Dynamic Earth Pressure Distribution Behind Flexible Retaining Walls

by

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THE problem of lateral earth pressures is still not fully understood though the first classical earth pressure theory by Coulomb dates back to 1773. Even to-day, the retaining walls are designed using active earth pressure from Coulomb's theory. This is in spite of the fact that actual rupture surfaces developed are curved while Coulomb had assumed a straight rupture surface in his theory. The confidence of the designers is derived out of the model test results which supported the wedge theory. However, the validity of application of these model ests in practice is a point worth examining because of some discrepancies. The problem becomes much more complicated if dynamic earth pressures are to be considered. Here, in addition to the shortcomings of the procedure the assumptions of dynamic forces and the centroid of dynamic increment in the pressure add up to the ambiguity of the problem. However, some model tests [Y. Ishii, H. Arai & H. Tsuchida, (1960), T.V.A., (1950)] have shown that the magnitude of dynamic increment can be obtained with reasonable accuracy from "pseudostatic" methods. Therefore, the immediate problem is the determination of the point of application of this increment which will be dependent of the type of wall deformation. There is a lack of information in this regard and here an attempt is made to determine the point of application of the dynamic increment on flexible walls by model tests.

The importance of the problem will be apparent on examination of the design of massive retaining walls in seismic zones. To quote a few, the retaining walls at Farakka and Beas in India will be as much as 27.9 and 25.0 m high. The designs are based on a conservative assumption of the centroid of dynamic earth pressures as per the code of practice (IS : 1893-1970) and any experimental evidence as to the most probable

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This paper was received on 9 August 1973. It is open for discussion up to January 1975.

pressure distribution will not only rationalize the design considerably but will affect the economy of construction as well.

Important studies on dynamic earth pressures behind retaining walls have been carried out by Mononobe-Okabe (1929); L.S. Jakobsen (T.V.A. 1950); Ishii, Arai and Tsuchida (1960); Matsuo and Ohara (1960) and more recently by Prakash and Basavanna, (1969). The first and the last are purely analytical studies being modifications of Coulomb's theory where an inertia force is taken in addition to static forces. Experimental studies so far conducted are limited to rigid walls.

Considering high retaining walls for an illustration, the strain in the backfill is most likely to be due to flexural bending of the wall because of the possibility of extremely rigid foundations. However, any strain in the foundation soil will result in the rotation and translation of the wall as a body. If the foundation is not very rigid, the deformation due to rotation is likely to obliterate any deformation due to bending.

In a model study for determining the effect of wall deformations on dynamic earth pressures, it is not advisable to introduce flexural bending deformation as well as movement of the wall on a body simultaneously. The solution to this problem is to conduct separate tests on two walls—one having bending deformations only and the other undergoing deformations as a rigid body. In this paper the results of tests on a flexible wall are reported. Some data from the tests on rigid walls are also reported to emphasize some points regarding the behaviour of flexible walls.

Test Set-up

Test Bin

A large bin 5.2 m × 2.8 m × 1.2 m high mounted on a shaking table which can be set into motion by the impact of a pendulum is available at the school. The size allows a fairly large wall to be used for the tests thereby reducing the possibilities of errors inherent in small sized apparatus.

The Wall

A high cantilever wall on rigid foundation can best be represented in a model by metal wall rigidly fixed to the base and having sufficient thickness to permit comparable deflections, so as to induce strains of similar order in the backfill.

Thus it is clear that the problem in modelling the wall is that of obtaining comparable deflections. Since the walls are designed for active earth pressures, comparable deflections in the model and the prototype can be obtained by considering the deformations required for the development of active conditions.

Terzaghi has given rough quantitative values of amounts of yield needed for the two types of active cases (arching active and totally active) in the case of one typical dense sand [S. Prakash and P. Nandakumaran, (1969)]. These are :

(1) If the mid-height point of the wall moves outward a distance roughly equal to 1/20 of 1 percent of the wall height, an arching active case is attained.

(2) If the top of the wall moves outward an amount roughly equal to 1/2 of 1 percent of the wall height, the totally active case is attained.

The above values are valid only for rigid walls. But, a reasonable modification of these values for flexible walls seems to be the best way of obtaining the thickness of the model wall. Accordingly it was arbitrarily assumed that the deformation of the mid-height of the wall be 1/4 of 1 percent of the height of the wall for active conditions. This assumption is discussed later in this paper.

To have a fairly large height to length ratio of the wall, a wall height of 1.0 m was adopted. The thickness of this cantilever for deflections of 1/400 times the height at mid-height was found out as 1.0 cm. For this computation, the load on the wall was taken as a linearly varying load computed from Rankine's theory.

