A Note on Elastic Surface Settlement in Sand

by

B.K. Ramiah*

L.S. Chikkanagappa**

A. Prabhakara***

Introduction

THE solution for displacement due to a point load acting on a semi-infinite mass is available in literature [Timoshenko & Goodier, (1951)]. This solution has been extended to include distributed loads over circular and rectangular areas [Timoshenko & Goodier 1951, Harr (1966)], by superposition. These solutions are based on the assumption that the modulus of elasticity of the mass is constant with depth. But, in soil mechanics problems, one usually encounters with soil mass (which is usually assumed to be as an elastic-half space), where the modulus of elasticity E, increases with depth. It has been shown that in cohesionless soils, that the modulus of elasticity is proportional to the depth below the surface of the ground [Adrian Pauw (1952)]. Experimental results indicate that the elastic properties of a soil are a function of over burden pressure and the modulus of elasticity vary linearly with depth (Richart, Hall and Woods 1970). In the present investigation, the basic equation for displacement is modified to include the variation of E with depth linearly and an expression has been obtained for a point load P, acting on a semi-infinite mass. The other assumptions in deriving the basic equation remains unaltered.

Analysis

If a point load 'P' acts on a semi-infinite mass, then the displacement component can be obtained from (Timoshenko and Goddier 1951)

$$\frac{\partial w}{\partial z}=\frac{P}{2\pi E}\Big[3(1+\nu)r^2z(r^2+z^2)^{-5/2} -\{3+\nu(1-2\nu)\}z(r^2+z^2)^{-3/2}\Big] \quad \dots (1)$$

* Principal, Professor and Head, Department of Civil Engineering, University Visvesvaraya College of Engineering, Bangalore-560001.

** Lecturer in Civil Engineering, Department of Civil Engineering, University Visvesvaraya College of Engineering. Bangalore-560001.

*** Lecturer in Civil Engineering, Department of Civil Engineering, University Visvesvaraya College of Engineering, Bangalore-560001.

This paper was received on 12 April 1973. It is open for discussion up to April 1975.

where r = Radial distance from the origin = $\sqrt{x^2+y^2}$

z = Depth below the surface

ν = Poisson's ratio

w = Vertical displacement.

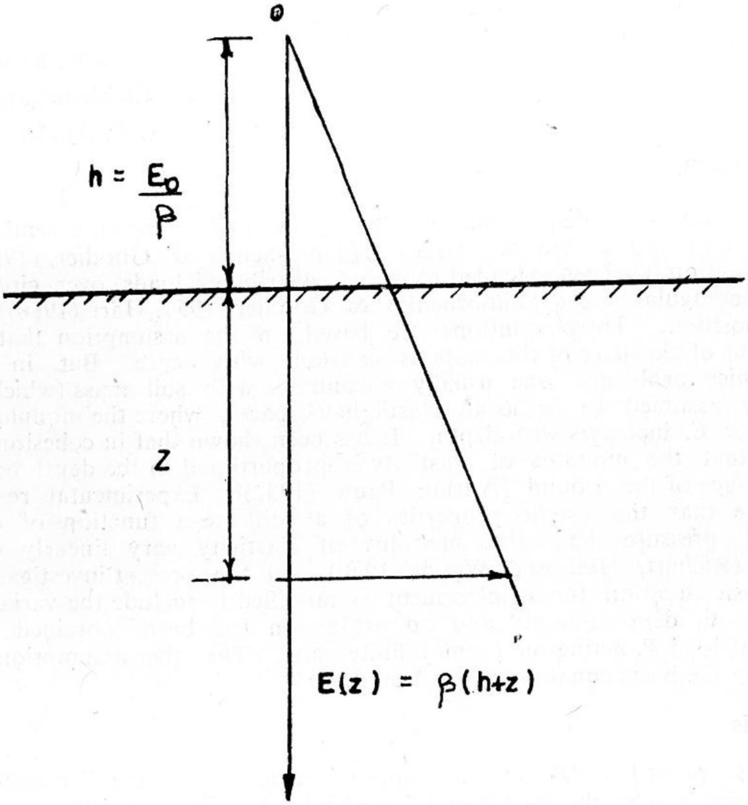

FIGURE 1 (a) : Variation of modulus of elasticity with depth Z.

