Concept of Flow Parameters and their Applications in Soils

by
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Introduction

MOST of the present solutions have been worked out for rigid plastic behaviour, whereas the elastico-plastic or elastico-visco-plastic scheme of the soil under the footing will best correspond to reality (Houska', 1969). Here it is necessary to mention that the author has taken up this minimum realistic treatment for the soil under the footing as the elastico-plastic scheme. All the present approaches lead to ultimate pressures, at the same time, disregarding altogether deformations in the soil as well as at the surface. Large scale tests in Berlin (Degebo, 1961) have shown clearly that failure does only occur in originally dense soils whereas in loose material, extremely large displacements are observed without any tendency to cause a failure so that allowable settlement and not the failure does limit the bearing capacity. Moreover, failure theories so far developed do not reflect real conditions of soils. Evidently, displacement and compaction which first Ohde in 1950 tried to take into consideration, are of primary importance. Later, Lorenz (1962) and Gudehus (1966) followed the classification approach viz, determining the stress distribution under a certain load, finding a stress-strain relation for a specific material and then determine the settlement of foundation by integrating over the relevant area. However, the stress-strain relation "Material law" of soils involves too many parameters of which a significant part depends on displacement of the material as well as on density changes.

Concerning the stress-strain behaviour it must be noted that according to the methodology of the theory of plasticity upto the present, the theoretical solutions have been worked out for rigid plastic behaviour, whereas soils would require a more realistic treatment as elasto-plastic, with strain-hardening or strain-softening, as the minimum adequate form of representation (De Mello, 1969). The use of computational models as exemplified by whitman and Hoeg in 1965 and others, opens new vistas along this line.

So far, the application of shear strength parameters c and ϕ is well known in soil mechanics, but the concept of flow parameters in soils is probably a new method of approach. In fact, there exist stress-strain characteristics of all types of soils. On one hand, it is not possible to

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describe all types of soils by both c and ϕ, on the other hand, it is always possible to characterize any type of soils by means of flow parameters as because, it gives an idea of stress-strain behaviour of in-situ soils similar to that of the load-settlement relation out of a load lest. Flow parameters can be described as unique properties of soils under any condition. De Mello pointed out the choice of applicable "Flow criterian" through Mohr-coulomb equation $s = c + \sigma \tan \phi$.

Thus, to evade some of the difficulties, an analytical-graphical approach has been suggested by the author in order to predict simultaneously the bearing capacity and settlement of saturated $c-\phi$ soils from the yield criteria of flow curves and flow parameters. Treating soils as elastico-plastico materials, the author has obtained flow curves and flow parameters from the probable stress-strain relations attained under the pre-consolidation pressures. In this approach, field confining pressures are assumed to be equal to the pre-consolidation pressure of soils ($Ko = 1$). The author's method of determining pre-consolidation pressure (Chakraborty, 1970, 1972) by following equal incremental pressure and equal time interval in the triaxial drained tests has been indirectly supported by the resistance concept formulated by Janbu, (1969). The author in his analysis, attempted to use the concept of theory of deformation of soils. It can be judged that the theory of deformation can yield good results in the case of cohesive soils/clays/silts (Houska, 1970).

Stress-strain relations

Stress-strain relationship for elastico-plastic substance (Nadai 1963) is

$$\in = \psi\left[\sigma_1 - \mu(\sigma_2 + \sigma_3)\right] \qquad (\ldots 1)$$

In conventional triaxial tests, σ_2 is equal to σ_3. The equation (1) can now be written by considering the change in the volume of soils as follows :—

$$\in = \psi\left[\sigma_1 - 2\ \mu\ \sigma_3\right] \qquad (\ldots 2)$$

where $\in$ = Axial strain

ψ = Flow function $= \psi' + \psi''$

ψ' = A constant quantity $= \frac{1}{E}$

ψ'' = A variable quantity

σ_1 = Major principal stress

σ_3 = Minor principal stress

μ = Poisson's ratio (0.4 to 0.5 for soils).

The value of E is determined by drawing initial tangent to the stress-strain curve. In tha triaxial stress condition, μ is assumed to be equal to 0.5 throughout the test for an undrained condition. i.e., there is no volume change in soils (Terzaghi and Peck, 1948). The equation (2) can further be simplified as

$$\in = \psi\left[\sigma\sigma_{31} -\right] \qquad (\ldots 3)$$

where $\sigma_1 - \sigma_3$ = Deviator stress or difference of stress.