Pressure Measurements

Eight pressure cells were used to measure the pressures in static as well as dynamic conditions. Since commercially available pressure cells were unsuitable in this study for various reasons a new type of earth pressure cell was developed, Figure 1. The sensing diaphragm is a beam spring and the strains are measured by employing a wire resistance strain gauge. Details of this pressure cell are given elsewhere [P. Nandakumaran

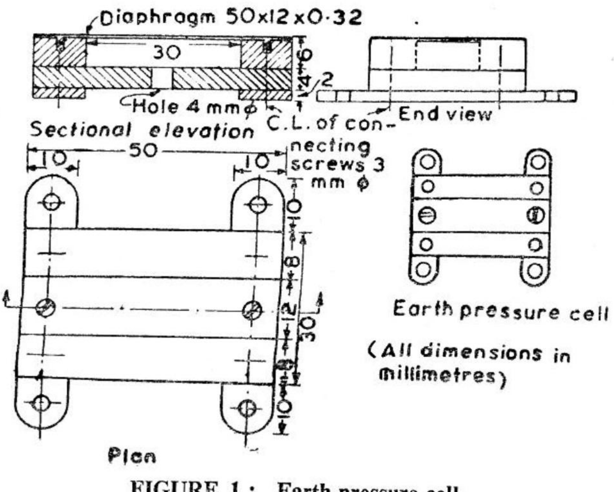


FIGURE 1 : Earth pressure cell.

and H.C. Dhiman, (1970)]. Eight of such pressure cells were housed in rectangular holes cut in three lines in the middle section of the wall. They were kept at different elevations ranging from 15 cm to 85 cm from the top at 10 cm interval and in such a way that the diaphragms of the cells were flush with the face of the wall.

To check the pressures measured using the pressure cells the bending moments on the wall were computed from strain measurements and were compared with the bending moments computed from the observed earth pressure diagram. For this purpose eight wire resistance strain gauges were pasted directly on the outside of the wall at the same elevations as the pressure cells. To avoid the possible discrepancies in assuming the fixity at the base and the effect of cutting holes the wall was calibrated in position. A known load, applied at three points so as to be fairly uniformly distributed along the length of the wall, was applied at the top of the wall for calibration, Figure 2. The relation between the strains and the bending moments for all the strain gauges was found to be linear and identical, Figure 3.

As a further check on the measured pressures the deflection pattern of the wall was measured using five dial gauges and this was compared with the deflection diagram computed from the measured pressures.

For the measurement of dynamic earth pressures only pressure cells were used and the signals from them were amplified by means of a Brush Universal Amplifier and recorded on a self-writing oscillograph.

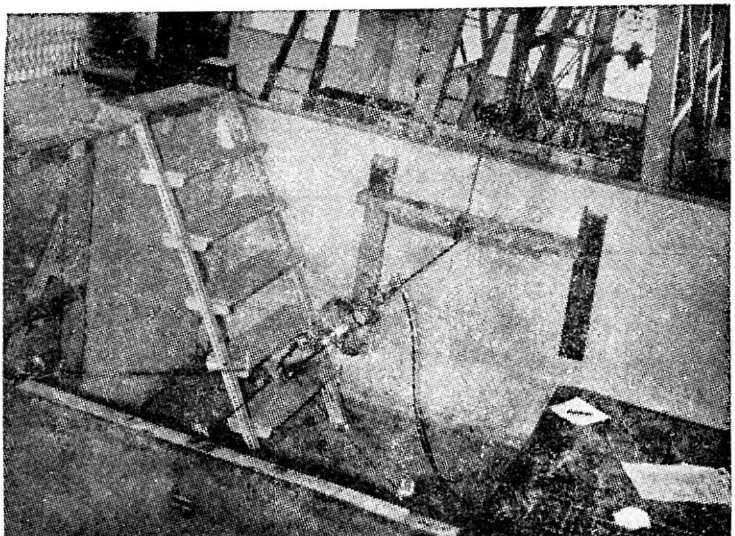


FIGURE 2 : Test set-up for calibration of model wall.

