Partially Penetrating Well in a Semi-Infinite Media with Initial Gradient

by

A. Arumugam*

Introduction

Several investigators have observed that in many soils, especially in densely packed fine grained soils, flow starts only when the hydraulic gradient exceeds a certain value known as the initial gradient or limiting gradient or threshold gradient (Miller and Low 1963, Olsen 1965, Swartsendruber 1962). Several hypotheses have been put forward to explain this non Darcian Flow in the laminar region and they can be grouped as follows (Basak and Madhav 1973).

1. Non-Newtonian liquid Viscosity hypothesis
2. Bingham Plastic fluid and "Quasi-Crystalline" water structure hypothesis
3. Electro kinetic coupling hypothesis
4. Wall force hypothesis
5. Domain structure hypothesis and
6. Transient particle arrangement hypothesis.

Review and detailed discussion of these hypotheses is given by Basak and Madhav (1973).

The value of the initial gradient depends on the soil and its state. For instance the value of the initial gradient, I_o, may be of the order of 2-3 for natural clays and 30-60 for very dense clays (Polubrinova-Kochina, 1962). The following table gives the value of I_o for various types of soils :

TABLE I

Sr. No.	Investigator	*Sample Characteristics*	*Conditions of Flow*	Magnitude as Observed by Olsen (1965)
1.	Von Engelhardt and Tunn (1955)	Sand stone with clay fraction less than 5%	Saturated	$0<I_o<500$
2.	Lutz and Kemper (1959)	Unconfined samples of Bentonite, Halloysite and Bladen clay pastes	Saturated	$50<I_o<200$
3.	Miller and Low (1963)	(*i*) Bentonite Pastes (*ii*) Confined samples of Li and Na Montmorillonite	Saturated	$0<I_o<70$
4.	Hansbo (1960)	Natural clays and confined samples	Saturated	$0<I_o<2.2$

*Research Scholar, Department of Civil Engineering, I.I.T., Kanpur, India.

The Paper is open for discussion till August 1975.

Sandy loam and to a certain extent sandy soils are also known to possess initial gradient.

Consequences of initial gradient are of potential interest in several disciplines. Drainage and water movement in clay soils are of primary importance to soil science, Hydrology and Soil engineering. Consolidation of clay deposits is of utmost interest and fundamental importance to Soil Mechanics. Non-Darcy behaviour will also affect the flow through unsaturated soils which is of primary importance to agriculture and hydrology. Recent analysis revealed the influence of the initial gradient on the spacing of trenches and also on the reduction in discharge (Valsangkar and Subramanya, 1972).

Many empirical expressions have been suggested to describe the relation between the velocity and the hydraulic gradient for soils which possess threshold gradient. Choosing the relationship between the velocity and the hydraulic gradient proposed by Puzerevskaya (1931) (Polubrinova-Kochina 1962)

$$V = K\left(\frac{dh}{ds} - I_o\right) \qquad \ldots(1)$$

where V = Gross seepage velocity

K = Coefficient of permeability

$\frac{dh}{ds}$ = Hydraulic gradient and

I_o = Initial gradient below which there is no flow,

equations have been derived for unconfined and confined flow into a trench and also radial seepage for fully penetrating well with confined conditions (Valsangkar and Subramanya 1972).

In this paper, an analysis has been made for the simplest case of a partially penetrating well in a semi infinite aquifer having initial gradient, choosing Equation (1) to describe the relation between the velocity and the hydraulic gradient and assuming Dupit-Forcheimer assumption to be valid.

Analysis

In practice, one often encounters wells that extend partially through an aquifer. The simplest case of partially penetrating well occurs when a well just penetrates the top surface of a semi-infinite medium. The seepage in this case is characterized by spherical flow (Harr 1962).

