An Elastic Analysis for the Laterally Loaded Single Pile

by

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Introduction

ANALYSES of the behaviour of piles subjected to lateral loads have generally employed the theory of subgrade reaction. Despite the mathematical convenience of the subgrade reaction theory, the consequent assumption of the soil as a Winkler or spring medium is unsatisfactory as the continuity of the soil mass is not taken into account. A more satisfactory analysis in which the soil is assumed to be an elastic continuum has been developed recently which is, however, applicable only for soils having a constant soil modulus with depth. In the present paper an analysis based on the elastic theory allowing for a variation of the soil modulus with depth is presented and comparisons between the results obtained from an experimental investigation on model piles loaded laterally in sand with the corresponding solutions obtained from the present theory and from the elastic theory assuming a constant soil modulus have been given.

A comprehensive solution to the problem of the laterally loaded pil- has basically two parts. First it is necessary to obtain complete informae tion describing the behaviour of the soil ; secondly it is necessary to determine the pile behaviour. In the present analysis the deformations within the soil have been evaluated from the equation of Mindlin (1936) for horizontal displacement due to a horizontal load within a semi-infinite mass whereas, the pile displacements have been obtained from the equation of flexure of a thin strip expressed in finite difference form. The analysis presented in this paper is similar in principle to that employed by Spillers and Stoll (1964), however, in using the Mindlin equation for obtaining the deformations within the soil, the soil modulus E_s is allowed to vary with depth (to simulate the medium better) with the added assumption that the stress distribution amongst the different segments remain unchanged.

The elastic theory has been applied to some test studies conducted by Murthy (1964) with model piles in sand, loaded laterally at the ground surface. Theoretical solutions have been obtained for an assumed linear variation of soil modulus with depth and also for a constant modulus with depth. The respective theoretical distributions obtained for the deflection, moment, shear and soil reaction have been compared with those reported from the experimental investigation.

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Elastic Analysis

The pile is assumed to be a thin vertical strip of length, L and width b, having a constant flexural stiffness E_pI_p. The soil is assumed to be an ideal homogeneous, isotropic or anisotropic, semi-infinite, elastic material having parameters μ and E, the Poisson's ratio μ, remaining a constant and the Young's modulus, E, remaining constant or varying with depth. To simplify the analysis possible horizontal shear stresses developed between the soil and the sides of the pile are not taken into account. The pile is divided into n elements as shown in Figure 1, all elements being of equal length t. Each element is acted upon by a uniform horizontal stress p, which is assumed constant across the width of the pile and which is approximated to a concentrated force P, acting at the centre of the element.

Assuming that purely elastic conditions prevail within the soil, the horizontal displacements of the soil and the pile are equated at the element centres. This results in $n-2$ independent equations in terms of the loads P_i, which along with the boundary conditions are sufficient to solve for all the unknown soil reactions, $P_1, P_2 \ldots P_n$

Pile Displacements

The basic beam equation used is,

$$E_pI_p\frac{d^4y}{dx^4} = -p \quad \ldots (1)$$

where p = distributed load on pile.

Expressing the beam equations in finite difference form, for any point i, on the pile we have,

$$\left(\frac{dy}{dx}\right)_i = \frac{y_{i-1}-y_{i+1}}{2t} = \theta \quad \ldots (2)$$

$$\left(\frac{d^2y}{dx^2}\right)_i = \frac{y_{i-1}-2y_i+y_{i+1}}{t^2} = \frac{M}{E_pI_p} \quad \ldots (3)$$

$$\left(\frac{d^3y}{dx^3}\right)_i = \frac{y_{i-2}-2y_{i-1}+2y_{i+1}-y_{i+2}}{2t^3} = V \quad \ldots (4)$$

$$\left(\frac{d^4y}{dx^4}\right)_i = \frac{y_{i-2}-4y_{i-1}+6y_i-4y_{i+1}+y_{i+2}}{t^4} = \frac{-p}{E_pI_p} \quad \ldots (5)$$

Boundary conditions at the bottom of the pile are

(*a*) Moment $(M_B) = 0$

(*b*) Shear $(V_B) = 0$ ... (6)

