.3	15.227	13.097	10.370
			0.4	15.001	12.878	10.209
			0.5	14.769	12.653	10.044
			0.6	14.529	12.421	9.873
			0.7	14.284	12.189	9.697
			0.8	14.031	11.958	9.521
			0.9	13.772	11.728	9.344
			1.0	13.513	11.497	9.168

TABLE 3 (Continued)

β_s. in degrees	$\frac{B_f}{H_s}$	$\frac{\beta_1 H_s}{2c_{Hs}}$	$\frac{q_0}{\gamma H_s}$	Values of N_s $k=0.8$	$k=1.0$	$k=2.0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15	0.50	0	0.1	6.2118	5.5296	4.1649
			0.2	6.2095	5.5275	4.1634
			0.3	6.2071	5.5254	4.1618
			0.4	6.2046	5.5232	4.1602
			0.5	6.2020	5.5209	4.1585
			0.6	6.1994	5.5186	4.1568
			0.7	6.1967	5.5162	4.1551
			1.0	6.1881	5.5086	4.1495
		0.2	0.1	11.063	9.966	7.756
			0.2	10.741	9.680	7.539
			0.3	10.398	9.375	7.309
			0.4	10.034	9.050	7.062
			0.5	9.647	8.704	6.798
			0.6	9.234	8.336	6.517
			0.7	8.797	7.944	6.216
			0.8	8.331	7.527	5.895
			0.9	7.836	7.083	5.551
			1.0	7.346		
		0.4	0.1	15.179	13.245	10.320
			0.2	14.696	12.790	9.975
			0.3	14.186	12.309	9.610
			0.4	13.646	11.800	9.222
			0.5	13.076	11.262	8.811
			0.6	12.474	10.693	8.375
			0.7	11.838	10.091	7.911
			0·8	11.166	9.454	7.419
			0.9	10.457	8.817	6.926
			1.0	9.757	8.187	6.437

TABLE 3 (Continued)

β_s, in degrees	$\frac{B_f}{H_s}$	$\frac{\beta_1 H_s}{2c_{Hs}}$	$\frac{q_o}{\gamma H_s}$	Values of N_s $k=0.8$	$k=1.0$	$k=2.0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
30	0.25	0	0.1	6.2023	5.5248	4.1650
			0.2	6.2001	5.5230	4.1645
			0.3	6.1958	5.5222	4.1639
			0.4	6.1783	5.5214	4.1634
			0.5	5.9376	5.5206	4.1628
		0.2	0.1	9.2267	8.5550	6.9721
			0.2	8.9348	8.2935	6.8816
			0.3	8.6217	8.0113	6.6956
			0.4	8.2858	7.7067	6.4496
			0.5	7.9259	7.3778	6.1781
		0.4	0.1	11.587	10.754	8.9837
			0.2	11.198	10.404	8.7105
			0.3	10.782	10.029	8.4132
			0.4	10.339	9.6276	8.0890
			0.5	9.8678	9.1965	—
45	0.25	0	0.1	6.0051	5.5012	4.1580
			0.2	5.7907	5.4981	4.1562
			0.3	5.5597	5.2851	4.1551
			0.4	5.3121	5.0532	4.1543
		0.2	0.1	7.7384	7.3553	6.4713
			0.2	7.4358	7.0792	6.2446
			0.3	7.1144	6.7819	5.9908
			0.4	6.7741	6.4626	5.7055
			0.5	6.4152	6.1203	—
		0.4	0.1	9.4381	8.9839	7.9402
			0.2	9.0521	8.6309	7.6508
			0.3	8.6446	8.2539	7.3302
			0.4	8.2156	7.8514	6.9732
			0.5	7.7650	7.4221	—

TABLE 3 (Continued)

