Short Communications

Analysis of Relief Wells in Embankments

by

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Introduction

THE purpose of relief wells is to control underseepage and prevent sand boils in the vicinity of earth dams and levees. Seepage under the embankments in alluvial river valleys can be a serious problem, particularly where a rather thin stratum of relatively impermeable soil underlain by a pervious substratum exists, as shown in Figure 1. Although the interception of seepage and reduction in hydrostatic pressures can be achieved by a variety of means, such as land side berms, ponding basins, drainage trenches, sub-levees, and pressure relief wells, the latter are particularly effective. One special advantage of relief well systems is the ease with which they can be expanded if an initial installation should not furnish the needed control.

The construction of levee systems along the Missouri and lower Mississippi Rivers has focussed considerable interest on this problem, and since about 1940, extensive model and field studies have been conducted. Based mostly on these studies, Middlebrooks and Jervis (1947) and Turnbull and Mansur (1961) developed formulae for the design of fully and partially penetrating relief wells. Such problems of seepage control were also encountered in Tarai area of Uttar Pradesh, particularly Sarda Sagar and Nanak Sagar dams. Some basic studies by means of electrical analogy and hydraulic models were carried out at Uttar Pradesh Irrigation Research Institute (1961). More recently, Cedergren (1967) obtained some approximate solutions.

Unless the period of operation is relatively short or the spacing of relief wells is so large that their zones of influence do not effectively overlap, the drawdown due to an individual well is affected by the neighbouring wells. Although majority of existing procedures for well groups do consider this interference, they do not take into account the unsteady response of the system. A steady state approach serves only as a first approximation in many cases, and it is often used simply for sake of expediency. Since the engineer is required in many practical problems to balance technical

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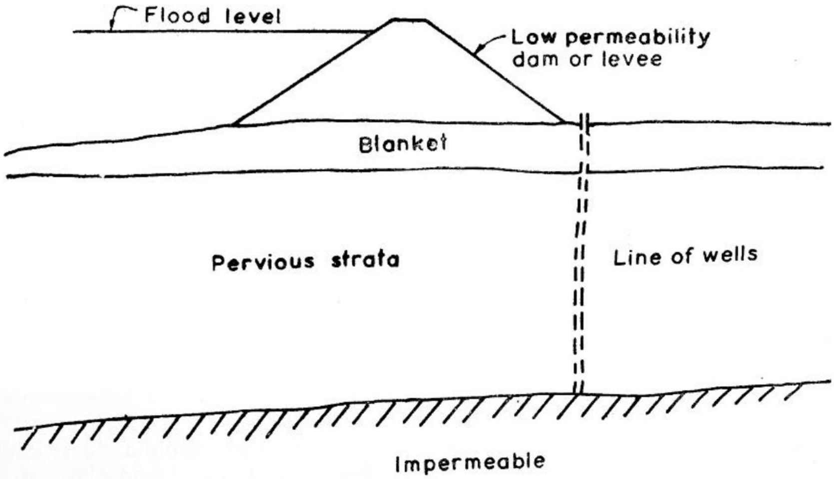

FIGURE 1 : Seepage conditions under embankments.

need against economical considerations, the issue becomes one of "how fast" versus "how much". The rate at which pressure relief is accomplished may be critical on some jobs, such as flood embankments, and relatively fast systems may be needed. However, "relatively fast" is a very subjective term, and, in general, few attempts have been made in the past to quantify time considerations, except on intuitive basis. Accordingly, this study will investigate the time-dependent drawdown associated with points in the vicinity of relief well groups arranged in a line array on the land side of earth embankments.

Theoretical Development

As shown in Figure 2, consider a straight line array of p wells, which fully penetrate an isotropic, homogeneous and pervious stratum, subjected to artesian pressure. Let the well spacing be l, radius of each well be r_w, and let each well discharge at the same rate, Q. Furthermore, assume that the number of wells is odd ($p=2n+1$), let the outer constant potential boundary be semi-circular; locate the origin of the well system at the centre well (with the wells being numbered from the centre as 0, ± 1, ± 2, ± 3..........$\pm n$), and direct the x and y axes perpendicular and parallel, respectively to the line of wells. Now, using the principle of superposition, the drawdown after time t at any arbitrary point can be expressed as (Rao et al, 1971)

$$s=\frac{Q}{8\pi Bk}\sum_{j=-n}^{n} -E_i\left(-\frac{r_j^2}{4at}\right) \quad \ldots(1)$$