Substitution of these in Equations (3) and (4) leads to,

$$y_{-1} = 2y_1-y_2$$
$$y_{-2} = 2y_{-1}-2y_2+y_3 \quad \ldots (7)$$

Writing Equation (5) about point 1 and simplifying,

$$2y_1-4y_2+2y_3 = \frac{-p_1t^4}{E_pI_p} \quad \ldots (8)$$

where p_1 = distributed load along pile at $i=1$

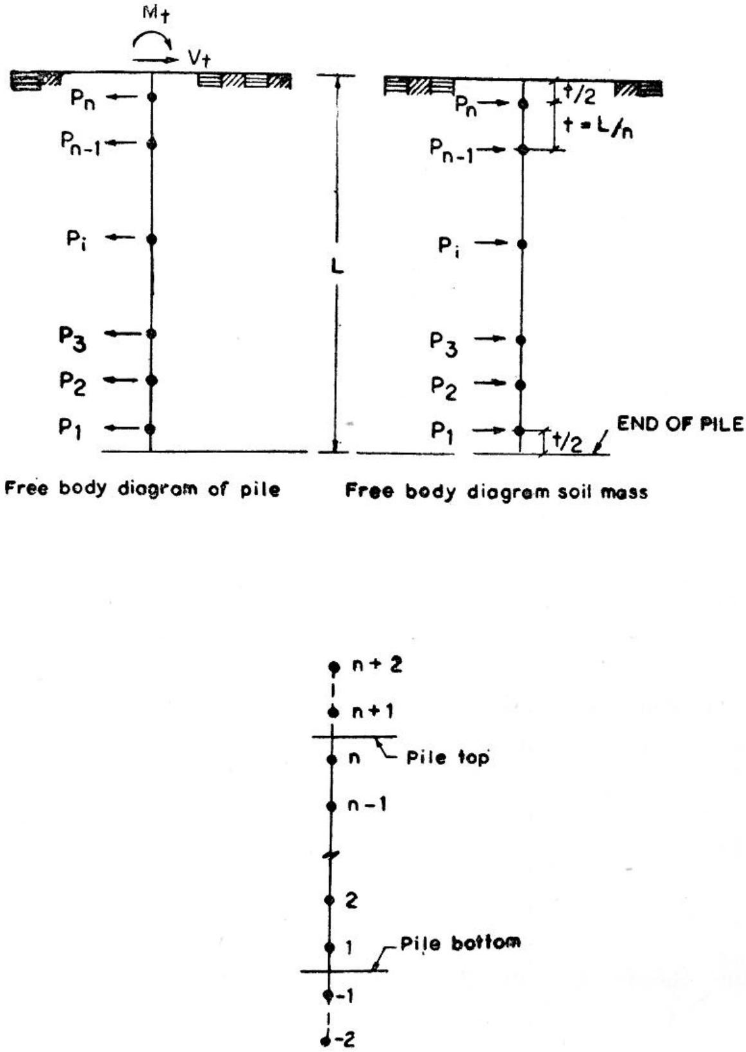

FIG 1

putting $(p_1 \times t)$ = concentrated load, P_1

and $t^3/E_p I_p = R$

Equation (8) may be re-written as,

$$y_1 - 2y_2 + y_3 = \tfrac{1}{2} P_1 R \qquad \ldots \quad \ldots \quad \ldots \quad (9)$$

Writing Equation (5) about point 2 and simplifying,

$$y_2 - 2y_3 + y_4 = -(P_2 R + P_1 R) \qquad \ldots \quad \ldots \quad \ldots \quad (10)$$

Similarly for $i=3$, we obtain,

$$y_3-2y_4+y_5=-\left(\frac{3}{2}P_1R+2P_2R+P_3R\right) \quad \ldots (11)$$

For $i=4$,

$$y_4-2y_5+y_6=-(2P_1R+3P_2R+2P_3R+P_4R) \quad \ldots (12)$$

Hence for the n th element we can write

$$y_n-2y_{n+1}+y_{n+2}=-\left(\frac{n}{2}P_1R+(n-1)\,P_2R+(n-2)\,P_3R+\ldots\ldots P_nR\right) \quad \ldots (13)$$

Putting,

$$P_1R/2=a_1$$
$$2P_1R/2+P_2R=a_2$$
$$3P_1R/2+2P_2R+P_3R=a_3$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$nP_1R/2+(n-1)\,P_2R+(n-2)\,P_3R+\ldots\ldots P_nR=a_n \quad \ldots (14)$$

We can write the general expression for the displacement of any point on the pile as,

$$y_i=2y_{i+1}-y_{i+2}-a_i \quad \ldots (15)$$

Boundary conditions at the top of the pile are,

(a) Moment $=M_t$ ... (15)

(b) Shear $=V_t$ ... (16)

putting, $$\frac{M_t t^2}{E_pI_p}=B_2 \quad \ldots (17)$$

and, $$\frac{2V_t t^3}{E_pI_p}=B_3 \quad \ldots (18)$$

Writing Equations (3) and (4) for the top of the pile

$$y_{n-1}-2y_n+y_{n+1}=B_2$$
$$y_{n-2}-2y_{n-1}+2y_{n+1}-y_{n+2}=B_3 \quad \ldots (19)$$

or,

$$y_{n+1}=2y_n-y_{n-1}+B_2$$
$$y_{n+2}=y_{n-2}-4y_{n-1}+4y_n+2B_2-B_3 \quad \ldots (20)$$

Hence we obtain the pile displacements at the element centres for the n elements as,

$$y_1=2y_2-y_3-a_1$$
$$y_2=2y_3-p_4-a_2$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$y_n=2y_{n+1}-y_{n+2}-a_n \quad \ldots (21)$$

Substituting for y_{n+1} from Equation (20) in the $(n-1)^{th}$ equation of equation (21),

$$y_{n-1} = 2y_n + y_{n-1} - 2y_n - B_2 - a_{n-1}$$

or

$$a_{n-1} = -B_2 \qquad \dots (22)$$

Similarly substituting for y_{n+2} from (20) in the n^{th} equation we get,

$$y_{n-2} - 2y_{n-1} + y_n - a_n = B_3$$

Substituting for y_{n-2} from the $((n-2)^{th}$ equation the above equation becomes