Any type of soils can be characterized by the Equation (2) or (3) under triaxal stress conditions so far as their stress-strain characteristics are concerned. It is now possible to plot $\psi - \in$ relation by considering both drained and undrained conditions for triaxially tested soil samples from stress-strain relations under the preconsolidation pressures (not shown in the figures).

Under any confining pressure applied to soil samples and by considering any state of soils, there exists ψ and $\in$ relation as shown in the Figure 1. The author has done extensive works on ψ and $\in$ relations for various soils by considering each soil under different testing conditions, but desires to present results of undrained triaxial tests and in these tests the Equation (3) is used (Chakraborty, 1970).

Introduction of flow curves and flow parameters

Considering experimental stress-strain relations under preconsolidation pressures of saturated cohesive soils whose c and ϕ values vary from 0.15 to 1.10 kg/cm² and 2° to 12.5° respectively under triaxial undrained tests at $\dot{\in} = 1$ percent per minute. The values of ψ have been calculated for different percentages of strain, $\in$. This ψ—$\in$ relation is defined as the flow curve (Figure 2). It is observed from the curves that ψ increases with the increase of strain. It is further observed that ψ increases rapidly in the range of zero to about 1 precent of strain. $\psi - \in$ relation becomes linear beyond 1 percent of strain [Figures 2 (a) and 2 (b)]. The starting point of the linear portion at T is characterised as the beginning of the plastic flow. The plastic flow continues upto the point V. After the point V, viscous flow begins. The line TV which makes an angle α_1 with the horizontal is called the angle line of plastic flow. Standard method of determination of α_1 is as shown in the figure 2. The flow function corresponding to the point T is defined as the limiting point of elastico- plastic condition of soils as designated by ψ_1'. The strain corresponding to ψ_1' is $\in_1'$. ψ_1' is treated as the yield criteria of —$\in$ curve.

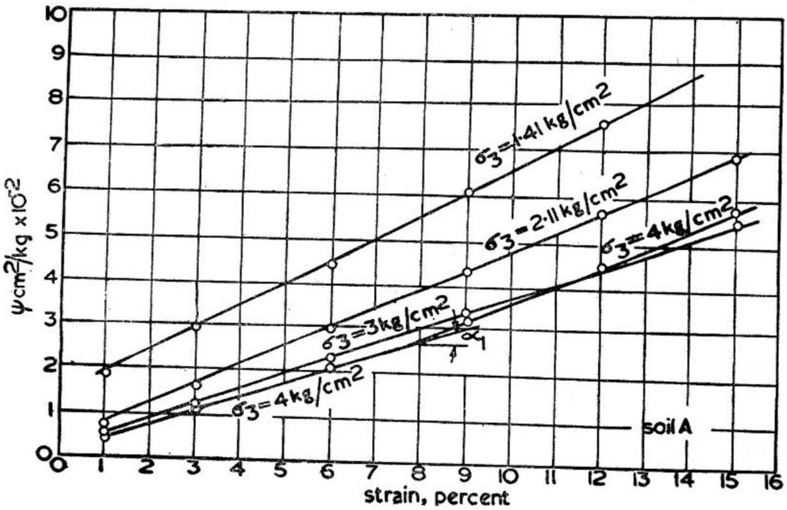


FIGURE 1 : ψ — $\in$ relation under different Confining pressures.