where, r_j, the distance between (x, y) and the centre of jth well is given by

$$r_j^2=x^2+(y-jl)^2 \quad \ldots(2)$$

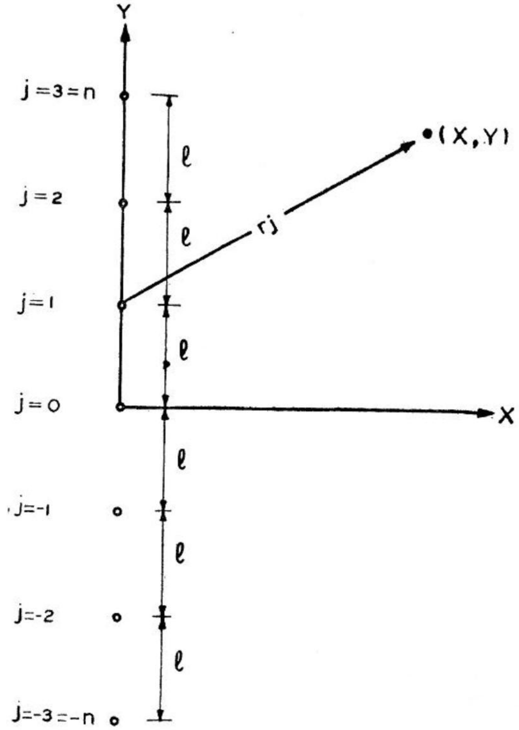

FIGURE 2 : Schematic diagram for typical well group in a straight line array.

In Equation (1), $a = Bk/S$, where, B is the thickness of the pervious stratum, and k and S are the coefficients of permeability and storage, respectively. Coefficient of storage may be defined as the amount of water released from storage per unit surface of an aquifer per unit decline of water. Although rigid limits cannot be established, the storage coefficient of artesian aquifers may range from as little as 0.00001 to probably as much as 0.005. E_i is the exponential integral function representing the non-steady state of flow towards a well in the well-known Theis equation.

At the face of the centre well, where largest drawdown occurs, $x = r_w$ and $y = 0$. Therefore, Equation (1) becomes,

$$s = \frac{Q}{8\pi\, Bk} \sum_{j=-n}^{n} -E_i\left(\frac{-r_w^2}{4\, at} - \frac{j^2 l^2}{4\, at}\right) \qquad \ldots(3)$$

Denoting $\alpha = r_w^2/(4\, at)$, the exponential integral may be approximated by

$$-E_i(-\alpha) = ln\ (0.562/\alpha) \qquad \ldots(4)$$

for small values of α ; ln is the natural logarithm. With the introduction of

$$\beta = \frac{l^2}{4\,at} \qquad \ldots(5)$$

the drawdown may be expressed as

$$s = \frac{Q}{8\pi\, Bk}\left[ln\ \frac{0.562}{\alpha} + 2F(\beta,\, n) \right] \qquad \ldots(6)$$

where,

$$F(\beta,\, n) = \sum_{j=1}^{n} -E_i(-\beta\, j^2) \qquad \ldots(7)$$

This leads to the definition of a dimensionless drawdown parameter, given by,

$$\frac{2\ skB}{Q} = \frac{1}{4\pi}\left[ln\ \frac{0.562}{\alpha} + 2F(\beta,\, n) \right] \qquad \ldots(8)$$

The parameter, β may be defined as a time parameter for investigating the transient nature of flow; note that this parameter is also affected by the well spacing, l.

The above treatment is restricted by certain conditions, obtaining in the field. If the relief well is drained at a specified level, depending on the outfall available, the well has a constant drawdown with varying discharge. The analysis of a problem of this nature is rather rigorous, and is not amenable to simple solution. Again, the head behind the embankment increases gradually, during which time the relief well pressures keep on adjusting to the head. This variation in head with time has not been considered in the present study.

Parametric Study and Results

Equation (8) was studied for a variety of straight line well systems and times of operation. The different systems consisted of 3, 5, 7, 9, 11, 13, 21 and 41 wells of 30 cm diameter uniformly spaced at 3, 8, 15, 30, 45, 60, 90, 120 and 150 metres, and fully penetrating a pervious stratum with coefficient of permeability equal to 35×10^{-4} cm per sec and storage coefficient of 0.002; the combinations are reasonably representative of those which are commonly encountered in typical relief well systems. Drawdowns are computed for pumping times of 0.01, 0.1, 1, 10 and 100 days. Hydraulic losses which occur at the well face have not been taken into consideration.

The exponential integral involved in various computations was adapted for computer solution by expanding it into a convergent infinite series.

Figure 3 shows the relationship between the drawdown in the centre well, time and spacing in terms of dimensionless parameters for different well systems. It may be observed that the effect of number of wells on drawdown is not significant at small times of pumping ; this is physically correct, because at small values of time the respective cones of depression of the individual wells in a group do not interfere with one another. Continued pumping, however, leads to a rapid increase in drawdown, as evidenced by logarithmic ordinates. With a knowledge of the properties of foundation material (such as the coefficients of permeability and storage), the number of wells, well spacing and the capacity of pumps, Figure 3 can be used for preliminary design purposes to determine the drawdown at a given time ; alternatively, the well spacing needed to effect a given change in the water-level may be calculated. Similar relationships may be obtained for well arrays of other dimensions.