$$a_n - a_{n-2} = B_3 \qquad \dots (23)$$

Hence, we can write the general expression for the first $(n-2)$ equations as,

$$y_i = 2y_{i+1} - y_{i+2} - a_i \qquad \dots (24)$$

The $(n-1)^{th}$ equation assumes the form

$$a_{n-1} = B_2 \qquad \dots (25)$$

and the n^{th} equation becomes

$$a_n - a_{n-2} = B_3 \qquad \dots (26)$$

Soil Displacements

Assuming the soil to be an elastic half-space and using the equation of Mindlin (1936) for horizontal displacement due to a horizontal load within a semi-infinite mass we obtain the horizontal displacement $\bar{y}_{ij}$ of the soil at a point i, along a vertical line adjacent to the pile surface, at a depth Z_i below the ground surface due to a horizontal load P_j, located at a depth C_j and acting on an element j as,

$$\bar{y}_{ij} = \frac{P_j}{16\pi(1-\mu)G}\left\{\frac{3-4\mu}{|Z_i - C_j|} + \frac{1+2(1-\mu)(1-2\mu)}{Z_i + C_j} + \frac{2C_jZ_i}{(Z_i+C_j)^3}\right\} \dots (27)$$

Allowing G, to vary with depth we can re-write the above equation as,

$$\bar{y}_{ij} = \frac{P_j}{16\pi(1-\mu)G_j}\left\{\frac{3-4\mu}{|Z_i - C_j|} + \frac{1+2(1-\mu)(1-2\mu)}{Z_i + C_j} + \frac{2C_jZ_i}{(Z_i+C_j)}\right\} \dots (27a)$$

where G_j = shear modulus of soil adjacent to the pile element j

μ = Poisson's ratio of soil

and G_j is given by,

$$G_j = \frac{E_j}{2(1+\mu)}$$

where E_j = Young's modulus of soil adjacent to pile element, j.

Putting $(3-4\mu) = d$

$1+2(1-\mu)(1-2\mu) = e$

$16\pi(1-\mu) = k$

Equation (27*a*) becomes,

$$\bar{y}_{ij}=\frac{P_j}{kG_j}\left\{\frac{d}{|Z_i-C_j|}+\frac{e}{Z_i+C_j}+\frac{2C_jZ_i}{(Z_i+C_j)^3}\right\} \quad \ldots(27b)$$

The displacement at the point i at a depth Z_i due to all elements of the pile is therefore,

$$\bar{y}_i=\frac{1}{k}\sum_{j=1}^{n}\frac{P_i}{G_j}\left\{\frac{d}{|Z_i-C_j|}+\frac{e}{(Z_i+C_j)}+\frac{2C_jZ_i}{(Z_i+C_j)^3}\right\} \quad \ldots(28)$$

The soil and the pile reactions act at the centre of each of the elements of length t, into which the pile is divided as shown in Fig. 1. Hence there are n loads $P_1, P_2 \ldots\ldots P_n$ acting at depths, $C_1=(n-\frac{1}{2})\ t$; $C_2=(n-\frac{3}{2})t$; $C_3=(n-\frac{5}{2})t$;......; $C_n=\left(n-\frac{2n-1}{2}\right)t$ respectively. Each load represents a load over an area $(t\times b)$ where b, is the width of the pile. If this area is replaced by an equivalent circular area of radius,

$$a=\sqrt{\frac{t\,b}{\pi}}$$

and the singular term,

$$\sqrt{\frac{1}{\{y^2+(Z-C)^2\}}}$$

in the more general Mindlin expression, from which Equation (28) is derived is averaged over this area, the term

$$\frac{1}{|Z-C|}$$

becomes $2/a$. This value is used in the displacement Equation (28) when $Z_i=C_j$.

Hence we obtain the soil displacements at the centres of the pile elements as,

For $i=1$ at depth $Z_1=(n-\frac{1}{2})\ t$,

$$\bar{y}_1=\frac{1}{k}\left[\frac{P_1}{G_1}\left\{2d(\pi/tb)^{1/2}+\frac{e}{(2n-1)t}+\frac{2(2n-1)\ (2n-1)t^2}{4(2n-1)^3t^3}\right\}\right.$$
$$+\frac{P_2}{G_2}\left\{\frac{d}{t}+\frac{e}{(2n-2)t}+\frac{2(2n-3)\ (2n-1)\ t^2}{4(2n-2)^3t^3}\right\}$$
$$+\frac{P_3}{G_3}\left\{\frac{d}{2t}+\frac{e}{(2n-3)t}+\frac{2(2n-5)\ (2n-1)t^2}{4(2n-3)^3\ t^3}\right\}$$
$$\ldots\quad\ldots\quad\ldots\quad\ldots\quad\ldots\quad\ldots$$
$$+\frac{P_j}{G_j}\left\{\frac{d}{(j-1)\ t}+\frac{e}{(2n-j)t}+\frac{2(2n-2j+1)\ (2n-1)\ t^2}{4(2n-j)^3\ t^3}\right.$$
$$\ldots\quad\ldots\quad\ldots\quad\ldots\quad\ldots\quad\ldots$$
$$\left.+\frac{P_n}{G_n}\left\{\frac{d}{(n-1)\ t}+\frac{e}{(n)\ t}+\frac{2(2n-1)\ t^2}{4(n)^3t^3}\right\}\right] \quad \ldots(29)$$