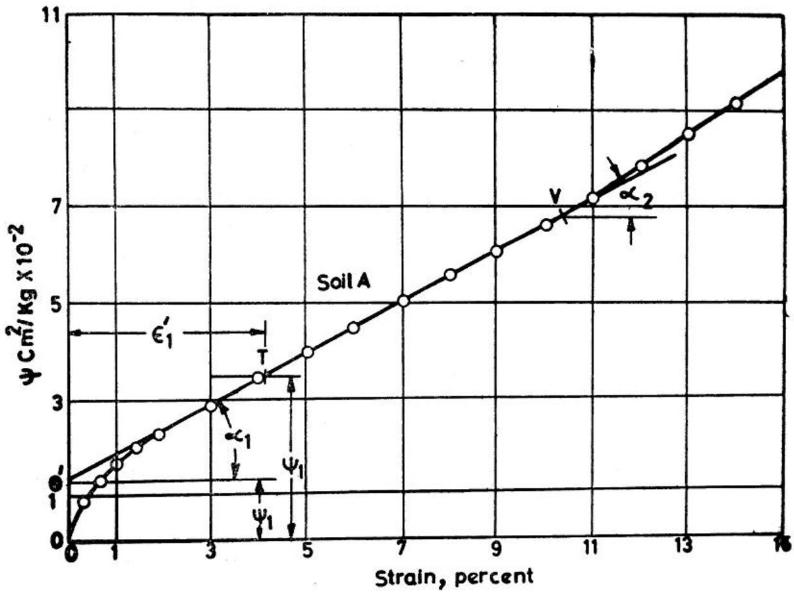


FIGURE 2 : **Introduction of a flow-curve.**

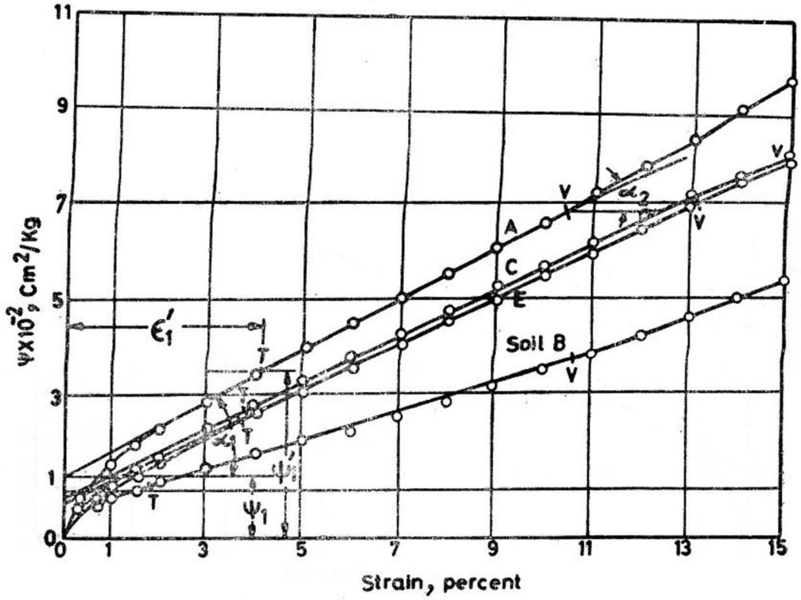
A flow constant ψ_1 is obtained at O' by extending the line TV to the ordinate of ψ (figure 2). The slope of the line $O'TV$ is defined as the plastic flow index and beyond the point V it is defined as the viscous flow index with an angle α_2. Like shear strength parameters c and ϕ, ψ_1 and α_1 are defined as flow parameters. All types of soils can be characterised by flow parameters. These parameters are the unique properties of $\psi-\in$ curve for a particular soil under a definite confining pressure.

Application of flow parameters and flow curves

(*A*) Flow parameters can be related to shear strength parameters mathematically as

$$\left[\psi_1,\ \alpha_1\right]=\frac{1}{f\ [c,\phi]}$$

Similar to the Coulomb's equation, $\psi=\psi_1+\in\tan\alpha_1$; ψ_1 and α_1 are found out from the figures 2 (a) and 2 (b). Another method of determining the the modulus of elesticity E of soils is that E is equal to $\frac{1}{\psi_1}$(Table 1). ψ_1 is greater than ψ' which is calculated by knowing the initial tangent modulus of the stress-strain relation. This indicates that E value from $\psi-\in$ curve is lower than the E value from the stress-strain curve. Values of E from $\psi-\in$ curve are more rational, because these are found out by considering stress-strain relations upto peak stresses q (upto the point V). From Table 1 it is seen that α_1 increases with the decreasing value of q and viceversa. This is because of the characteristic behaviour of general flow curves. There exist approximately linear relationships between ψ_1 and q in the log-log scale (Figure 3) and α_1 versus q in the natural scale (Figure 4) respectively.



FIGUE 2 (a) : Flow Curves of differnt soils.

