Vertical Stress Increase in Elastic Half Space due to Moment at Surface

by

D. Babu Rao*

Structures are often subjected to lateral loads, which in turn, cause moments on the foundations. In computing settlement due to the moment, determination of vertical stresses in the soil mass is required. Solutions in the form of equations, tables and figures are found in the literature to determine vertical stresses for a variety of loading conditions. However, a convenient solution to the problem of moment acting at the surface is not available. The purpose of this note is to derive an expression for vertical stress increase due to moment applied at the surface of an elastic half space, and to present an influence chart for determining the stress increase.

By equating the moment to a couple (with a very small lever arm) acting on the straight boundary of an infinite plate, Timoshenko and Goodier (1951) have given the following stress function for the two-dimensional problem in polar coordinates :

$$\phi = \frac{M}{\pi}(\theta + \sin\theta \cos\theta) \qquad ...(1)$$

where M is the moment of the applied couple and θ is the angle as shown in Figure 1. The radial, circumferential and shearing stresses are given, respectively, by

$$\sigma_r = \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}$$

$$= -\frac{4M}{\pi r^2}\cos\theta \sin\theta \qquad ...(2)$$

$$\sigma_\theta = \frac{\partial^2\phi}{\partial r^2} = 0 \qquad ...(3)$$

$$\tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)$$

$$= \frac{2M}{\pi r^2}\cos^2\theta \qquad ...(4)$$

By considering the element shown in Figure 1, the vertical stress may be written as

$$\sigma_z = \sigma_r \cos^2\theta + 2\,\tau_{r\theta}\sin\theta\cos\theta$$

$$= \frac{8M}{\pi r^2}\cos^3\theta\sin\theta \qquad ...(5)$$

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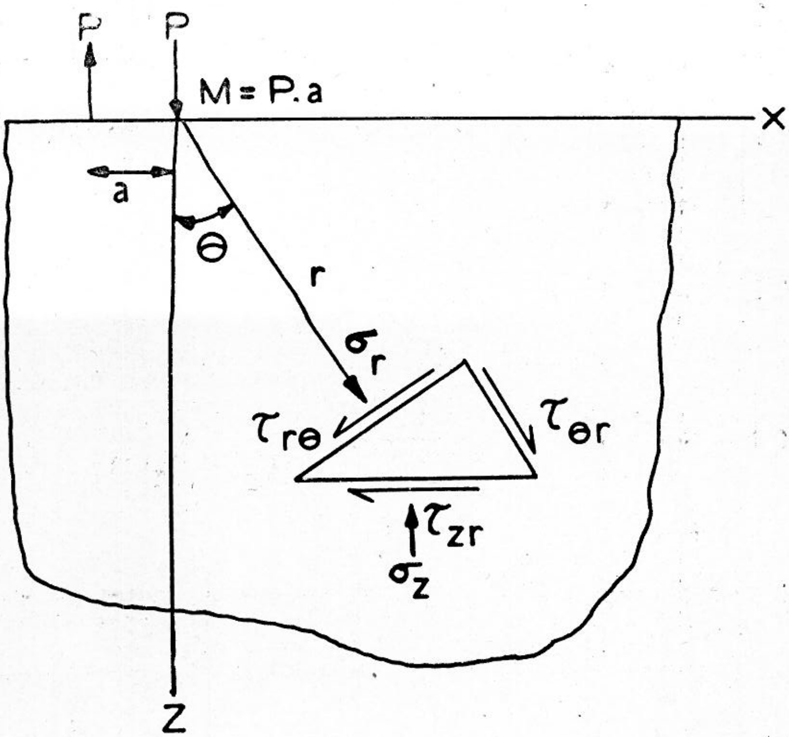

FIGURE 1. Coordinate system

Letting $n = x/z$ and noting that Sin $\theta = x/r$ and Cos $\theta = z/r$, Equation (5) may be rewritten as

$$\sigma_z = \frac{M}{z^2} \quad \frac{8n}{\pi(n^2+1)^3} \qquad \text{...(6)}$$

Introducing the influence value I as a function of x and z, Equation (6) takes the form

$$\sigma_z = \frac{M}{z^2} . I \qquad \text{...(7)}$$

Figure 2 gives the influence values for various ratios of x and z. It may be noted that the maximum value of I is 0.66 for $x/z = 0.45$.