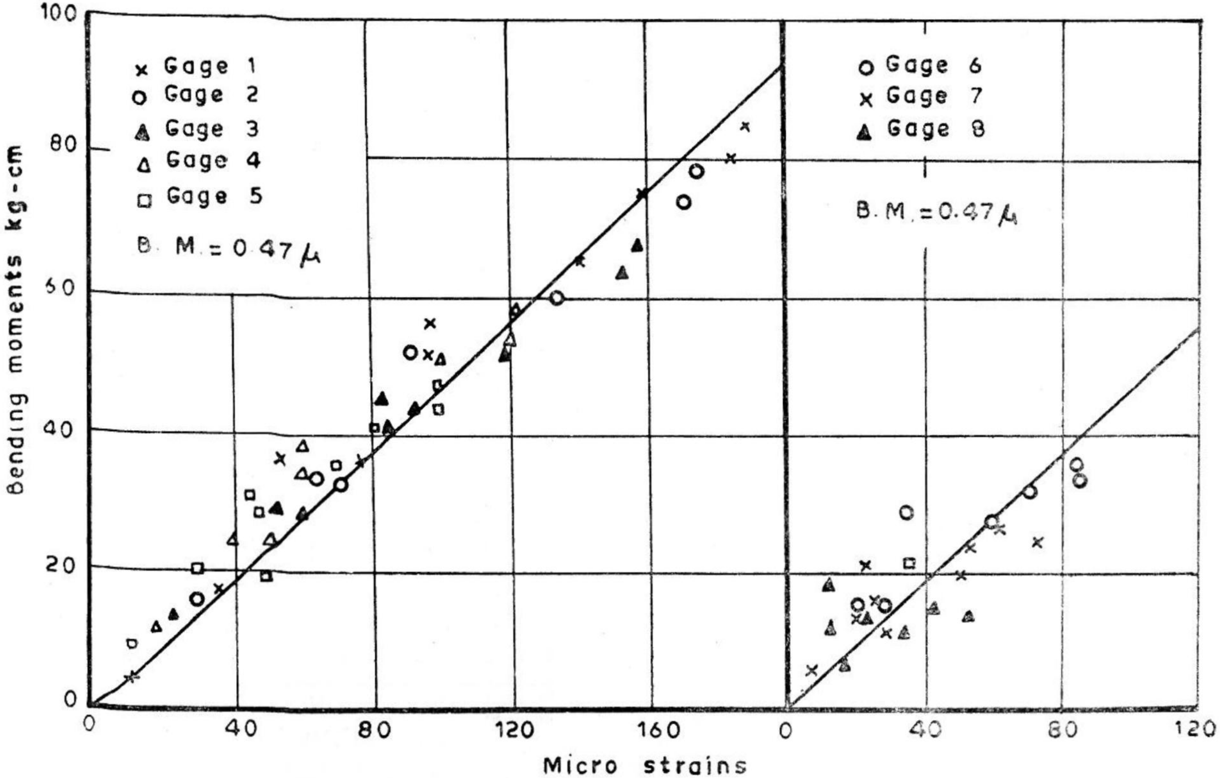


FIGURE 3 : Calibration of wall bending moment versus strain.

Soil Used

Air dried, clean Ranipur sand was used in these experiments. The salient properties of the sand are as given below :

(*a*) Soil type : SP (Poorly graded sands, little or no fines according to Indian Standards Classification).

(*b*) Uniformity coefficient=2.10

(*c*) Effective size D_{10}=0.13 mm

(*d*) Specific gravity of solids S_s=2.66

(*e*) Relative density at the test condition=56 percent

(*f*) Grain size distribution of this sand is shown in Figure 4.

The minimum void ratio of the sand (0.575) was determined by horizontal vibrations of the sand kept in submerged state inside a standard proctor mould fixed on a shaking platform while the maximum void ratio (0.86) was obtained by dropping small quantities of sand in water kept in a measuring jar.

Ranipur sand has fairly high shearing resistance. The angle of internal friction is 38.5° at relative density of 31.5 percent and 42° at a relative density of 70.25 percent [J. Narain, S. Saran and P. Nandakumaran, (1969)]. The value of the angle of internal friction in the test condition was 40 percent.

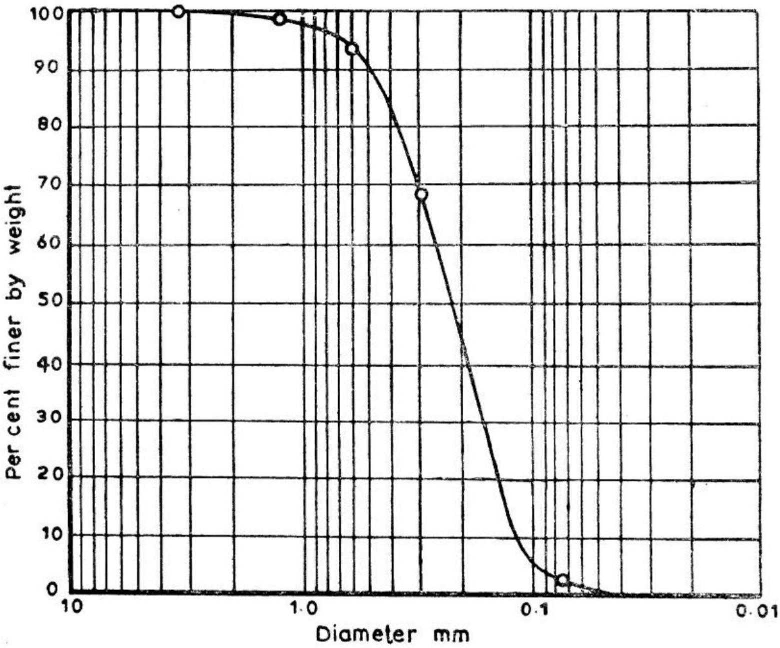


FIGURE 4 : Grain size distribution curve.