Substituting L/n for t, the above equation becomes,

$$\bar{y}_1=\frac{1}{k}\left[\frac{P_1}{G_1}\left\{2d\left(\frac{\pi n}{Lb}\right)^{1/2}+\frac{en}{(2n-1)L}+\frac{(2n-1)(2n-1)(n)}{2(2n-1)^3L}\right\}\right.$$
$$+\frac{P_2}{G_2}\left\{\frac{dn}{L}+\frac{en}{(2n-2)L}+\frac{(2n-3)(2n-1)(n)}{2(2n-2)^3L}\right\}$$
$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$
$$+\frac{P_j}{G_j}\left\{\frac{dn}{(j-1)L}+\frac{en}{(2n-j)}+\frac{(2n-2j+1)(2n-1)(n)}{2(2n-j)^3L}\right\}$$
$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$
$$\left.+\frac{P_n}{G_n}\left\{\frac{dn}{(n-1)L}+\frac{e}{L}+\frac{(2n-1)(n)}{2(n)^3L}\right\}\right] \qquad \ldots(30)$$

Again putting,

$$2d\left(\frac{\pi n}{Lb}\right)^{1/2}=l_1$$

$$\frac{dn}{L}=l_2 \quad \frac{en}{(2n-1)L}=x_1 \quad \frac{(2n-2)(2n-1)(n)}{2(2n-1)^3L}=r_{11}$$

$$\frac{dn}{2L}=l_3 \quad \frac{en}{(2n-2)L}=x_2 \quad \frac{(2n-2)(2n-1)(n)}{2(2n-1.5)^3L}=r_{21}$$

$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$

$$\frac{dn}{(j-1)L}=l_j \quad \frac{en}{(2n-j)L}=x_j \quad \frac{(2n-i)(2n-j)(n)}{2\left[2n-\left(\frac{i+j}{2}\right)\right]^3L}=r_{ij}$$

We can re-write Equation (30) as,

$$\bar{y}_1=\frac{1}{k}\left\{\frac{P_1}{G_1}\left(l_1+x_1+r_{11}\right)\right.$$
$$+\frac{P_2}{G_2}\left(l_2+x_2+r_{31}\right)$$
$$+\frac{P_3}{G_3}\left(l_3+x_3+r_{51}\right)$$
$$+\quad \cdots \quad \cdots \quad \cdots$$
$$+\frac{P_j}{G_j}\left(l_j+x_j+r_{2j-1,1}\right)$$
$$+\quad \cdots \quad \cdots \quad \cdots \quad \cdots$$
$$\left.+\frac{P_n}{G_n}\left(l_n+x_n+r_{2n-1,1}\right)\right\} \qquad \ldots(31)$$

Similarly for $i=2$ at depth $Z_2=(n-\frac{3}{2})\,t$ we obtain,

$$\bar{y}_2=\frac{1}{k}\left\{\frac{P_1}{G_1}\left(l_2+x_2+r_{13}\right)\right.$$

$$+\frac{P_2}{G_2}\left(l_2+x_3+r_{33}\right)$$
$$+\frac{P_3}{G_3}\left(l_2+x_4+r_{53}\right)$$
$$+\ \dots\quad\dots\quad\dots\quad\dots$$
$$+\frac{P_j}{G_j}\left(l_{j-1}+x_{j+1}+r_{2j-1,3}\right)$$
$$+\ \dots\quad\dots\quad\dots\quad\dots$$
$$+\frac{P_n}{G_n}\left(l_{n-1}+x_{n+1}+r_{2n-1,3}\right)\bigg\} \qquad \dots(32)$$

For $i=m$ at depth, $Z_m=\left\{n-\left(\frac{2m-1}{2}\right)\right\}t$

$$y_m=\frac{1}{k}\left\{\frac{P_1}{G_1}\left(l_m+x_m+r_{1,2m-1}\right)\right.$$
$$+\frac{P_2}{G_2}\left(l_{m-1}+x_{m+1}+r_{3,2m-1}\right)$$
$$+\frac{P_3}{G_3}\left(l_{m-2}+x_{m+2}+r_{5,2m-1}\right)$$
$$+\ \dots\quad\dots\quad\dots\quad\dots$$
$$+\frac{P_m}{G_m}\left(l_1+x_{2m-1}+r_{2m-1,2m-1}\right)$$
$$+\frac{P_{m+1}}{G_{m+1}}\left(l_2+x_{2m}+r_{2m+1,2m-1}\right)$$
$$+\ \dots\quad\dots\quad\dots\quad\dots$$
$$\left.+\frac{P_n}{G_n}\left(l_{n-m+1}+x_{n+m-1}+r_{2n-1,2m-1}\right)\right\} \qquad \dots(33)$$

For the n^{th} element at depth $Z_n=\frac{1}{2}t$,

$$y_n=\frac{1}{k}\left\{\frac{P_1}{G_1}\left(l_n+x_n+r_{1,2n-1}\right)\right.$$
$$+\frac{P_2}{G_2}\left(l_{n-1}+x_{n+1}+r_{3,2n-1}\right)$$
$$+\frac{P_3}{G_3}\left(l_{n-2}+x_{n+2}+r_{5,2n-1}\right)$$
$$+\ \dots\quad\dots\quad\dots\quad\dots$$
$$\left.+\frac{P_n}{G_n}\left(l_1+x_{2n-1}+r_{2n-1,2n-1}\right)\right\} \qquad \dots(34)$$

Assuming elastic conditions as prevailing within the soil, the soil and pile displacements may be equated at the element centres. Thus the Equations in (21) can be written in terms of the soil displacements, which in turn can be expressed in terms of the loads P_1, $P_2 \dots\dots P_n$, using the above equations. The resulting equations are given below.