Figure 1. Illustrates such a case

where r_w = Radius of well

R = Radius of the inflow surface,

H_w = Head at r_w

H_R = Head at R,

ρ (rho) = Any radius

H = Head at ρ

Q_s = Discharge at any radial distance,

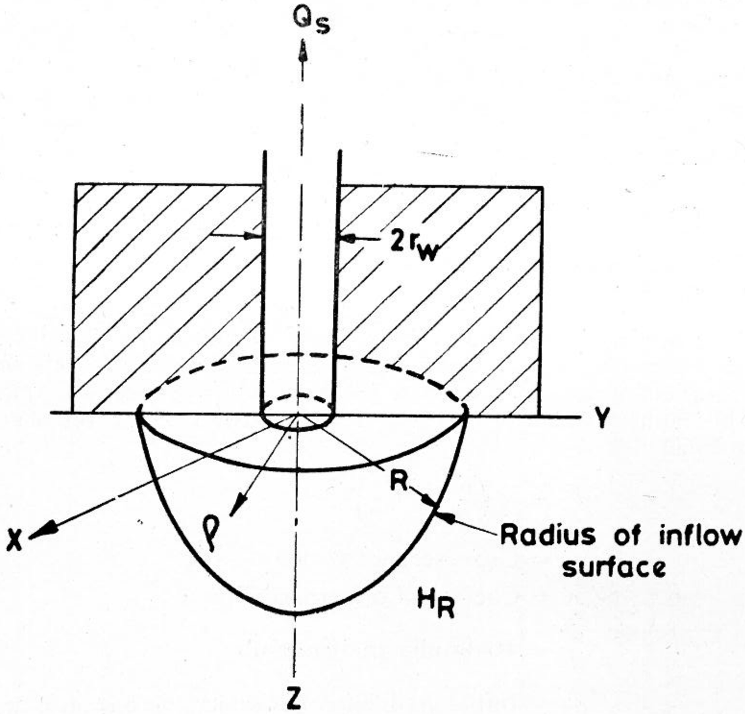

FIGURE 1.

If Q_s is the discharge at any radial distance ρ the average velocity $V\rho$ is

$$V_\rho = \frac{Q_s}{2\pi\rho^2} \qquad \ldots(2)$$

From Equations (1) and (2), the governing differential equation is obtained as

$$\frac{Q_s}{2\pi\rho^2} = K\left(\frac{dH}{d\rho} - I_o\right) \qquad \ldots(3)$$

Separating the variables, Equation (3) can be written as

$$\frac{Q_s}{2\pi K} \cdot \frac{d\rho}{\rho^2} = dH - I_o\, d\rho \qquad \ldots(4)$$

On integration, Equation (4) yields

$$\frac{Q_s}{2\pi K} \cdot \left(-\frac{1}{\rho}\right) + A = H - I_o\,(\rho) \qquad \ldots(5)$$

Where A is the constant of integration. In order to evaluate A, the boundary condition that at $\rho = r_w$, $H = H_w$ is substituted in Equation (5).

Accordingly,

$$A = \frac{Q_s}{2\pi K r_w} + H_w - I_o r_w$$

Equation (5) then becomes

$$\frac{Q_s}{2\pi K}\left(\frac{1}{r_w}-\frac{1}{\rho}\right)=(H-H_w)-I_o\,(\rho-r_w) \qquad ...(6)$$

Equation (6) gives the head at any radial distance in terms of other parameters.

Equation (6) is valid for any radial distance. Therefore, substituting $\rho = R$, Equation (6) modifies to the form

$$\frac{Q_s}{2\pi K}\left(\frac{1}{r_w}-\frac{1}{R}\right)=(H_R-H_w)-I_o\,(R-r_w) \qquad ...(7)$$

$\Rightarrow R \gg r_w$ Equation (7) can be simplified as

$$\frac{Q_s}{2\pi K}\left(\frac{1}{r_w}\right)=(H_R-H_w)-I_o\,(R-r_w) \qquad ...(8)$$

Dividing both sides by $(R-r_w)$, Equation (8) is rendered dimensionless

$$i.e.\ \frac{Q_s}{2\pi K.\ r_w\,(R-r_w)}=\frac{(H_R-H_w)}{(R-r_w)}-I_o \qquad ...(9)$$