Sand Placements

Sand was placed in the tank in 10 cm layers. Each layer was compacted by six blows of a wooden mallet on a wooden plank 90.0 cm $\times$ 30.0 cm placed on the surface of the sand. Five calibrated thins were kept during the filling of each layer and the density was determined after each layer was completed. It was observed that the method adopted gives the same reproducible densities throughout the deposit.

Test Procedure and Test Results

The zero readings of all the eight pressure cells, all the eight strain gauges and the five dial gauges were taken and then the wall was backfilled with the sand. When the backfilling was complete, the final readings of all the cells, strain gauges and the dial gauges were taken. The differences between the final readings and the initial readings furnished the data to compute the earth pressures, bending moments and deflections at various elevations of the wall.

Lines of dyed Ranipur sand incorporated in the backfill during the filling of sand were used to observe the development of rupture in the soil behind the wall.

Just before shock loading of the table the dial gauges were removed. The table was then set into motion by a single impact of a pendulum. Simultaneous records of the acceleration of the table, acceleration of the wall and increase in earth pressure at one elevation were made during the shock. By imparting identical shocks to the table eight times, the increase in pressures at all the elevations and hence the pressure distribution diagram was obtained.

The earth pressure obtained from tests in which four different table accelerations were employed are given in Table I. In Figure 5, the comparison of computed and observed bending moments and the comparison of measured and computed wall deflections are shown for Test No. 4 along with the measured earth pressure. Similar results were obtained for all tests [S. Prakash and P. Nandakumaran, (1969)]. All the particulars and results of tests are listed in Table I.

TABLE I

Particulars of Test Data.

Test series No.	Total static pressure g/cm of wall	Static E. P. coeffi-cients	Point of application above base cm	Acceleration in Test (Peak) g	Total dynamic increment g/cm of wall	Point of appli-cation base cm	Dynamic increment static pressure
1	2627.0	0.3343	37.60	4.29	1961.0	54.65	0.750
2	2752.5	0.3520	36.00	3.32	1659.5	50.30	0.604
3	2600.0	0.3322	34.25	3.34	1680.0	50.05	0.646
4	2697.0	0.3392	36.00	4.55	2177.0	48.30	0.807