For $i=1$ we obtain,

$$\bar{y}_1-2\bar{y}_2+\bar{y}_3+a_1=0$$

or the above equation can be re-written in terms of the loads P_1, P_2, $P_3 \ldots P_n$, using Equations (33) and (14) as,

$$\frac{P_1}{G_1}\left\{(l_1-2l_2+l_3)+(x_1-2x_2+x_3)+(r_{11}-2r_{13}+r_{15})+\frac{KRG_1}{2}\right\}$$
$$+\frac{P_2}{G_2}\left\{(l_2-2l_2+l_2)+(x_2-2x_3+x_4)+(r_{31}-2r_{33}+r_{35})\right\}$$
$$+\frac{P_3}{G_3}\left\{(l_3-2l_2+l_1)+(x_3-2x_4+x_5)+(r_{51}-2r_{53}+r_{55})\right\}$$
$$+ \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$
$$+\frac{P_n}{G_n}\left\{(l_n-2l_{n-1}+l_{n-2})+(x_n-2x_{n+1}+x_{n+2})\right.$$
$$\left.+(r_{2n-1,1}-2r_{2n-1,3}+r_{2n-1,5})\right\}=0 \quad \ldots(35)$$

Similarly, for $i=2$, we obtain

$$\bar{y}_2-2\bar{y}_3+\bar{y}_4+a_2=0$$

or

$$\frac{P_1}{G_1}\left\{(l_2-2l_3+l_4)+(x_2-2x_3+x_4)+(r_{13}-2r_{15}+r_{17})\right.$$
$$\left.+\frac{2KRG_1}{2}\right\}$$
$$+\frac{P_2}{G_2}\left\{(l_1-2l_2+l_3)+(x_3-2x_4+x_5)+(r_{33}-2r_{35}+r_{37})+KRG_2\right\}$$
$$+\frac{P_3}{G_3}\left\{(l_2-2l_1+l_2)+(x_4-2x_5+x_6)+(r_{53}-2r_{55}+r_{57})\right\}$$
$$+\frac{P_4}{G_4}\left\{(l_3-2l_2+l_1)+(x_5-2x_6+x_7)+(r_{73}-2r_{75}+r_{77})\right\}$$
$$+\frac{P_5}{G_5}\left\{(l_4-2l_3+l_2)+(x_6-2x_7+x_8)+(r_{93}-2r_{95}+r_{97})\right\}$$
$$+ \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$
$$+\frac{P_n}{G_n}\left\{(l_{n-1}-2l_{n-2}+l_{n-3})+(x_{n+1}-2x_{n+2}+x_{n+3})\right.$$
$$\left.+(r_{2n-1,3}-2r_{2n-1,5}+r_{2n-1,7})\right\}=0 \quad \ldots (36)$$

For $i=m$,

$$\bar{y}_m-2y_{m+1}+y_{m+2}+a_m=0 \quad \text{or}$$
$$\frac{P_1}{G_1}\left\{(l_m-2l_{m+1}+l_{m+2})+(x_m-2x_{m+1}+x_{m+2})\right.$$
$$\left.+(r_{1,2m-1}-2r_{1,2m+1}+r_{1,2m+3})+\frac{mKRG_1}{2}\right\}$$

$$+\frac{P_2}{G_2}\Big\{(l_{m-1}-2l_m+l_{m+1})+(x_{m+1}-2x_{m+2}+x_{m+3})$$

$$+(r_{3,2m-1}-2r_{3,2m+1}+r_{3,2m+3})+(m-1)KRG_2\Big\}$$

$$+ \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$

$$+\frac{P_m}{G_m}\Big\{(l_1-2l_2+l_3)+(x_{2m-1}-2x_{2m}+x_{2m+1})$$

$$+(r_{2m-1,2m-1}-2r_{2m-1,2m+1}+r_{2m-1,2m+3})+KRG_m\Big\}$$

$$+\frac{P_{m+1}}{G_{m+1}}\Big\{(l_2-2l_1+l_2)+(x_{2m}-2x_{2m+1}+x_{2m+2})$$

$$+(r_{2m+1,2m-1}-2r_{2m+1,2m+1}+r_{2m+1,2m+3})\Big\}$$

$$+\frac{P_{m+2}}{G_{m+2}}\Big\{(l_3-2l_2+l_1)+(x_{2m+1}-2x_{2m+2}+x_{2m+3})$$

$$+r_{2m+3,2m-1}-2r_{2m+3,2m+1}+r_{2m+3,2m+3})\Big\}$$

$$+ \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$

$$+\frac{P_n}{G_n}\Big\{(l_{n-m+1}-2l_{n-m}+l_{n-m-1})+(x_{n+m-1}-2x_{n+m}+x_{n+m+1})$$

$$+(r_{2n-1,2m-1}-2r_{2n-1,2m+1}+r_{2n-1,2m+3})\Big\}=0 \quad \ldots (37)$$

For $i=n-1$, we have the equation,

$$a_{n-1}=-B_2$$

or from Equation (15) we can write,

$$\left\{\left(\frac{n-1}{2}\right)P_1+(n-2)P_2+(n-3)P_3+\ \ldots P_{n-1}\right\}R=\frac{M_t t^2}{E_p I_p} \quad \ldots (38)$$