If it is assumed that E varies linearly with depth Z as shown in Figure 1 (a), then the modulus of elasticity can be expressed as a function of depth Z as

$$E(z)=\beta(h+z) \qquad \text{... (2)}$$

Here, β is the rate at which the modulus increases with depth and $h=E_o/\beta$ where E_o is the modulus of elasticity at the surface. Introducing Equation (2) into Equation (1), the following expression is obtained.

$$\frac{\partial w}{\partial z}=\frac{P}{2\pi\beta}\left[3(1+\nu)r^2.\ \frac{z}{h+z}\ (r^2+z^2)^{-5/2} - \{3+\nu(1-2\nu)\}\ \frac{z}{h+z}\ (r^2+z^2)^{-3/2}\right] \qquad \text{... (3)}$$

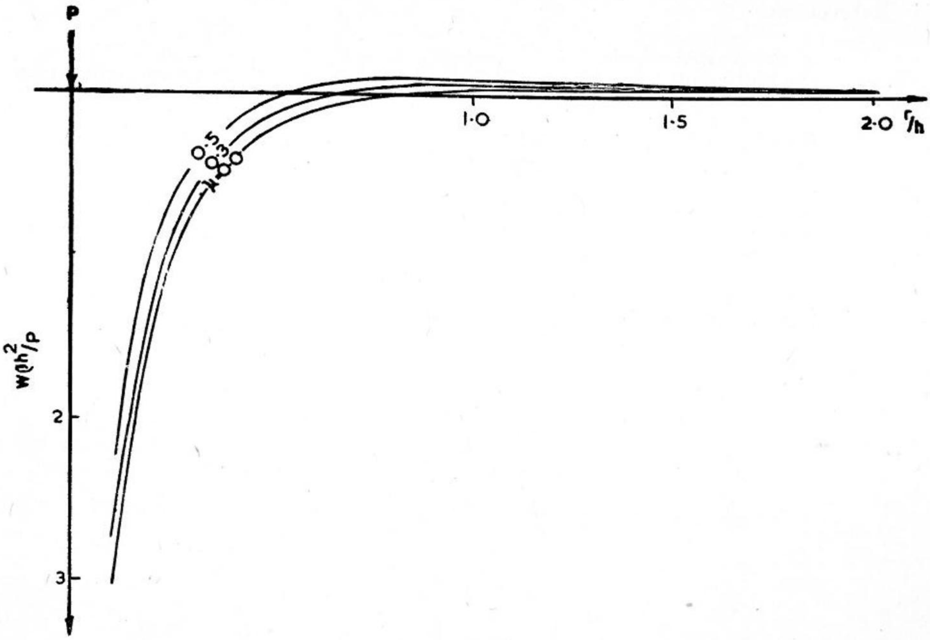

FIGURE 1 (b) : Surface settlements under concentrated load.

Integration of Equation (3) with respect to Z and letting $Z=0$ and rearranging the terms, gives the following expression for displacement.

$$\left(\frac{w\beta h^2}{p}\right)_{Z=0}=\frac{1+\nu}{2\pi}\left[\frac{1}{(1+m^2)^{5/2}}(\nu m^2-1.5+\nu)\right.$$

$$\left.\log_e\frac{\sqrt{1+m^2}+m}{\sqrt{1+m^2}-m}+\frac{2(1-\nu)}{m(1+m^2)}-\frac{3m}{(1+m^2)^2}\right] \quad \ldots (4)$$

where $m=r/h$

Plot of the Equation (4) is shown in Figure 1 (*b*), where the ratio r/h is plotted against $(\omega\beta h^2/P)$. It is interesting to note the bulging of the surface at distance of $r/h=0.54$ for $\nu=0.5$ and at $r/h=0.8$ for $\nu=0.0$. The values of β, ν and E_o must be determined experimentally, preferably by the use of dynamic soil tests in the field. [Adrian Pauw (1952)]. The settlement obtained from Equation (4), is less than that obtained from Boussinesq's equation, where E is constant with depth.

Conclusion

An expression for displacement due to a point load acting on semi-infinite mass with modulus of elasticity varying linearly has been derived and presented.

Acknowledgement

The authors express their thanks to the Bangalore University for providing facilities for research.

References

ADRIAN PAUW, (1952) : "A Dynamic Analogy for Foundation-Soil System" Symposium on Dynamic Testing of Soils, ASTM, special Technical publication No. 156.

HARR, M.F. (1966) : "Foundations of Theoretical Soil Mechanics" International Student Edition, Kogakusha Company Ltd., Tokyo.

RICHART, JR. F.E., HALL, JR., J.R, and WOODS, R.D. (1970) : "Vibrations of Soils and Foundations", Prentice Hall International series in Theoretical and Applied Mechanics.

TIMOSHENKO, S. and GOODIER, J.N. (1951) : "Theory of Elasticity" McGraw Hill Book Company, Inc.. Tokyo, Kogakusha Co.