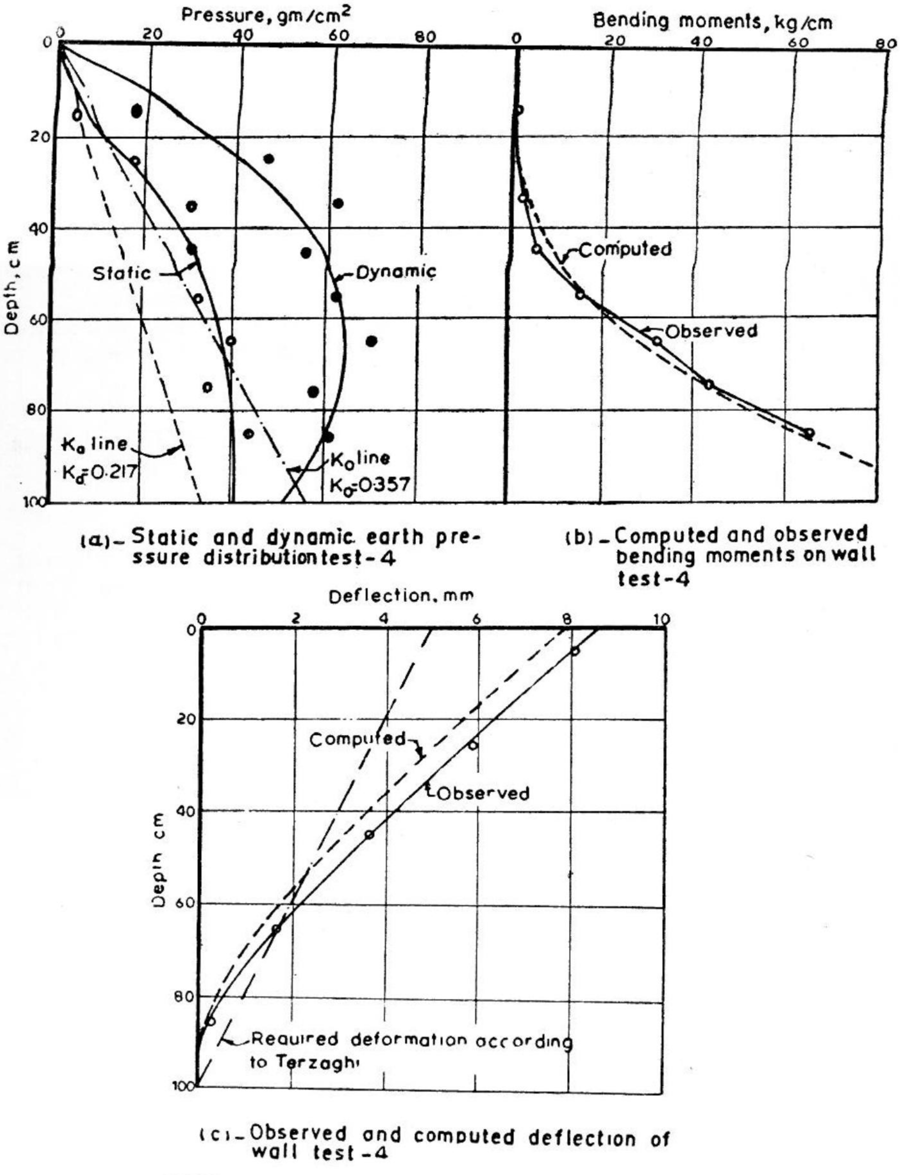


FIGURE 5 (a, b & c) : Test results—Test No. 4.

Discussion of Results

Static Pressures

The distribution of pressures in static conditions in one of the tests is shown in Figure 5 (*a*). It is seen that the pressure distribution is not hydrostatic but is curved with a maximum value of pressure at the base.

In Figure 5 (*a*), Coulomb's active earth pressures K_a and Jaky's K_o lines have also been shown. It will be seen that the pressures are more or less close to the Jaky's at-rest pressures. In Figure 5 (*c*) is plotted Terzaghi's deformation line required to produce active conditions behind a wall. The total deformation at the top of the wall and for a considerable depth below is more than that required to cause active conditions behind it according to the postulation by Terzaghi for rigid walls. This apparent discrepancy between the pressures corresponding to the deformation conditions existing in the model wall may be explained as follows :—

(*i*) Terzaghi had visualized the existence of a wall along with backfill without any deformation of the wall corresponding to 'at-rest' condition. Now, if the wall and backfill are deformed simultaneously, the deformations postulated by him should be sufficient to develop 'active' conditions resulting in subsequent reduction in pressures on the wall.

In practice, the 'at-rest' conditions cannot exist behind flexible retaining walls, backfilling is started after the wall has been constructed, resulting in deformations of the wall both below and above the level of the backfill. Thus the deformation of the backfill at any level does not equal the total deformation of the wall, the discrepancy being maximum at the top and zero at its bottom. It is, therefore obvious that actual deformations of the backfill may not be enough to cause active conditions although the wall has deflected by sufficient amount.

(*ii*) In a recent study of the cause of damage found on bridge abutments, Siedek (1969) has arrived at the conclusion from actual field measurements that at-rest earth pressures are acting on retaining walls founded on rock. This fact clearly shows that the results obtained in the comparatively small retaining wall in the laboratory can directly be used in field without any fear of scaling effects. The scaling effects are absent because the strains in the backfill in the laboratory model and the field are similar both in nature and in extent.

(*iii*) Results from tests on a rigid retaining wall model 1.0 m high and backfilled with the same sand at the same properties are shown in Figures 6 and 7. The 'at-rest' pressures measured [Figure 6 (*a*)] after the backfilling is complete show almost linear increase with depth and are almost equal to the values given by the equation $K_o = (1 - \sin \phi)$ except probably along the mid-height of the wall. This could be because of some experimental errors. The at-rest pressures reduced, on the wall being rotated about the toe and the resulting values [Figure 6(*b*)] are almost equal to Rankine's active earth pressure values. Probably this may be explained by the possibility of very small amount of friction between the polished steel face of the wall and the backfill. For attaining the active earth pressures, the wall had to be displaced to the tune of 0.4 to 0.5 percent of the wall height at its top. It will be noted that this agrees favourably with the values given by Terzaghi. Along with the development of active conditions a clear rupture surface also was observed, though the wedge thus formed was smaller than those from either Rankine's or Coulomb's theories—Figure 7. Such a trend was observed in an earlier study also in a smaller model [P. Nandakumaran, (1967)].