For the n^{th} element we have,

$$a_n-a_{n-2}=B_3$$

or from Equation (14),

$$P_1+2P_2+2P_3+2P_4+\ \ldots\ +2P_{n-1}+P_n)R=\frac{2V_t t^3}{E_p I_p} \quad \ldots (39)$$

Results and Discussion

The accuracy of results is found to depend markedly on the number of elements into which the pile is divided. To examine the influence of the number of elements on accuracy, solutions were obtained by Poulos (1971) for 6, 11, 21 and 31 elements. Assuming the correct displacements to be given by the Richardson's h^2 extrapolation of the results for 21 and 31 elements, Poulos found that the use of 21 elements gave a reasonable compromise between sufficient accuracy and excessive computer time. On this basis all the solutions given in this paper were obtained for 20 elements. The pile and soil characteristics used for the present investigation are given in the Appendix. Suitable computer programmes were prepared to develop the co-efficients of the corresponding 20 equations which were then solved simultaneously, using the standard IBM subroutine, "SIMQA", to obtain the values of the loads P_1, P_2, P_3......P_n. These values were then substituted back in the Mindlin equation to obtain the deflections at the corresponding element centres. Through a process of successive differentiation and employing the finite difference technique the distribution for the moment, shear and soil reaction were then obtained.

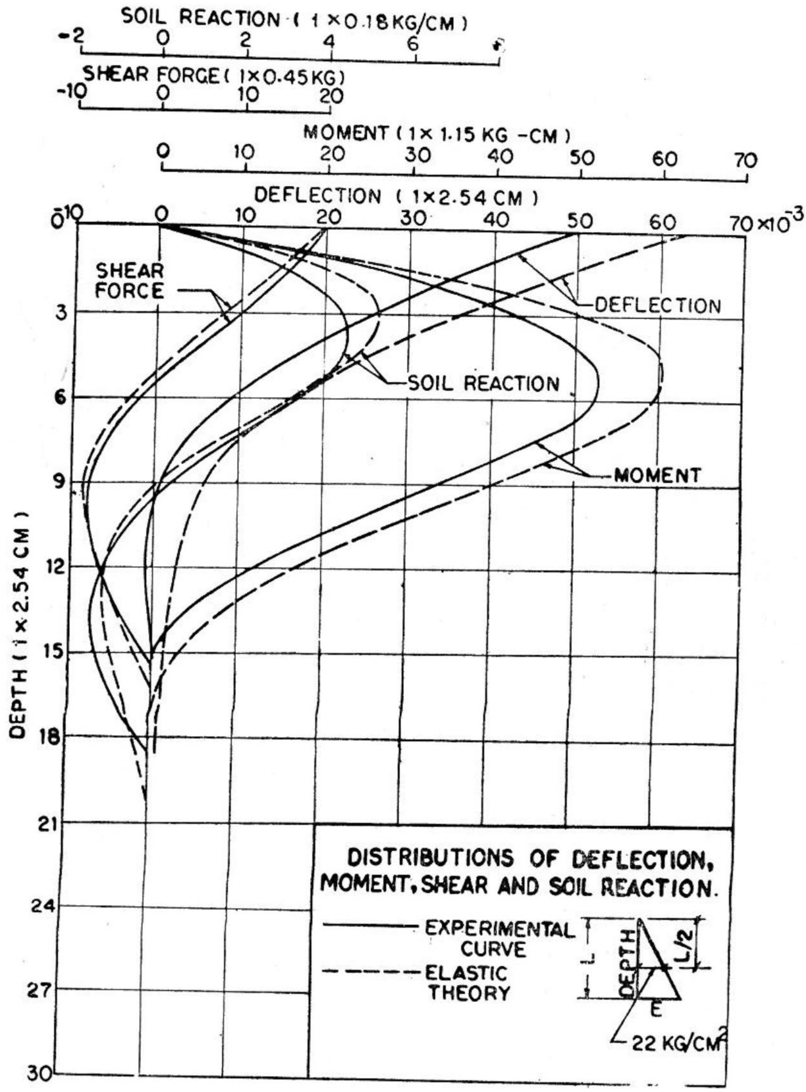

FIGURE 2

On the basis of certain simplified triaxial tests an average value of E equal to 22 Kg/cm² was assumed for the sand, as a first estimate of the actual value in the experimental investigation with model piles. This value is seen to fall, in the range of values suggested by Poulos (1971) for medium sand. A linear variation of E with depth, with a zero value at the ground surface and an average value of 22 Kg/cm² was then assumed. The experimental curves are shown in Figure 2. The distributions of shear and soil reaction as obtained from theory agree reasonably well with the experimental curves, however, the curves for moment and deflection do not compare so well. This is probably due to the approximations involved in assuming the average value of from the simplified triaxial tests. A linear variation with a slightly larger average value of E equal