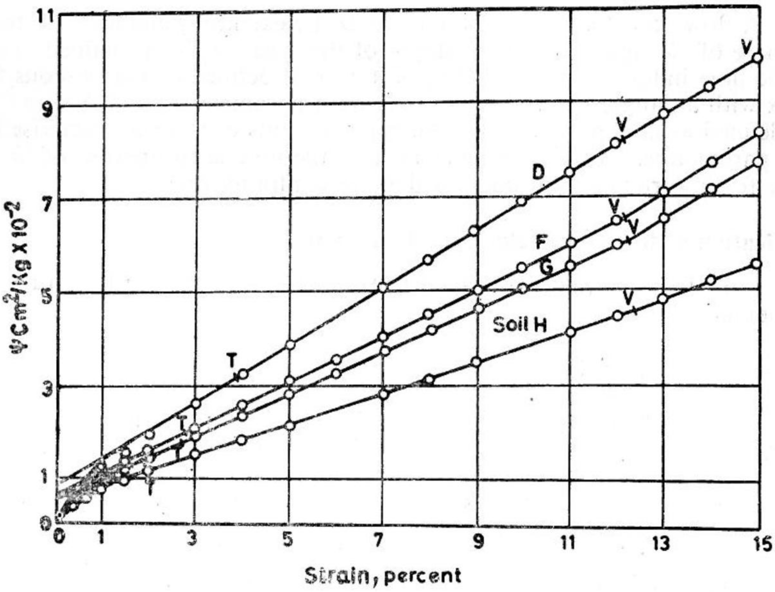


FIGURE 2 (b) : Flow curves of different soils.

ψ_1' being the yield criteria of $\psi-\in$ curve, q'_{safe} can be measured corresponding to the strain $\in_1'$ in the respective stress-strain relations of soils (Table II). Approximately linear relationships are possible between ψ_1 and q'_{safe} in the log-log scale, and α_1 and q'_{safe} in the natural scale (Figures 3 and 4) respectively.

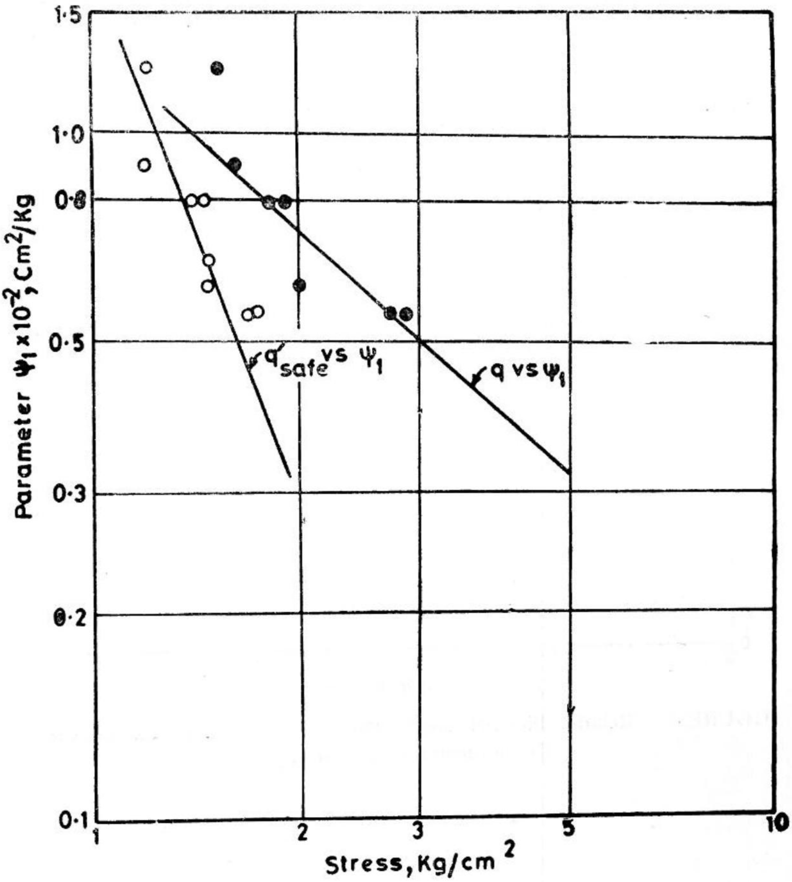


FIGURE 3: Relationships between (i) Parameter ψ_1 and peak stresses (q); (ii) Parameter ψ_1 and q'_{safe}

Table I

Undisturbed Soils	$\psi=1/E$ (cm^2/Kg) x 10^{-2}	Parameter ψ_1 (cm^2/Kg) x 10^{-2}	Parameter α_1 (degrees)	Maximum stress (q) from stress-strain curves Kg/cm^2
A	0.870	1.250	27.00	1.532
B	0.500	0.550	17.00	2.850
C	0.500	0.800	26.50	1.850
D	0.715	0.900	31.00	1.57
E	0.500	0.800	24.50	1.88
F	0.455	0.650	27.00	1.86
G	0.455	0.600	24.50	2.00
H	0.455	0.550	18.50	2.70