β_s, in degrees	$\frac{B_f}{H_s}$	$\frac{\beta_1 H_s}{2c_{Hs}}$	$\frac{q_o}{\gamma H_s}$	Values of N_s		
				$k=0.8$	$k=1.0$	$k=2.0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
60	0.25	0	0.1	5.2259	5.0672	4.1360
			0.2	5.0204	4.8752	4.0831
			0.3	4.8071	4.6724	4.00020
			0.4	4.5876	4.4598	3.99718
			0.5	4.3636	4.2387	—
		0.2	0.1	6.5360	6.3541	5.8904
			0.2	6.2625	6.0977	5.6674
			0.3	5.9816	5.8302	5.4221
			0.4	5.6952	5.5528	5.1523
			0.5	5.4051	5.2666	—
		0.4	0.1	7.8293	7.6238	7.1026
			0.2	7.4903	7.3056	6.8267
			0.3	7.1442	6.9759	6.5265
			0.4	6.7929	6.6357	6.1993
			0.5	6.4384	6.2864	—
75	0.25	0	0.1	4.4548	4.3936	4.1255
			0.2	4.2730	4.2184	4.0522
			0.3	4.0913	4.0407	3.8782
			0.4	3.9113	3.8624	3.6945
			0.5	3.7345	3.6851	3.5019
			0.6	3.5621	3.5102	3.3016
		0.2	0.1	5.4658	5.4027	5.2220
			0.2	5.2327	5.1774	5.0125
			0.3	5.0015	4.9511	4.7926
			0·4	4.7743	4.7259	4.5632
			0.5	4.5524	4.5023	4.3252
		0.4	0.1	6.4679	6.4024	6.2164
			0.2	6.1847	6.1283	5.9620
			0.3	5.9054	5.8546	5.6973
			0.4	5.6319	5.5835	5.4235
			0.5	5.3659	5.3166	5.1412
			0.6	5.1086	5.0556	4.8509

TABLE 4

Values of N_s for $\phi = 10°$ and $b/H_s = 0$

β_s, in degrees	$\frac{B_f}{H_s}$	$\frac{\beta_1 H_s}{2c_{Hs}}$	$\frac{q_o}{\gamma H_s}$	Values of N_s $k=0.8$	$k=1.0$	$k=2.0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
60	0.25	0	0.1	6.9550	6.7607	6.2837
			0.2	6.51917	6.3389	5.8827
			0.3	6.0887	5.9173	5.4655
			0.4	5.6662	5.4978	5.0312
		0.2	0.1	8.6712	8.4355	7.8672
			0.2	8.1166	7.9011	7.3675
			0.3	7.5706	7.3685	6.8504
			0.4	7.0358	6.8399	—
		0.4	0.1	10.3743	10.0966	9.4354
			0.2	9.7030	9.4516	8.8387
			0.3	9.0433	8.8101	8.2235
			0.4	8.3979	8.1743	—
75	0.25	0	0.1	5.5414	5.4728	5.2841
			0.2	5.2347	5.1712	4.9874
			0.3	4.9382	4.8673	4.6867
			0.4	4.6534	4.5901	4.3829
			0.5	4.3814	4.3139	—
		0.2	0.1	6.7969	6.7177	6.5126
			0.2	6.4127	6.3405	6.1461
			0.3	6.0427	5.9735	5.7774
			0.4	5.6886	5.6186	5.4073
			0.5	5.3513	5.2771	5.0359
		0.4	0.1	8.0454	7.9539	7.7286
			0.2	7.5847	7.5023	7.2933
			0.3	7.1423	7.0643	6.8576
			0.4	6.7196	6.6417	6.4221
			0.5	6.3178	6.2359	5.9868