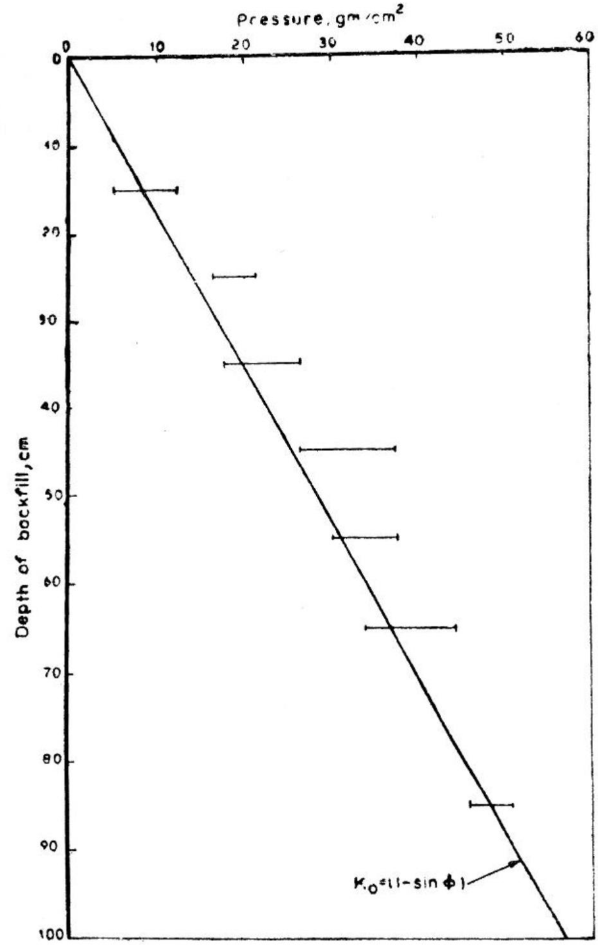


FIGURE 6 (a): At-rest pressure distribution on a rigid retaining wall.

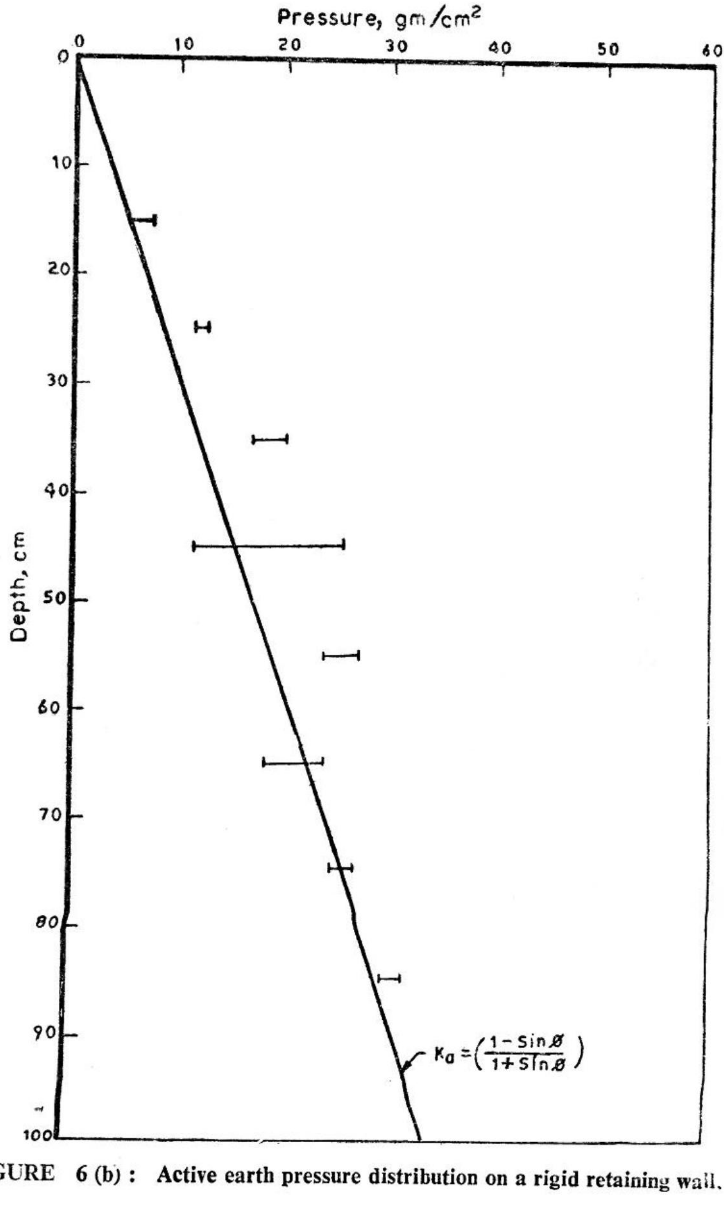


FIGURE 6 (b) : Active earth pressure distribution on a rigid retaining wall.