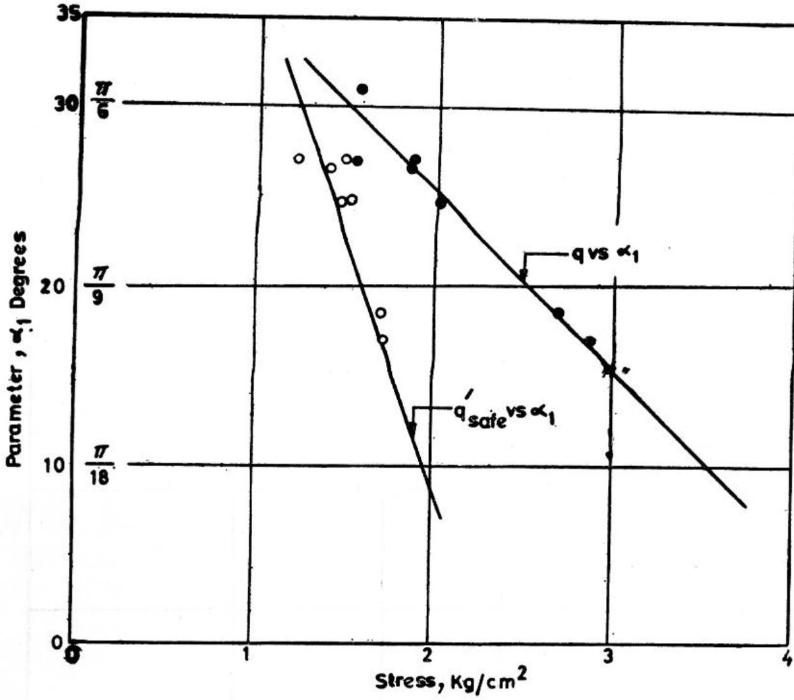


FIGURE 4 : Relationships between (i) Parameter α_1 and peak stresses (q); (ii) Parameter α_1 and q'_{safe}.

Table II

Undisturbed Soils	ψ_1' (cm²/Kg) x 10^{-2}	ϵ'_1 percent	q'_{safe} (Kg/cm²)
A	3.50	4.15	1.20
B	1.10	1.85	1.70
C	2.70	3.80	1.40
D	3.20	3.90	1.18
E	2.50	3.65	1.45
F	2.20	3.15	1.47
G	1.80	2.70	1.47
H	1.20	2.20	1.705

TABLE III

Comparison of bearing capacity based on model tests and triaxial tests in Soil A

Tests on saturated remoulded soil A	Applied consolidation pressure in the box (Kg/cm^2)	Period of consolidation (months)	q'_{safe} based on $\psi-\epsilon$ curve (triaxial test) Kg/cm^2	Yield stress value corresponding to the sharp break point (from stress versus settlement when plotted observations of model tests in the log-log scale), Kg/cm^2
M_1	1.00	3	0.47	1.40
M_2	1.50	3	0.77	1.70
M_3	2.00	3	0.90	2.25

As results of model tests are not the main aspect in the text of this paper, model test results are compared with that of q'_{safe} only for the saturated remoulded soil A under different consolidation pressures (Table III). In this table, it has been shown that q'_{safe} and the yield stress values out of model tests, increase with the increase in consolidation pressure. As model tests are not done in the undisturbed soils, it does not seem to be necessary to compare results for undisturbed soils. The Table III will fairly illustrate the comparison of bearing capacity for q'_{safe} and yield stresses under the similar condition of saturated remoulded soil A. Under the single consolidation pressure of say, 1.00 Kg/cm^2 for the remoulded soil A, three tests have been carried out, each taking minimum of three months for consolidating the soil. M_1 as indicated in Table III, is the mean value of three very close values of the yield stress. The factor of safety for, M_1, M_2, and M_3 ranges from 2.5 to 3 as compared to q'_{safe}.

From the experimental investigations of saturated cohesive soils tested in the laboratory, the following expressions are given for the soils mentioned in different figures :

(*i*) $\alpha_1 = a - bq$ (Figure) 4)

where α_1 *is in radian and* q *is in* Kg/cm^2,

$a = 0.7935$, $b = 0.1745$

(*ii*) $q'_{safe} = a_1 - b_1\,\alpha_1$ (Figure 4)

where q'_{safe} is in Kg/cm^2, α_1 is in radian

$a_1 = 2.33$ and $b_1 = 2.12$

From (*i*) if peak stresses (q) be determined from the stress-strain relations under the pre-consolidation pressures it is easily possible to find out the value of α_1 approximately without plotting $\psi-\epsilon$ curve. Once the value of α_1 is known q'_{safe} is approximately determined from (*ii*)

(*iii*) $q = c\,\psi_1^{-n}$ (Figure 3)

where q is in Kg/cm^2, ψ_1 is in cm^2/Kg.