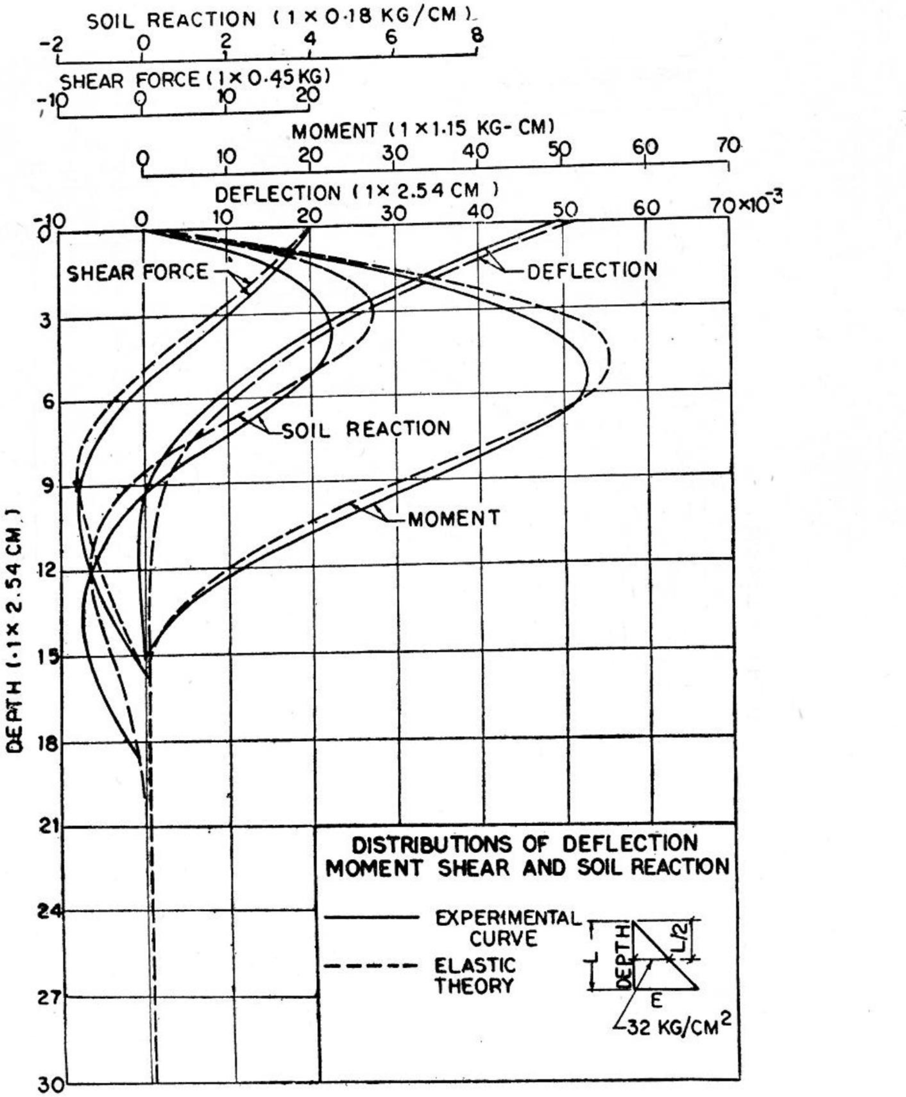

FIGURE 3

to 32 Kg/cm² [still within the range of values suggested by Poulos (1971) for medium sand] was also assumed and the corresponding distributions are seen to have a better overall agreement with the experimental curves. The comparisons are shown in Figure 3. A more elaborate triaxial test programme would probably help in assessing a more exact value of *E*.

The distributions obtained for a constant *E* with depth equal to 22 Kg/cm² is seen to be in considerable error, in that it greatly underestimates the moments and deflections and gives inadmissible soil reaction curves which predict large pressures on the pile near the ground surface, which in reality cannot develop. The comparisons between the experimental and theoretical curves are shown in Figure 4 and 5. The assumption of a linear variation of *E* with depth is seen to be a reasonable assumption for piles loaded in sand and the incorporation of this variation in the Mindlin equation does not apparently lead to any serious error as has been the contention of Poulos (1971).