Representing the dimensionless discharge as Q^* and the average gradient as I_{av}, Equation (9) transforms to

$$Q^* = I_{av}-I_o \qquad ...(10)$$

where the dimensionless discharge Q^* is $\dfrac{Q_s}{2\pi K r_w\,(R-r_w)}$

and the average gradient I_{av} is $\dfrac{H_R-H_w}{R-r_w}$

The importance of the threshold gradient, I_o, can be seen from the Equation (10). As long as $I_{av} \leqslant I_o$ no discharge can take place. In otherwords, the minimum value for I_{av} for discharge to commence is governed by the value of I_o. In practice, the value I_{av}, *i.e.* the ratio $\dfrac{H_R-H_w}{R-r_w}$ very rarely exceeds 1. Fixing this value as the upper bound for I_{av} one can say that for satisfactory yield for a well that just penetrates an aquifer having initial gradient, the value of the initial gradient should be far less than unity. Otherwise, if the soil possess high initial gradient, one has to maintain large difference in head between the inflow surface and the well. Taking the case of densely compacted clay, whose initial gradient value is of the order of 30 (Polubrinova-Kochina, 1962), the minimum value of the ratio $\dfrac{H_R-H_w}{R-r_w}$ should be little above 30, so that there may be some discharge. In otherwords, the head difference between the inflow surface and the well, H_R-H_w, should be atleast 30 times the radius of the inflow surface (Neglecting for simplicity the radius of the well). If the radius of the inflow surface, for example, is 100 metres. then the head difference should atleast have a value of 3,000 metres. Hence fixing fairly low values of 0.1, 0.2 for I_o, for the purpose of analysis, the effect of I_{av} on the dimensionles discharge parameter Q^* can be studied. Figure 2 indicates the relationship between I_{av} and Q^*. Since the value of I_{av} at zero discharge represents the initial gradient, the intercept of the curve on the x axis gives the value of the initial gradient.

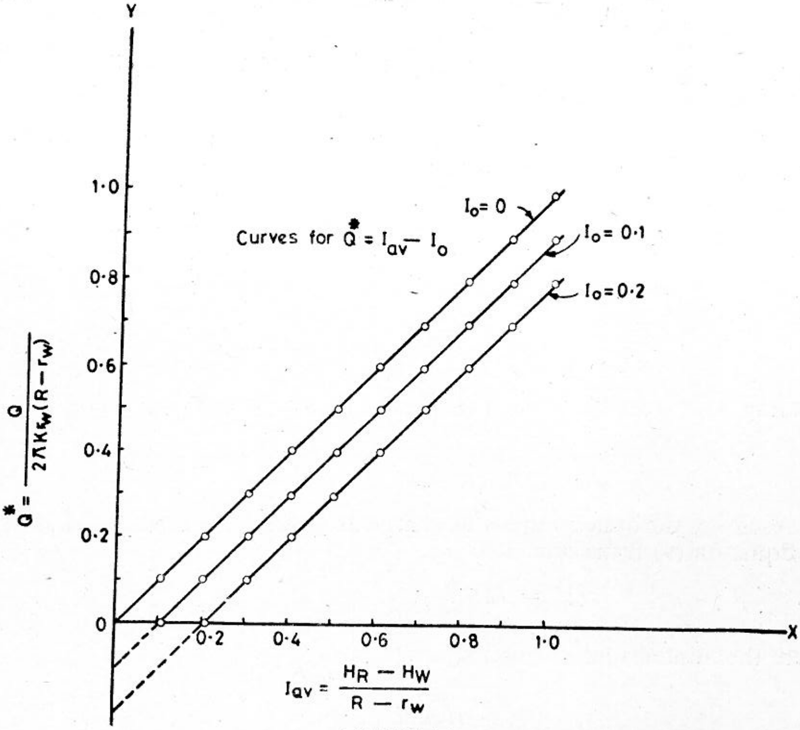

FIGURE 2.

Comparison between spherical and radial Flow

Denoting Q_r as the discharge, producing simple radial flow (completely penetrating a layer having initial gradient and of thickness D) and Q_s as the discharge producing spherical flow (Just penetrating a semi-in-finite medium having the same value of initial gradient) the ratio of these factors for the same potential drop will be interesting to note.

$$\frac{Q_s}{Q_r} = \frac{2\pi K r_w \{(H_R - H_w) - I_o\ (R - r_w)\}}{2\pi K D\ \{(H_R - H_w) - I_o\ (R - r_w)\ \}}\ x\ ln\left(\frac{R}{r_w}\right)$$

$$= \frac{r_w}{D} \times \ln\left(\frac{R}{r_w}\right) \qquad ...(11)$$

The value of the ratio Q_s/Q_r remains the same for zero initial gradient soils (Harr 1962). Further, if the ratio Q_s/Q_r is taken to represent the efficiency, it is seen that the efficiency of the spherical flow system is dependent on the ratio r_w/D. Since in general the radius of well, r_w, is very much smaller compared to the thickness of the medium it is evident that the spherical flow well is highly inefficient in soils having initial gradient. The same conclusion holds for Darcy flow conditions also. By putting zero value for the initial gradient in Equation (10) the standard expression for the discharge of a well that just penetrates the top surface of a semi-infinite medium without initial gradient is obtained (Harr 1962).