Acknowledgement

The problem presented herein was exposed by Dr. R.W. Christensen of University of Wisconsin, Madison, U.S.A. This is gratefully acknowledged.

Reference

TIMOSHENKO, S. and GOODIER, J.N. (1951) : "*Theory of Elasticity*". McGraw-Hill Book Company, Inc., New York, p. 88

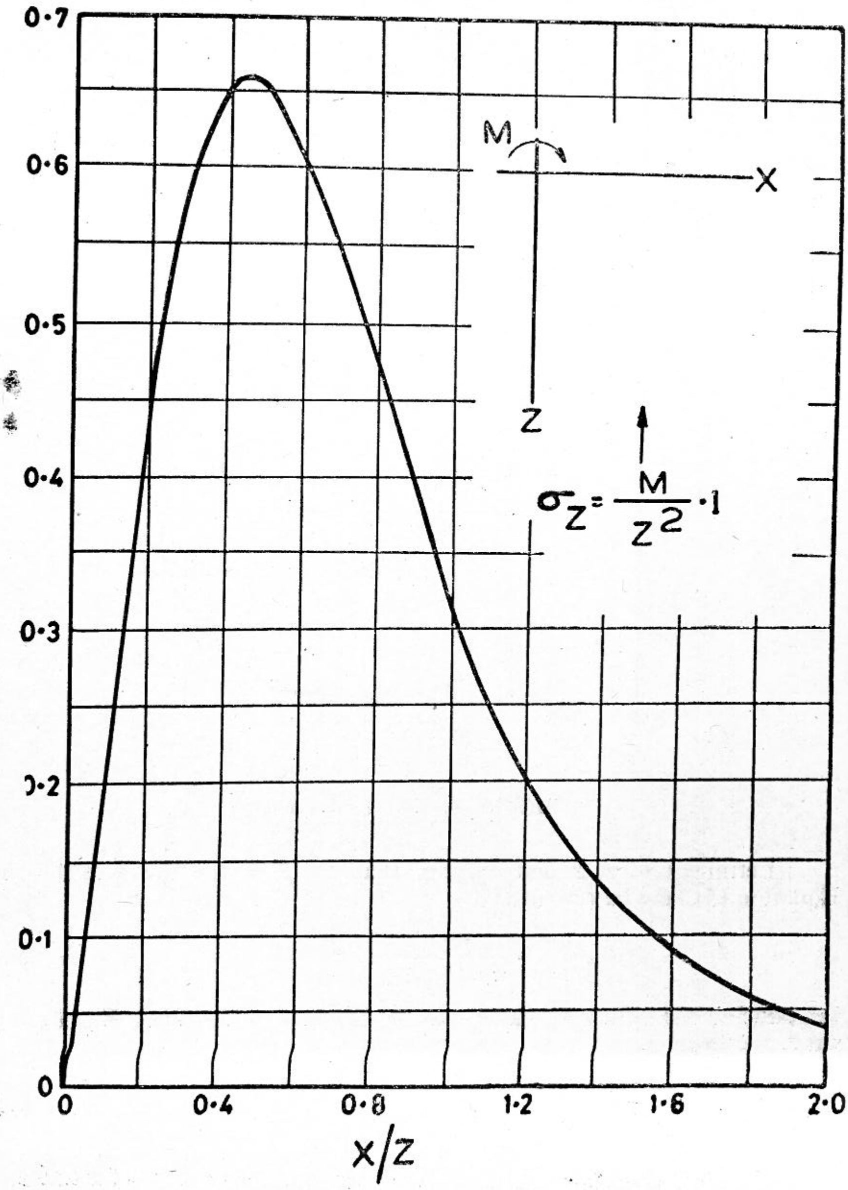

FIGURE 2. Influence Chart

1. Free Surface Seepage Through Foundation and Berm of Cofferdams

C.S. Desai

(*Indian Geotechincal Journal, Vol. 5, No. 1, January 1975, pp* 1-10)

A numerical procedure based on the finite element method is developed and employed for steady free surface flow through foundation of and berm around cofferdams Results are presented in terms of equipotentials and dissipation of applied heads. Criteria are derived for selection of optimum meshes and to assure numerical convergence and accuracy. Use of the procedure for design-analysis is discussed.