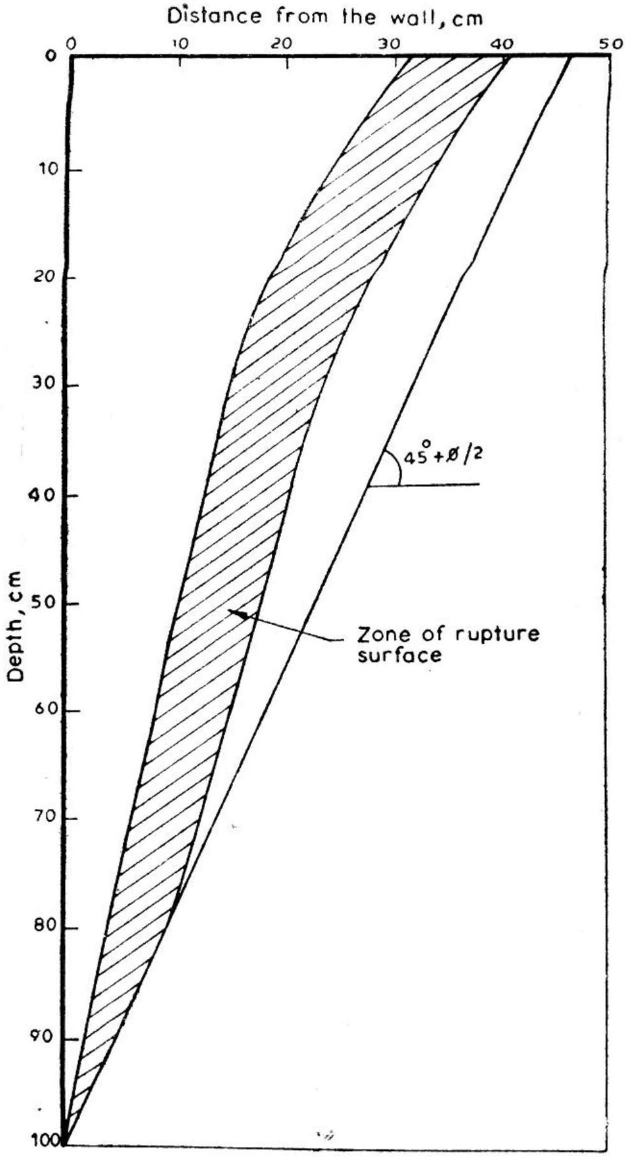


FIGURE 7 : Zone of rupture surfaces (rupture wedge formed behind the rigid retaining wall).

The above observations compare and contrast the behaviour of a 'rigid' retaining wall on very rigid foundations. From these tests and from the observations of damaged bridge abutments by Siedek (1969) it becomes apparent that the case of rigid foundations calls for additional care and that the designs must be based on 'at-rest pressures' rather than the 'active' pressures. By adopting a similar procedure for backfilling, the laboratory tests have also been utilised to highlight the importance of the deformation in the backfill irrespective of the deformation of the wall.

For dynamic tests, shock loading was used after this stage of the test.

Bending Moments

From the static pressure distribution curve, the bending moments at various elevations of the wall can be computed.

The bending moments computed assuming it to be a cantilever beam and observed (as computed from the strain gauge readings) are shown in Figure 5 (*b*) for Test No. 4. For other tests, similar results were obtained. The difference between the computed and measured values of bending moments, is negligible in all cases. Thus the pressures measured on pressure cells are reliable.

Deflections of the Wall

The actually measured deflection diagram of the wall is only slightly different from the computed deflections of the wall from the pressure distribution diagram as seen for Test No. 4 in Figure 5 (*c*). Similar data was obtained in Test No. 3. Thus the pressure measured are accurate and the pressure cells are dependable for earth pressure measurements.

Dynamic Pressures

The initial conditions for dynamic tests are not active conditions but some state between 'at-rest' and active states. As described earlier this magnitude is nearly equal to the 'at-rest' pressure postulated by Jaky. Because of the similarity in nature of wall deformations and backfilling procedures, these initial conditions will hold good for a cantilever retaining wall on rigid foundations. The impact loads applied on the wall had to have higher magnitude of accelerations than generally considered suitable because of the short duration of the load. As is well understood, the magnitude of earth pressures is inevitably a function of the strain in the backfill, which in turn, is dependent on the movement of the wall. Therefore, for a realistic loading the magnitude as well as the duration of the dynamic loads become the two most important criteria to be adopted. These criteria can be taken care of by considering the spectral acceleration as well as the damage potential (the area of the accelerogram above the 'yield acceleration') of the applied load. The spectral acceleration values are evidently much larger in this case than the values usually required but the damage potential values are very much comparable to that of an actual earthquake and normal values of yield accelerations. Therefore, the accelerations reported are only the maximum peak values and in the view of identical (generally) duration of the loads have been used for the sake of comparison. Thus, though the magnitude of pressures might well be difficult to compare with those from usually available methods, the qualitative nature of the pressure distribution diagram is very much realistic. The paper intends to highlight the nature of the pressure distribution diagram itself rather than the magnitude of the pressures, the latter being most commonly obtained from the modified version of Coulomb's formula.

During the shock loading, the dynamic increment (the increase in pressure due to the shock) along the height was measured. As described later under the sub-heading rupture surfaces in the paper, any rupture did not develop in the backfill though the wall moved out, thereby indicating that the pressures *did not drop to active values during any part of the* test. Therefore, it is reasonable to assume that the dynamic increment is the result of the soil-structure interaction during the shocks and hence may be valid even if the initial conditions were to be different, viz., the state of active equilibrium behind the wall.