case $\frac{b}{H_s} = 0$ only. These values of N_s represent the minimum values obtained from toe circle failure, base circle failure and slope failure cases. All these values are obtained for $\mu = 45° - \frac{\phi}{2}$, in order to compare the value of the bearing capacity obtained from this analysis with that obtained from the analysis of bearing capacity failure. It is seen from these tables that, as the value of $\frac{q_o}{\gamma H_s}$ increases, the value of N_s decreases for given values of β_s, k and $\frac{\beta_1 H_s}{2c_{Hs}}$ due to the fact that the surcharge contributes more to the disturbing moment. As $\frac{\beta_1 H_s}{2c_{Hs}}$ (i.e. cohesion increasing with depth) increases, N_s increases and when k changes from 0.8 to 2 it decreases. It is also observed from these tables that as β_s increases N_s decreases. To show graphically the influence of $\frac{q_o}{rH_s}$, k and $\frac{\beta_1 H_s}{2c_{Hs}}$ the values of N_s are presented in Figures 5 to 8 for extreme values of k and $\frac{\beta_1 H_s}{2c_{Hs}}$ and $\beta_s = 15°$, 30° and 75°. Figures 5 and 6 are for surcharge for finite value of B_f whereas Figures 7 and 8 are for infinite values of B_f. From Figures 5 and 6 it is seen that as $\frac{q_o}{\gamma H_s}$ changes from 0 to 1.0, when $\frac{\beta_1 H_s}{2c_{Hs}} = 0$, the decrease in N_s is negligible for $\phi = 0°$, $\beta_s = 15°$ and $\frac{B_f}{H_s} = 0.25$. Whereas when $\frac{\beta_1 H_s}{2c_{Hs}} = 0.4$ the corresponding decreases in N_s are about 15 and 14.5 per cent for $k = 0.8$ and 2.0, respectively. For $\beta_s = 75°$, as $\frac{q_o}{\gamma H_s}$ increases from 0 to 0.6, when $\frac{\beta_1 H_s}{2c_{Hs}} = 0.0$, N_s decreases by about 23 and 20 per cent for $k = 0.8$ and 2.0, respectively. The corresponding values for $\frac{\beta_1 H_s}{2c_{Hs}} = 0.4$ are 24 and 25 per cent. For $\frac{q_o}{\gamma H_s} = 0.6, \beta_s = 15°, \phi=0°$ and $\frac{B_f}{H_s} = 0.25$, as $\frac{\beta_1 H_s}{2c_{Hs}}$ increases from 0 to 0.4, N_s increases by about 135 and 137 per cent for $k = 0.8$ and 2.0, respectively. The corresponding values when $\beta_s = 75°$ are about 43 and 47 per cent. When k changes from 0.8 to 2.0, the decreases in N_s for $\frac{q_o}{\gamma H_s} = 0.6$, $\beta_s = 15°$, $\phi = 0°$ and $\frac{B_f}{H_s} = 0.25$, are about 33 and 32 per cent for $\frac{\beta_1 H_s}{2c_{Hs}} = 0$ and 0.4, respectively. For $\beta_s = 75°$, the corresponding values are about 7 and 5 per cent. It is seen from Figures 7 and 8 that the same trend is noticeable for the case of surcharge on whole top of slope.

The values of N_s corresponding to $k = 1$, $\frac{\beta_1 H_s}{2c_{Hs}} = 0$ and $\phi = 0°$, obtained by the present analysis, are compared with those of Janbu (1954) in Table 5. It is seen from this table that the present N_s values are in good agreement with the values given by Janbu for unloaded surfaces of slope,

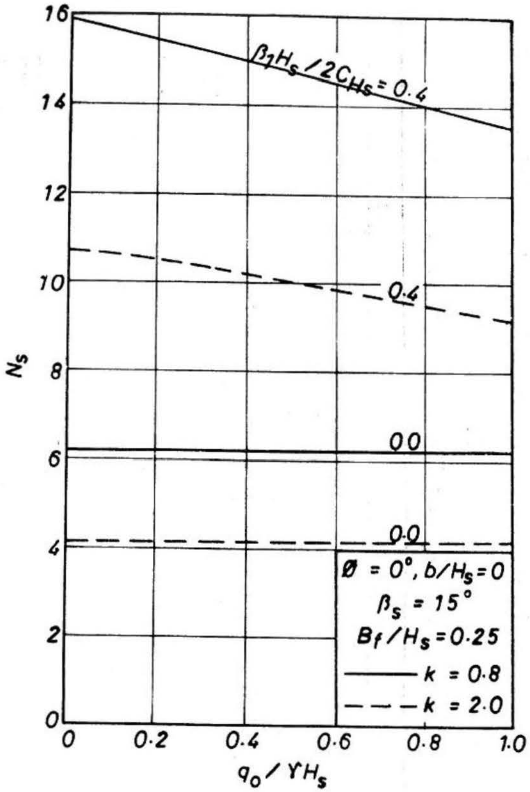

FIGURE 5. N_s vs $\frac{q_o}{\gamma H_s}$ for $\phi = 0°, \frac{B_f}{H_s} = 0.25$ and $\beta_s = 15°$