KEY WORDS : Free surface seepage; foundation of cofferdams; finite element method; design-analysis; numerical properties.

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2. Analysis of Foundation Mats Resting on a Layered Media

S.K. Saxena

(*Indian Geotechnical Journal Vol. 5, No. 1, January 1975, pp* 11-20)

Although much work has been done on the design of foundation mats, simple methods for the analysis of such structures are still rare. In this paper a simple method is presented for analysis of foundation mats resting on layered soils.

The slab can be represented by a physical model (discrete model) or by finite element divisions of a thin plate. The soils are treated as elastic isotropic solids. With these models expressed through equilibrium equations in matrix form, solutions are obtained in terms of deflections. The deflections are then used to compute stresses and bending moments in the slab.

A sample problem with three layers of soil is shown.

KEY WORDS : Mats; layered soils.

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3. Stresses in Soils Due to Axially Loaded Friction pile

N. Ray and V. Singh

(*Indian Geotechnical Journal, Vol. 5, No. 1, January 1975, pp* 21-43)

The paper presents an elastic analysis for determination of vertical stress-coefficients in soils due to axially loaded friction piles of varying length-diameter ratio using Mindlin expressions for a point load in a semi-infinite, homogeneous, isotropic, elastic medium obeying Hook's Law. The pile load is considered to be distributed uniformly with depth as a shear stress. Vertical stress coefficients are prepared in tabular form for practical use and rapid evaluation. The results of the vertical-stress-coefficients are compared with the Geddes' results graphically. The comparison shows that as the length-diameter ratio of the pile tends to infinity Geddes' solution is obtained as a limiting case of the present analysis. For lower values of this ratio, however, Geddes' results give conservative values for the vertical stress coefficients in the neighbourhood of the pile shaft. Use of the stress coefficient tables has been illustrated by working out the settlement of a pile group.

KEY WORDS : Axially loaded pile; soil; stresses; numerical analysis.

4. Partially Penetrating well in a Semi-Infinite Media with Initial Gradient

A. Arumugam

(*Indian Geotechnical Journal, Vol. 5, No. 1, January 1975 pp* 44-50)

Study of Non-Darcian flow behaviour in fine grained soils is of potential interest in several disciplines. Equation for partially penetrating well that just penetrates water bearing media having initial gradient is arrived at. The relationship between average gradient and discharge, percentage reduction in discharge and average gradient for a given initial gradient are studied. The effect of initial gradient on the yield of the well is brought out.

KEY WORDS : Non-Darcian flow; initial gradient; average gradient; partial penetration; spherical flow; radial flow; discharge.

5. A Transducer for Measuring Vertical Load Inside A Tri-Axial Cell

C.R. Gangopadhyay and S.C. Das

(*Indian Geotechnical Journal, Vol. 5, No. 1, January 1975, pp* 51-56)

The tri-axial cells that are commonly available in India do not have constant friction between the ram and the bushing of a cell, along the full length of travel of the ram, and the magnitude of friction varies with cell water pressure also. A study on a typical cell indicated that for a cell pressure between 1.75 to 7.00 Kg/cm^2, the maximum range of variation of piston friction, in terms of error in the measurement of axial stress on a standard 3.8 cm diameter soil specimen, was about 0.107 Kg/cm^2 during loading and 0.069 Kg/cm^2 during unloading.

The design and use of a beam-type transducer, provided with strain gauges, for measuring vertical load inside a tri-axial cell have been presented. The result of the shearing stage of an anisotropically cousolidated undrained test on a soft clay showed that the use of the transducer gave much more consistent result than that with a proving ring fixed outside the cell. The test result showed that the proving ring measurement over predicted undrained strength by about 7 per cent and under predicted secant modulus (at axial strain = 0.1 per cent) by about 40 per cent.

KEY WORDS : Axial load; proving ring; ramfriction; soil mechanics; strength; stress-strain modulus; transducer; tri-axial test.