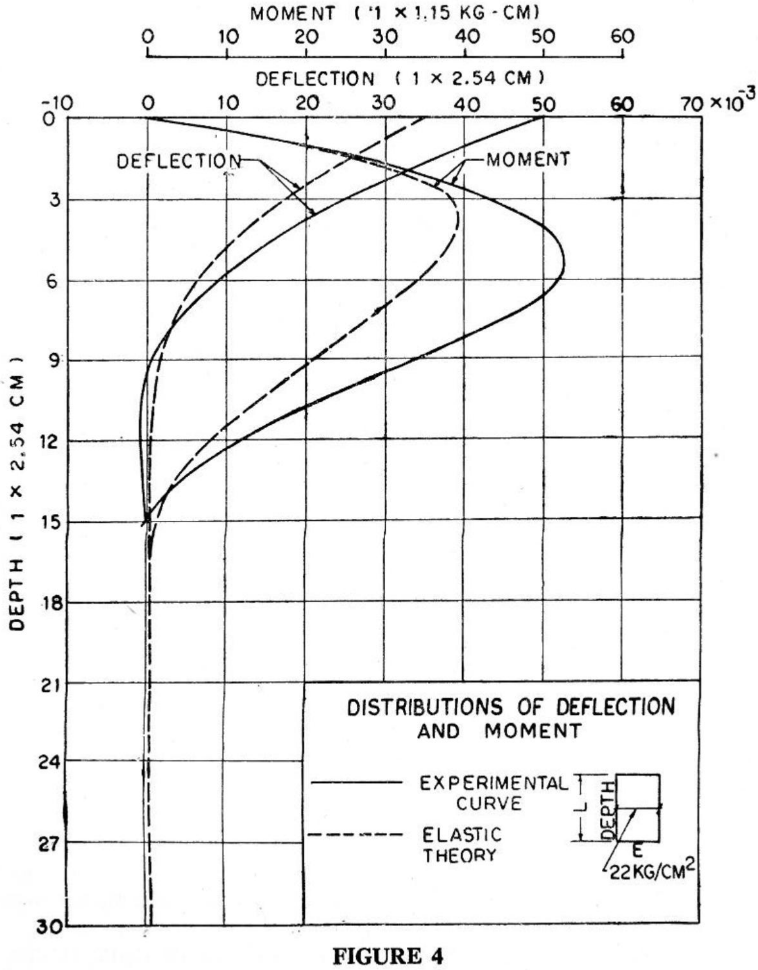

FIGURE 4

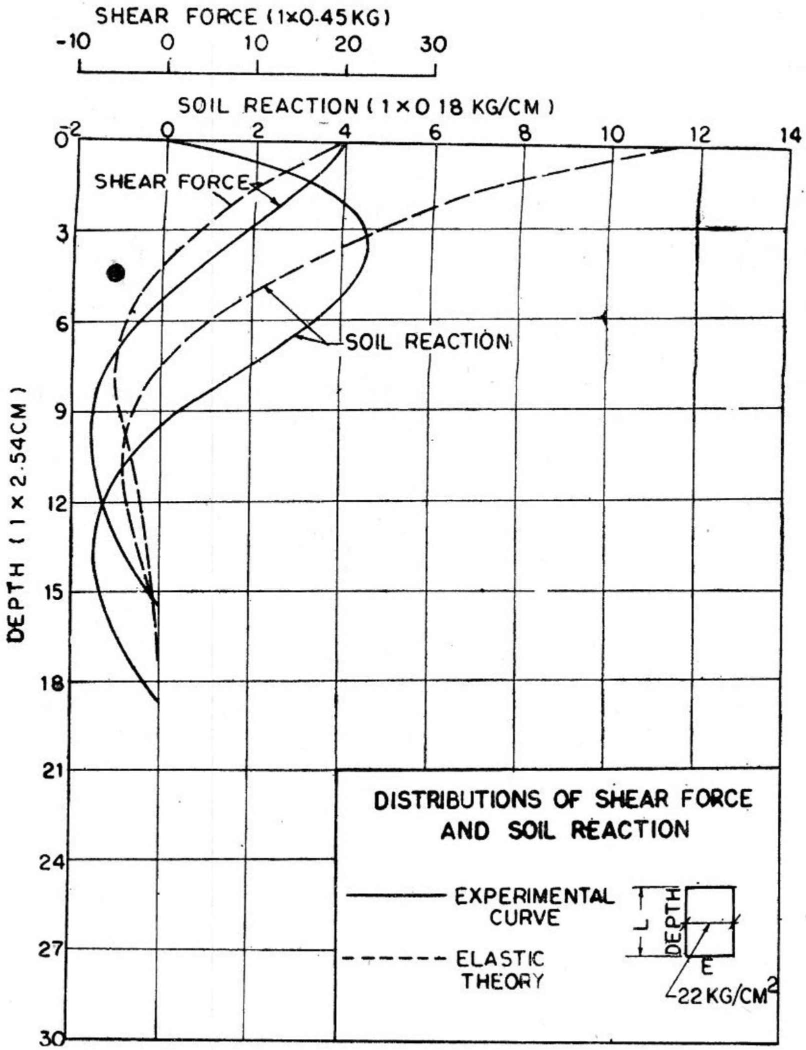

FIGURE 5

Conclusions

(1) The assumption of a constant soil modulus with depth is unrealistic for piles in sand and the elastic solutions based on this assumption give inadmissible soil reaction curves and the distributions of moment and deflection considerably underestimate the actual values.

(2) The method of analysis based on the elastic theory assuming a ʌariation of the modulus with depth is seen to be an acceptable method for predicting the behaviour of laterally loaded piles in sand. The assumed linear variation of the soil modulus with a zero value at the top

gave, in particular, admissible soil reaction curves and the moment and deflections were also estimated to a reasonable degree of accuracy.

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APPENDIX

Pile Characteristics

Type—Hollow pile of Aluminium alloy, ALCOA-6061-T6

Pile length = 76.2 cm.

Pile diameter = 1.905 cm.

Wall thickness = 0.089 cm.

Flexural stiffness = 15.06×10^4 Kg-cm^2.

Soil Characteristics

Soil Type—Ennore standard sand

Uniformity Co-efficient = 1.1

Specific gravity = 2.67

Laboratory density = 1.7 gms/c.c.

Poisson's ratio, = 0.4

Notation

L = embedded pile length

b = width of pile

E_p = Young's modulus of pile material

I_p = moment of inertia of pile section

E = young's modulus of soil

G = Shear modulus of soil

μ = Poisson's ratio of soil

y = horizontal displacement of pile

$\bar{y}$ = horizontal displacement of the soil

P_j = arbitrary horizontal force on pile

C_j = depth from ground surface to any horizontal force

Z_i = depth from ground surface to a point where deflection is desired

p = soil resistance

t = spacing of horizontal loads

n = number of forces taken to approximate the distributed pressure on the soil

θ = slope

M_t = applied moment

V_t = applied horizontal load.

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