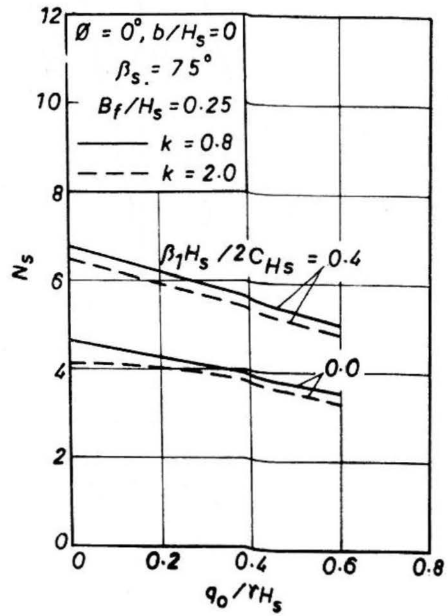

FIGURE 6. N_s vs $\frac{q_o}{\gamma H_s}$ for $\phi = 0°$, $\frac{B_f}{H_z} = 0.25$ and $\beta_s = 75°$

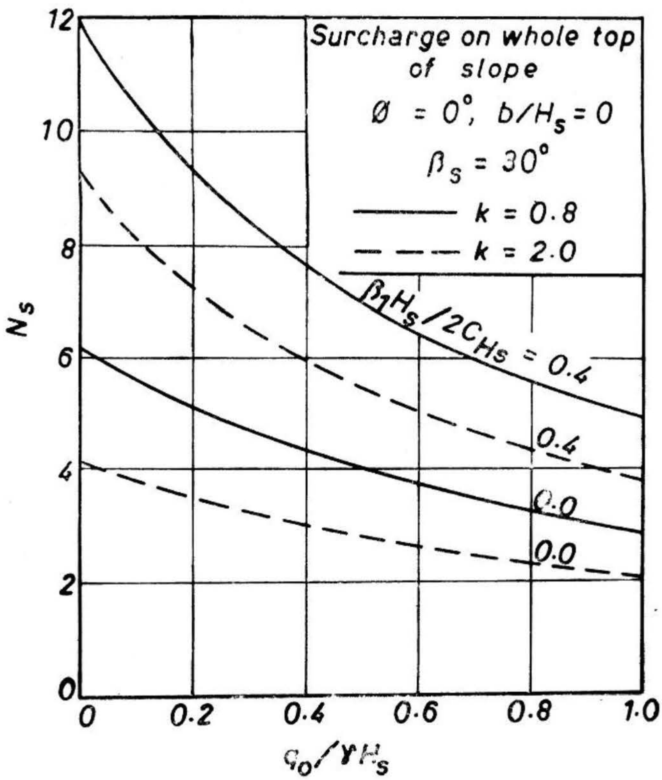

FIGURE 7. N_s vs $\frac{q_o}{\gamma H_s}$ for $\phi = 0°$ and $\beta_s = 30°$ (surcharge on whole top of slope)

TABLE 5

Comparison of N_s values for $\phi = 0°$ with those of Janbu (1954)

β_s, in degrees	$\frac{B_f}{H_s}$	$\frac{q_o}{\gamma H_s}$	Values of N_s: Present Analysis ($k=1$, $\beta_1 H_s/2c_{Hs} = 0$ and $b/H_s = 0$)	Values of N_s: Janbu (1954)
(1)	(2)	(3)	(4)	(5)
15	—	0	5.531	5.53
45	—	0	5.524	5.53
75	—	0	4.564	4.56
60	Surcharge on whole top of slope	0.2	3.968	3.97
60	,,	0.4	3.175	3.18
75	,,	0.2	3.391	3.39
75	,,	0.4	2.687	2.69
45	0.25	0.2	5.498	4.28
45	0.25	0.4	5.053	3.48
75	0.25	0.2	4.218	3.39
75	0.25	0.4	3.862	2.69

