Short Communications

Evaluation of Horizontal Subgrade Modulus for Clays

by

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Review of Literature

THE use of piles in the foundations of structures to resist lateral loads was recognised from very early times. Culman (1866) gave a method of computing the load on piles when the direction of the resultant of the forces acting on the foundation is inclined. The early ideas were to keep the piles in the same direction as the inclined loads with a view to make the loads on the piles axial. Winkler's (1867) hypothesis of subgrade reaction involving the soil support being replaced by independent closely spaced elastic springs marks the beginning of the concept of lateral resistance of piles. Vesic (1961) established the validity of Winkler's hypothesis. The theory of lateral load resistance of piles is based on the equation for a beam on an elastic foundation

$$EI\frac{d^4y}{dz^4}=-kDy \qquad \ldots (1)$$

The solution of Equation (1) would call for a mathematically convenient function for subgrade modulus kD.

Essentially the subgrade modulus kD can be evaluated either from the theory of elasticity or from load tests on piles. The field method being indicated pertains to the latter group. Co-efficient of horizontal subgrade reaction is defined as the ratio between the reaction pressure per unit of area of the pile surface at any point and displacement of that point

$$k=\frac{p}{y} \qquad \ldots (2)$$

Terzaghi (1955) holds, that for clays for which modulus of elasticity is practically independent of depth, coefficient of horizontal subgrade reaction will remain independent of depth and as such can be described by Equation (2) ; for sands however, as the modulus of elasticity linearly

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increases with depth, coefficient of horizontal subgrade reaction can be described by

$$k = nZ = \frac{p}{y} \qquad \dots (3)$$

The real variation of k with depth could however be different (Rifaat 1935, Palmer and Thompson 1948, Reese and Matlock 1956, Terzaghi 1955) brings out from the concept of pressure bulbs, the fact that the coefficient of horizontal subgrade reaction diminishes with increase in the width D of the pile surface, viz., for clays

$$k = \frac{1}{D} k_1$$

and for sands

$$k = \frac{1}{D} n_1 z \qquad \dots (4)$$

The non-linear nature of the pressure deflection diagram (Figure 1) suggests that the coefficient of horizontal subgrade reaction varies with deflection as well; larger values being obtained at small deflections and vice versa.

In the literature of Soil Mechanics the formulae devised by Granholm (1929), Biot (1937), Hamilton Gray (see Glick 1948), Jampel (1949) from the theory of elasticity enable evaluation of subgrade modulus kD for clay.

Method of Evaluating Subgrade Modulus from Field Tests

Terzaghi (1955) describes a method of calculating horizontal subgrade modulus from tests on rigid or short piles whose length l is not greater than $\frac{1.5}{\alpha}$.

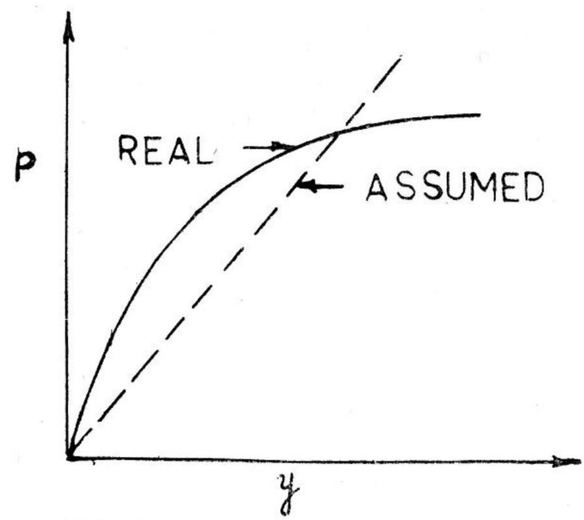

FIGURE 1 : Pressure deflection diagram.

The method being indicated is based on the assumptions that the coefficient of horizontal subgrade reaction is independent of depth and deflection of any point on pile surface and differs from that of Terzaghi in that it requires observational data on flexible or long piles where length l of the pile is to be greater than $\frac{3.0}{\alpha}$. A step by step procedure of the method is given below :

1. Drive a few test piles whose length l is to be greater than $\frac{3.0}{\alpha}$; since α is not known at the start, the length l is to be approximately chosen.
2. The lateral load is applied by jacking against an adjacent pile, the extent of lateral load and deflection to be reached in the test being those expected on the piles after completion of structure.
3. The load can be applied either at ground level or a little above it depending on whether there shall be only shear or shear and moment on the finished pile at ground level.
4. Calculate kD from Equation (10) and calculate α.
5. Check if the length l of the pile chosen is greater than $\frac{3.0}{\alpha}$, if not, the test may have to be repeated with a suitable revised length.