$c = 0.076$ and $n = 1.125$

(*iv*) $q'_{safe} = c_1 \psi^{-n_1}$ (Figure 3)

where q'_{safe} is in Kg/cm^2, ψ_1 is in cm^2/Kg.

$c_1 = 0.190$ and $n_1 = 0.406$

Constants a, a_1, b, b_1, c, c_1, n and n_1 are given for the soils tested. These constants vary within very narrow limits depending on the history of geology of soils (Chakraborty, 1970, Kumar, 1972). From (*i*) to (*iv*), it is observed that q'_{safe} can be found out approximately either by knowing α_1 or by knowing ψ_1 though more correct values of q'_{safe} can be obtained from the yield criteria of $\psi - \in$ curves.

(*B*) *Determination of allowable bearing capacity and allowable settlement of foundations from flow curves and flow parameters.*

Allowable bearing capacity can be calculated as

$$q_{allow} = q'_{safe} + \gamma D_f + \tfrac{1}{2}\gamma B N_\gamma$$

where D_f = depth of the foundation below ground level

$\frac{1}{2}\gamma B N_\gamma$ = Terzaghi's bearing capacity term added for saturated cohesive soils whose ϕ' exceeds $10°$.

The factor of safety within the range of 2.5 to 3 is implied in the expression of q_{allow} as has already been verified by model tests in the laboratory (Chakraborty, 1970, 1970A).

Based on the equivalent active zone concept as proposed by the author (Kumar, 1972) the allowable settlement of a shallow foundation for cohesive soils can be found out on the basis of axial yield strain equal to $\in_1'$ in the flow curve. The settlement in a homogeneous strata may be computed by assuming the significant depth of influence of stress to be equal to 0.5 × depth of active zone having equal to 1.5 times the width (B) of the foundation. Within this depth of soil, the stress distribution is assumed to be uniform, that is to say, the stress distribution being asymptotic upto the depth equal to $1.5B$, has been distributed approximately uniformly over the depth of soil equal to $0.75\ B$. Thus allowable settlement of a probable stress-strain relation of an over-consolidated and undisturbed saturated cohesive soil is as follows :—

$$\rho_{allow} = 0.5 \times 1.5B \times \in_1' = 0.75B\ \in_1'.$$

Lambe's work (1964) on "Methods of Estimating Settlements", suggests that settlement is equal to

$$\rho = \sum_{1}^{n} H_n \in_n$$

where the strain beneath the footing is considered in n parts of thickness H_n each and $\in_n$ is the corresponding average strain associated with the vertical stress caused by the surface loading in each stratum. In Lambe's work, the determination of $\in_n$ for each stratum will involve complecacy and reasonable determination of settlement may not be possible. If one believes in the theory of deformation characteristics of soils, the author's method of simultaneous determination of allowable bearing capacity and allowable settlement stand as they are so far as the practical view points are concerned.

Conclutions

(1) $\psi - \in$ curves which are derived from the probable stress-strain relations represent the elastico-plastic behaviour of soils under the pre-consolidation pressures.

(2) Concept of flow parameters and flow curves render new method of approach by-passing the existing failure criteria.

(3) Simultaneous determination of allowable bearing capacity and allowable settlement of foundations is possible by considering deformation characteristics of in-situ saturated cohesive soils, the concept of which has already been supported by many investigators.

(4) The presented method renders relatively safer result having a factor of safety ranging from 2.5 to 3 even under the saturated remoulded condition, under statics of soil media representing both homogeneous and heterogeneous strata.

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APPENDIX

Determination of q'_{safe}

Referring to Figure 2, first of all observations of deviator stress and strain percent are converted into ψ and strain percent. This can be done by referring to the equation 3. Now, from a plot between ψ versus strain percent, point T is located. This is the starting point of the linear portion of the curve OTV. The flow function corresponding to the point T, is defined as the limiting point of elastico-plastic condition of soils. This is designated as ψ'_1 . ψ'_1 is treated as the yield criteria of $\psi - \in$ curve. The strain corresponding to ψ'_1 is designated as $\in'_1$. Knowing $\in'_1$ from the $\psi - \in$ curve, deviator stress is determined from the deviator stress Vs. strain % curve. This deviator stress is designated as q'_{safe}. Thus, q'_{safe} is determined from the yield criteria of $\psi - \in$ curve.

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