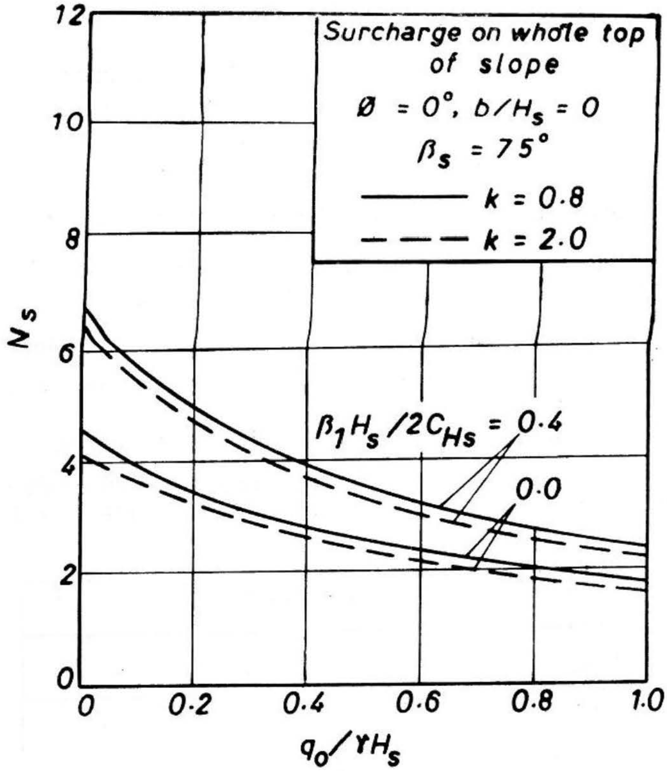

FIGURE 8. N_c vs $\frac{q_o}{\gamma H_s}$ for $\phi = 0$ and $\beta_s = 75°$ (surcharge on whole top of slope)

TABLE 6

Comparison of N_c values obtained from Slope Stability Analysis for $\phi = 0°$

β_s, in degrees	N_s	Present Analysis ($k=1$, $\beta_1 H_s/2c_{Hs} = 0$, $b/H_s = 0$)		Meyerhof (1957) (Surcharge on whole top of slope)
		B_f/H_s	N_c	N_c
15	5	0.25	0.53	0.52
60	5	0.25	0.67	0.10
60	4.5	0.25	1.52	0.50
75	4.5	0.25	0.18	0.0
75	4.0	0.25	1.28	0.50
75	3.5	0.25	2.10	0.75

i.e. for $\frac{q_o}{rH_s} = 0$.

The bearing capacity q_o' for given values of the parameters used in the present investigation of slope stability analysis may be estimated as

$$q_o' = N_s \frac{q_o}{\gamma H_s} \quad \cdots (10)$$

From the above expression, for given value of N_s, q_o' may be evaluated using the values of $\frac{q_o}{\gamma H_s}$ corresponding to the given value of N_s.

In order to compare the values of q_o' calculated from slope stability analysis with those of Meyerhof (1957), the values of N_s for $\phi = 0°$, $\frac{b}{H_s} = 0$ $\frac{\beta_1 H_s}{2c_{Hs}} = 0$ and $k = 1$ are plotted against $\frac{q_o}{\gamma H_s}$ in Figure 9. From this figure, the values of q_o' for $\phi = 0°$ (i.e. N_c) are calculated for different values of N_s and are compared with those given by Meyerhof (1957) in Table 6. It

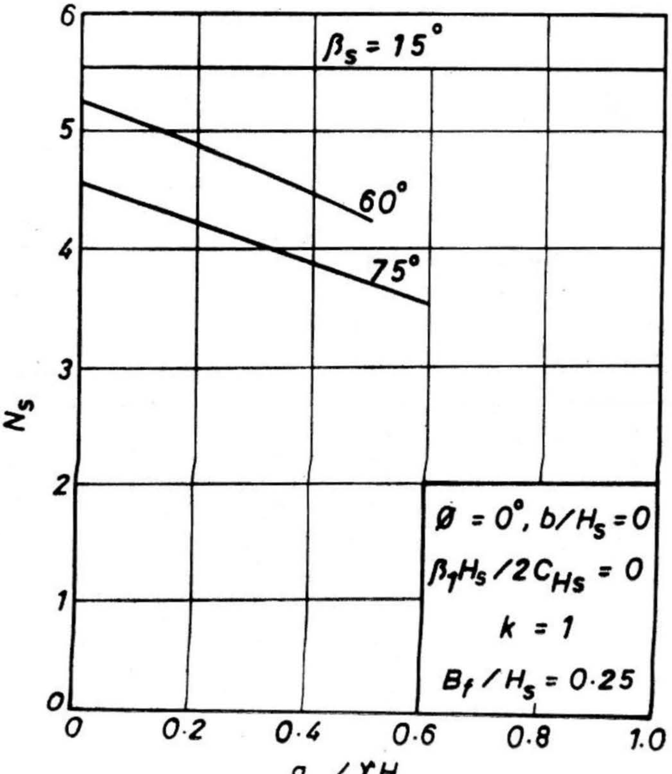

FIGURE 9. N_s vs $\frac{q_o}{\gamma H_s}$ for $\phi = 0°$, $k = 1$ and $\frac{B_f}{H_s} = 0.25$

is observed from this table that the values of N_c obtained from the present investigation are higher than those of Meyerhof, since Meyerhof's values correspond to a foundation of infinite width. Note that in the present analysis the width of foundation is finite.