Referring to Figure 2 and from the theory of infinite beams on elastic medium, we have

$$y=e^{+\alpha Z/\sqrt{2}}\left(A\cos\frac{\alpha Z}{\sqrt{2}}+B\sin\frac{\alpha Z}{\sqrt{2}}\right)$$

$$+\,e^{-\alpha Z/\sqrt{2}}\left(C\cos\frac{\alpha Z}{\sqrt{2}}+D\sin\frac{\alpha Z}{\sqrt{2}}\right) \qquad \ldots (5)$$

consisting of undamped and damped sine waves. With increase in the length of the pile, factor $e^{+\alpha Z/\sqrt{2}}$ increases rapidly assuming enormous values for sufficiently long piles which does not fit in with the condition of stable equilibrium. As such the part of the Equation (5) comprising undamped sine waves may have to be disregarded resulting in

$$y=e^{-\alpha Z/\sqrt{2}}\left(C\cos\frac{\alpha Z}{\sqrt{2}}+D\sin\frac{\alpha Z}{\sqrt{2}}\right) \qquad \ldots (6)$$

Inserting boundary conditions, viz, at

$$Z=0, \qquad y=Y_o \qquad \text{and } M=M_o$$

into Equation (6)

$$y=e^{-\alpha Z/\sqrt{2}}\left(Y_o\cos\frac{\alpha Z}{\sqrt{2}}+\frac{M_o}{EI\alpha^2}\sin\frac{\alpha Z}{\sqrt{2}}\right) \qquad \ldots (7)$$

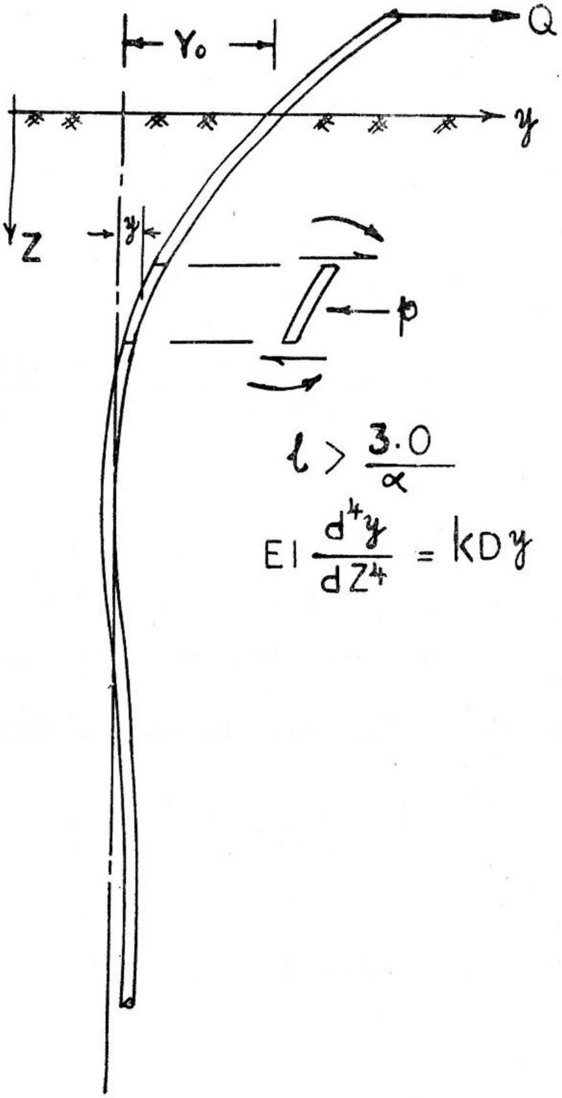

FIGURE 2 : Vertical pile subjected to horizontal load Q.

From conditions of static equilibrium, we have

$$\int_0^l kD\, y\, dz = Q \qquad \dots \quad (8)$$

Integrating Equation (8) after substituting for y from Equation (7) and putting $e^{-\alpha l/\sqrt{2}} = 0$, we get

$$kD = Ql\,\frac{1}{\sqrt{2}}\left(\frac{Y_o}{\alpha}+\frac{M_o}{EI\alpha^3}\right) \qquad \cdots (9)$$

Calling $A_e = \frac{1}{\sqrt{2}}\left(\frac{Y_o}{\alpha}+\frac{M_o}{EI\alpha^3}\right)$ as equivalent area of resistance of the pile surface, we have

$$kD = \frac{Q}{A_e} \qquad \cdots (10)$$

In Equation (10) kD corresponds to the state of static equilibrium.

Numerical Examples

Example 1

A lateral load test is conducted on a typical test pile by applying a horizontal load at the ground level with a view to calculate horizontal subgrade modulus. The following data is recorded.

$l = 10$ m

$I = 930$ cm^4

$E = 2 \times 10^6$ kg/cm^2

$Y_o = 0.5$ cm

$Q = 415$ kg

Substituting $Q = 415$ kg ; $M_o = 0$; $Y_o = 0.5$ cm

$EI = 930 \times 2 \times 10^6 = 18\ 6 \times 10^8$ kg cm^2 in Equation (10),

we have

$$kD = \frac{415}{0.5/\sqrt{2}\ \sqrt[4]{\frac{kD}{18.6\times 10^8}}}$$

from which $kD = 10$ kg/cm^2

$$\alpha = \sqrt[4]{\frac{10}{18.6\times 10^8}} = \frac{1}{117}\ \text{cm}^{-1}$$

$$\frac{3.0}{\alpha} = \frac{3.0}{1/117} = 351 \text{ cm}$$

But $l = 10$ m > 351 cm

Example 2

A typical test pile is subjected to a lateral load of 588 kg at 1.7 cm above ground level so as to calculate horizontal subgrade modulus. The following data is available.