Application

In order to illustrate applications of the procedure developed above, consider a dam or levee shown in Figure 1. The pervious substratum is 30 m thick with the coefficients of permeability and storage being 50×10^{-4} cm per sec and 0.0005, respectively. The head between high flood level and discharge level of the wells is, say, 30 m, which is the head causing artesian pressure. It is required to compare several alternatives regarding

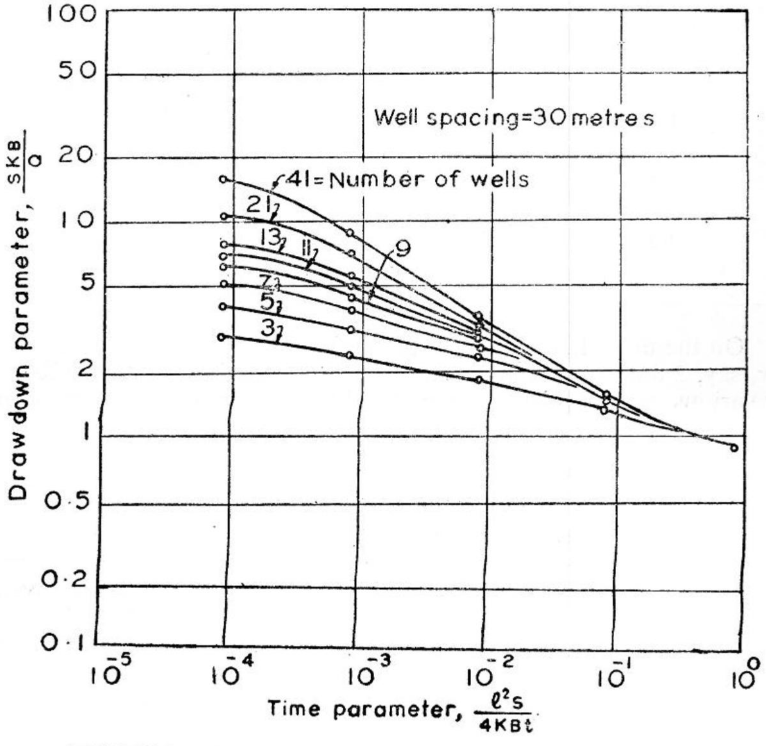

FIGURE 3 : Parametric relationship for various well systems.

pumping time, well capacity and spacing requirements. In the case of flood protection embankments, the rate at which artesian pressure can be dissipated is sometimes critical in safeguarding the embankment as well as the neighbouring area.

Figure 3 may be used to make preliminary determinations. Since the required drawdown, s, at the centre of relief well array is 30 m, the drawdown parameter, $2\ skB/Q$, is $2(30)\ (50 \times 10^{-4})\ (30)\ (60)/(100\ Q)$, or $5.4/Q$. The time parameter, $l^2S/(4\ KBt)$ is $(100)\ l^2\ (0.0005)/(4 \times 50 \times 10^{-4} \times 30\ t \times 60)$, or $l^2/(720\ t)$. In these expressions, Q is the discharge of each well in kilolitres per minute, and l and t are well spacing and period of pumping in metres and minutes respectively. If the length of embankment is, say, 1200 m, 21 wells in a line at a spacing of 60 m will be required, and the corresponding time parameter for a proposed discharge of 1 kilolitre per minute, from *Figure 3, is* 1.5×10^{-3}. *Thus, the required* drawdown is obtained in about 2.4 days of continuous pumping. Other alternatives for different values of pumping capacity, well spacing and pumping time are given in Table I.

TABLE I

Typical Well Systems for Constant Pumping Capacity of Each Well.

Pumping Capacity of Each Well (kilolitres per minute)	Number of Wells in Each Line	Pumping Time (days)
1	21	2.4
	13	15.3
	9	72.5
1.5	21	0.7
	13	2.7
	9	8

On the other hand, if the specified reduction in head is required after, say, 2 days of pumping, the time parameter is equal to $l^2/(207 \times 10^4)$, and various well systems which satisfy this requirement are given in Table II.

TABLE II

Typical Well Systems for Constant Pumping Time of 2 Days.

Number of Wells in Each Line	Discharge Rate for Each Well (kilolitres per minute)
41	0.5
21	1.1
13	1.7

Summary

A procedure has been developed to determine the unsteady drawdown at any point in the vicinity of relief wells, installed along a straight line. Graphical relationship is given to inter-relate typical combinations of variables. It is suggested that such a procedure provides useful tool for the engineer, responsible for designing or analysing the transient response of relief well systems in embankments for storage or flood protection purposes. A representative pressure relief problem has been selected to illustrate some applications of the method.

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