As an example, a few values of q_o' for $\phi = 0°$ obtained from the analysis of bearing capacity failure (Mogaliah, 1974) and slope stability failure are given in Table 7.

TABLE 7

Comparison of q'_o values for $\phi=0°$ $b/H_s = 0$ and $\beta_1 H_s/2c_{Hs} = 0$

β_s, in degrees	k	Bearing Capacity failure ($N_s = 0$)		Slope Stability failure	
		D/B_f	q_o'	N_s	q_o'
30	0.8	0.251	5.13	6.0	2.8
60	0.8	1.333	6.29	4.5	2.0
60	1.0	1.335	5.76	4.5	1.5
60	2.0	1.352	4.72	4.0	1.20
75	0.8	3.525	8.90	4.0	1.40
75	1.0	3.530	8.32	4.0	1.28
75	2.0	3.535	7.14	4.0	0.60

Conclusions

On the basis of the numerical results presented, the following general conclusions are drawn.

For a foundation width less than slope height, the values of N_s are higher than those for foundation width greater than slope height. The effect of anisotropy and increase in cohesion with depth have considerable influence on N_s. As β_s increases N_s decreases for given values of k, $\frac{\beta_1 H_s}{2c_{Hs}}$ and ϕ.

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Notation

$\bar{a}$ = perpendicular distance from line of action of resultant force due to cohesion, C_d, to the centre of slip circle (Figure 4),

B_f = width of surcharge (foundation) on top of the slope,

b = distance from the edge of slope to the surcharge (foundation),

C_d = resultant force due to cohesion developed along slip surface,

c = cohesion on a plane corresponding to any value of ψ

c_H = cohesion corresponding to horizontal direction (cohesion value for which $\psi = 0°$),

c_{Hs} = cohesion corresponding to horizontal direction at top of the slope,

c_V = cohesion corresponding to vertical direction (cohesion value for which $\psi = 90°$),

c_{Vs} = cohesion corresponding to vertical direction at top of the slope,

$\bar{d}$ = perpendicular distance between the line of action of $(W+Q_o)$ and centre of failure surface (Figure 4),

$F_s = \frac{c_{Vs}}{\gamma H_s} N_s$ = factor of safety,

H_s = height of slope,

$k = \frac{c_V}{c_H}$ = coefficient of anisotropy,

$N_s = \frac{\gamma H_s}{c_{vs}} F_s$ = stabliity number,

n = ratio of distance from toe of slope to the end of base circle to height of slope (Figures 1 and 2).

P = resultant intergranular force acting along slip surface (Figure 2),

$Q_o = q_o B_f$ = total surcharge load (Figures 1 and 2),

q_o = surcharge acting over width B_f on the top of slope (Figures 1 and 2),

$q_o' = N_s \frac{q_o}{\gamma H_s}$,

R = radius of slip circle (Figure1),

ν = angle between the resultant intergranular force, P, and the line of action of $(W+Q_o)$ (Figure 3),

W = weight of soil mass above slip circle,

α = half the angle subtended by a circular failure are at its centre (Figures 1 and 2),

α_c = rate of variation of c_v with depth,

$\beta_1 = \frac{\alpha_c}{k}$,

β_s = inclination of slope with horizontal,

γ = unit weight of soil,

$\triangle$ = angle between the line of resultant cohesion developed and the horizontal (Figure 2),

ϵ = angle between the line passing through centre of slip surface and the point of intersection of forces and the vertical (Figure 4),

θ = angle made with vertical by a line joining the centre of slip circle to any point on slip circle (Figure 1),

λ = angle between the chord of a slip circle and the horizontal (Figure 1),

$\mu = \frac{\pi}{4} - \frac{\phi}{2}$ = angle between major principal stress and failure plane (Figure 1),

ϕ = angle of internal friction of soil, and

ψ = inclination of major principal stress with horizontal.