$l = 12$ m

$I = 1020$ cm^4

$E = 2 \times 10^6$ kg/cm^2

$Y_o = 0.6$ cm

We have $Q=588$ kg ; $M_o=588\times1.7=1000$ kg-cm substituting the data into the equation (10), we have

$$kD=\frac{588}{\frac{0.6}{\sqrt{2}}\left(\frac{kD}{1020\times2\times10^6}\right)^{\frac{1}{4}}+\frac{1000}{\sqrt{2}}\times1020\times2\times10^6\left(\frac{kD}{1020\times2\times10^6}\right)^{\frac{3}{4}}}$$

from which $kD=12$ kg/cm^2

$$\alpha=\sqrt[4]{\frac{12}{2\times10^6\times1020}}=\frac{1}{114.5}\text{ cm}^{-1}$$

$$\frac{3.0}{\alpha}=\frac{3.0}{1/114.5}=343.5\text{ cm}$$

$$l=12\text{ m}>343.5\text{ cm}$$

Conclusions

(1) Formula such as given in Equation (10) has been the long felt need of the field engineers in as much as the evaluation of subgrade modulus is a frequently encountered problem in the field.

(2) The assumptions made in deriving Equation (10) are reasonable since for greater length of the pile the deflections are comparatively small, more so if the deflection at ground level is smaller.

(3) Since the observational data is from a prototype pile, the resulting value of kD is quite reliable.

(4) The results of elastic theories are doubtful as it is difficult to measure correctly the elastic constants of the soil in the laboratory, besides the simplifying assumptions made in the theories.

Notations

D=Diameter of pile,

E=Modulus of elasticity of pile material,

I=Moment of inertia of the pile cross section,

k=Coefficient of horizontal subgrade reaction in case of pile of diameter D,

kD=Subgrade modulus,

k_1=Coefficient of horizontal subgrade reaction in case of pile of unit diameter,

l=Length of pile,

M=Bending moment in the pile at depth z,

n=Constant of horizontal subgrade reaction for sands in case of pile of diameter D,

n_1=Constant of horizontal subgrade reaction for sands in case of pile of unit diameter,

p=Reaction pressure on pile surface per unit of area,

Q=Horizontal load on pile,

Y_o=Deflection of pile at ground surface,

y=Deflection of the pile at any point down its length,

Z=Distance to any point on the pile down its length,

$$\alpha = \sqrt[4]{\frac{kD}{EI}}$$

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References

BIOT, M.A. (1937) : "Bending of an Infinite Beam on an Elastic Foundation". *J. Appl. Mech.* U.S.A., 4 (No. 1), A-1.

CULMAN, C. (1866) : *"Graphishe Static"*. Zurich.

GRANHOLM, H. (1929) : "On the Elastic Stability of Piles surrounded by a Supporting Medium". *Handigar Ingeniors Vetenskaps Akademien*, No. 89.

GLICK, G.W. (1948) : "Influence of Soft Ground on the Design of long piles". *Proc. 2nd Interuational Conference on Soil Mechanics and Foundation Engineering*, Vol. 4. p. 84.

JAMPEL, S. (1949) : "An Analysis of groups of piles". *Concrete and Constructional* Engg., 44, 201.

PALMER, L.A. and THOMPSON, J.B. (1948) : "The Earth pressures and Deflections along the Embedded Lengths of piles subjected to Lateral Loads". *Proc. 2nd Internationrl Confcrence on Soil Mechanics and Foundation Engineering*, Rottor Dam, Vol. 5, pp. 156-161.

RIFAAT, J. (1935) : "Die spundwand als Erd-druck problem". *A.G. Gebr. Leeman & Co.* Zurich.

REESE, L.C. and MATLOCK, H. (1956) : "Non-Dimensional solutions for Laterally Loaded Piles with Soil Modulus Assumed Proportional to Depth". *Eighth Texas Conference on Soil Mechanics and Foundation Engineeriug*, Austin.

TERZAGHI, K. (1955) : "Evaluation of Coefficients of Subgrade Reaction". *Geotèchnique*, Vol. 5.

VESIC, A.B. (1961) : "Beams on Elastic Subgrade and the Winkler's Hypothesis". *Proc. 5th International Conference on Soil Mechanics and Foundation Engineertng*, Paris, Vol. 1, pp. 845-850.

WINKLER, E. (1867) : "Die Lahre Von Elastizitat and Festigkeit (on Elasticity and Fixity)". Prague, pp. 182-184.