6. Mechanical Stabilisation of Lateritic Soil for Improving Subgrade

C.S. Mohan and K.V. Paul

(*Indian Geotechnical Journal, Vol. 5, No. 1, January 1975, pp* 57-65)

The gradation of the Calicut lateritic soil was modified using Rothfuch's specifications for producing maximum density. A study was made using cinder ash as an additive. The resulting changes in California Bearing Ratio, permeability, compression strength and swelling are presented.

KEY WORDS : California Bearing Ratio; cinder ash; compression strength; gradation; laterite; permeability; swelling.

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2.	Bibliography on Soil Stabilization—S.R. Mehra	Rs. 13.00
3.	Bibliography of Soil Stabilisation—Supplement No. 1.	Rs. 4.00
4.	Bibliography on Earth Dams and Embankments.	Rs. 3.00
5.	Proceeding of the 1st Asian Regional Conference of the International Society of Soil Mechanics and Foundation Engineering, 1960.	*
6.	Symposium on Foundation Engineering held at Bangalore in January 1961.	*
7.	Proceeding of the Seventh Annual Session of the Indian National Society of Soil Mechanics and Foundation Engineering held at Hyderabad in February 1964.	*
8.	Proceeding of the Symposium on Foundations of Power Houses and Heavy Machines held at Poona in March 1965	*
†9.	Proceedings of the Symposium on Earth and Rockfill Dams—Beas Dam, Talwara— November 1968.	Rs. 60.00
10.	Monograph on Soil Stabilization.	Rs. 5.00
	(Concession to Members 50%)	
11.	Proceedings of the Eleventh Annual General Seasion	*
12.	Proceedings of the Symposium on Shallow Foundations—Bombay, December 1970.	Rs 40.00
13.	Proceedings of the Technical Sessions held at the time of the 13th Annual General Meeting held in May 1972	*
†14.	Proceedings of Symposium on Rock Mechanics and Tunnelling Problems, Kurukshetra, December, 1973.	

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(*ii*) The price except where otherwise indicated is inclusive of packing and postage charges.

†Copies are available with the Publisher M/s Sarita Prakashan, 175, Nauchandi Grounds, Meerut, U.P.

Those who are interested in purchase of the above publications, many place order with Secretary, Indian Geotechnical Society (Formerly) Indian National Society of Soil Machanies and Foundation Engineering), c/o Institution of Engineers (India), Delhi Centre, Bahadur Shah Zafar Marg,New Delhi-110001.

7. Vertical Stress Increase in Elastic Half space Due to Moment at Surface

D. Babu Rao

(*Indian Geotechnical Journal, Vol. 5. No. 1, January 1975, pp* 66-68)

An expression is derived for vertical stress increase due to moment applied at the surface of an elastic half space. An influence chart is presented to facilitate computations.

KEY WORDS : Elasticity; influence chart; moment; stress

GUIDE TO AUTHORS

Technical articles of broad interest to civil engineers on Soil Mechanics and Rock Mechanics and their application to civil engineering problems are invited for publication in this Journal. Articles should normally report original and thought provoking problems encountered and solved in the field of research, design and construction. Articles should be brief and should not normally exceed 12 printed pages (24 typed pages—A4 size—double spaced typing) including illustrations and should be submitrep in triplicate to the Secretary, Indian Geotechnical Society, C/o Institution of Engineers (India), Delhi Centre Bahadur Shah Zafar Marg, New Delhi-110029. Short informative articles may be submitted for publication under "Short Communication".

The following basic requirements in the preparation of manuscript should be followed :

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(*a*) TERZAGHI, K. (1943): "*Theoretical Soil Mechanics.*" John Wiley and Sons, Inc., New York, p 210.

(*b*) TERZAGHI. K. (1935): "Critical Height and Factor of Safety of Slopes against Sliding". *Proc. Ist Int. Conf. Soil Mech. and Found. Engg.*, 1 : 156-161.

(*c*) TERZAGHI, K. and LACROIX, Y. (1964): "Mission Dam; an Earth and Rockfill Dam on Highly Compressible Foundation". *Geotechnique*, 14 : 1 : 13-50.

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