By comparing this standard expression with Equation (8) it becomes obvious that the effect of the initial gradient is to reduce the effective head. Consequently one can expect some reduction in discharge in case of soils having initial gradient. If α_s represents the percentage reduction in discharge, then

$$\alpha_s = \frac{Q_{so}-Q_s}{Q_{so}} \times 100$$

$$= \frac{2\pi K r_w \{(H_R-H_w)-(H_R-H_w)+I_o\ (R-r_w)\}}{2\pi K r_w \{(H_R-H_w)\}}$$

$$\alpha_s = \frac{I_o}{I_{av}} \qquad \ldots(12)$$

where,

α_s : Percentage reduction in discharge, $\frac{Q_{so}-Q_s}{Q_{so}}$

Q_{so} : Discharge under Darcy flow conditions

Q_s : Discharge under non-Darcy flow conditions

I_o : Initial gradient

I_{av} : Average gradient, $\frac{H_R-H_w}{R-r_w}$

From Equation (12) the effect of I_{av} on α_s can be studied for a given value of I_o. For a given I_o this equation is the equation of a rectangular hyperbola, having asymptotes $\alpha_s = 0$, and $I_{av} = 0$. A plot is made connecting α_s and I_{av} for I_o values of 0.1 and 0.2 (Figure 3).

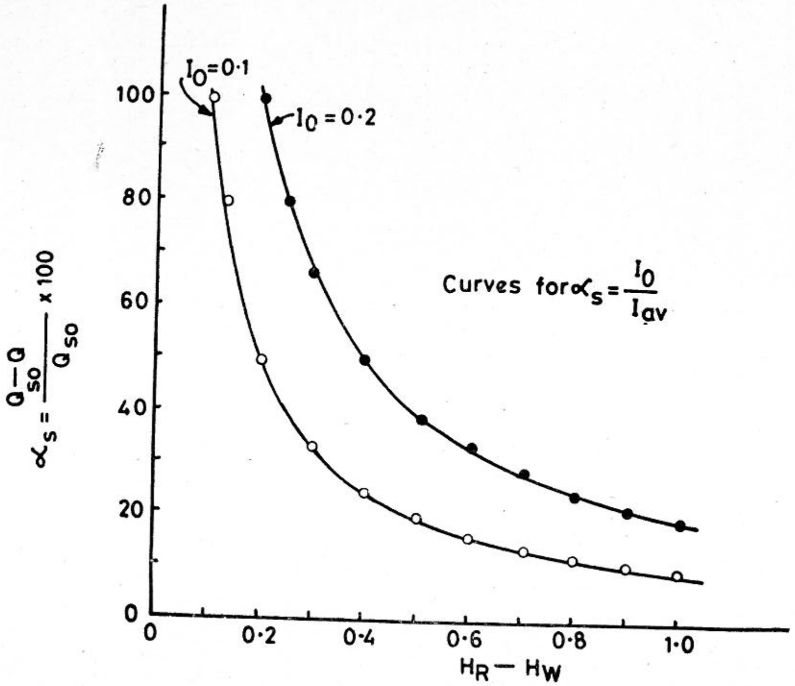

FIGURE 3.

Conclusions

Initial gradient is an important factor in seepage problems. Equation for partially penetrating well that just penetrates the bearing media having initial gradient is arrived at. In case of pure spherical flow for a given value of the initial gradient, the relationship between average gradient and discharge is linear. The effect of initial gradient is to reduce the effective head and consequently the discharge. For satisfactory yield the value of initial gradient of the aquifer material should be considerably less. The relationship between the percentage reduction in discharge, and average gradient for a given initial gradient is found to be hyperbolic.

Acknowledgement

The author wishes to express his thanks to Dr. M.R. Madhav for his valuable suggestions and encouragement.

Appendix. References

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