Another point of interest was the gradual outward movement of the wall. This can be explained in terms of the different magnitudes of resistance to motion of the wall towards and away from the backfill. The former consists of the stiffness of the wall and passive pressure from the backfill while the latter is much smaller and is only the stiffness of the wall minus the active earth pressure on the wall. This may be sufficient reason to assume that the wall does not vibrate and so the pressures do not get magnified on account of possible resonance during earthquakes. Because of the above two reasons, the dynamic increment distribution is realistic for design purposes.

The dynamic earth pressure distribution in the test No. 4 is shown in Figure 5 (*a*). The dynamic increment per unit of acceleration in all the tests is plotted in Figure 8. It will be seen from this figure that the point of application of the dynamic increment in pressure is independent of the acceleration and also that the magnitude of the dynamic increment is directly proportional to the acceleration.

Point of Application of Earth Pressures

The point of application of static pressures varies from 0.3425 to 0.0376 times the height of wall and therefore, this, along with the distribution diagram indicates that a hydrostatic pressure distribution for cantilever walls with rigid base may not be realistic.

The dynamic increment has been found to have a centroid varying between 0.483 to 0.5465 times the height of wall. This range is well within experimental errors. However, it is to be stressed that the point of application of dynamic increment is well below 0.66 H as is recommended by Indian Standards Institution (IS : 1893-1970).

For cantilever walls, a point of application of 0.55 H for dynamic increment can be taken with confidence.

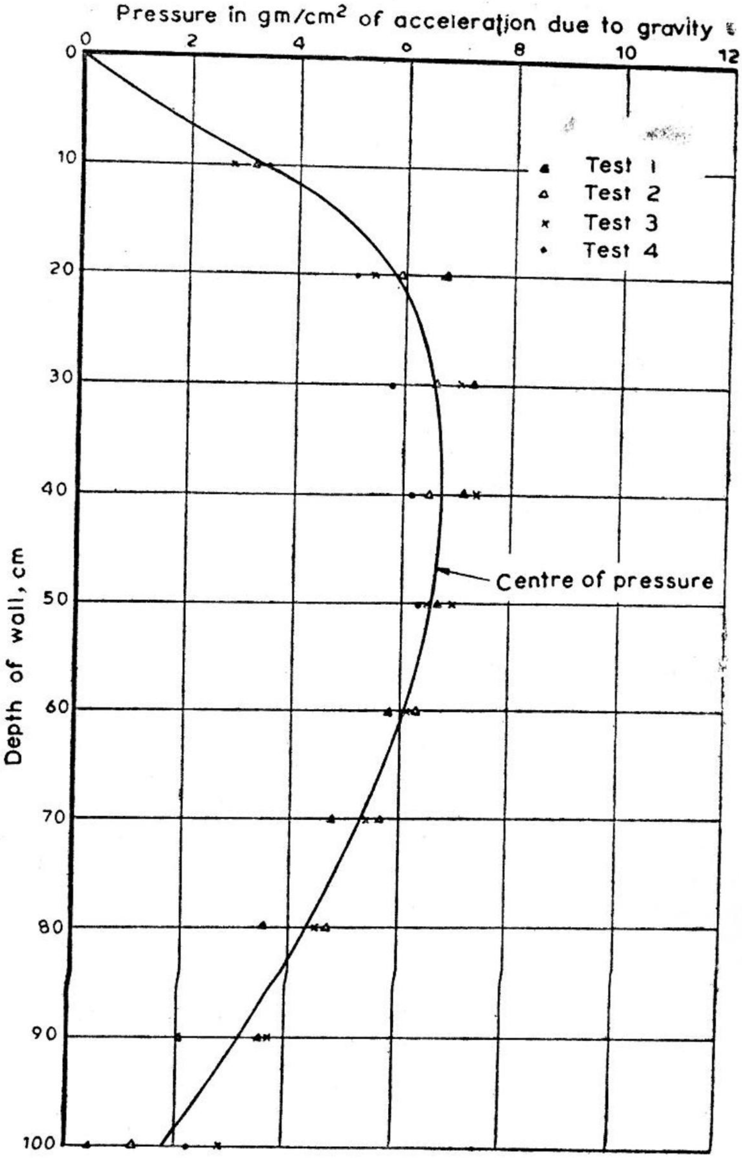


FIGURE 8 : **Distribution diagram of dynamic increment per unit of acceleration due to gravity with height of wall.**

Rupture Surfaces

Due to the inadequate displacement of the wall as described previously, no rupture surface developed before the shock loading. But the backfill was compacted and hence the sand surface settled due to the vibrations caused by the shocks. In spite of the outward movement of the wall as already described no rupture surface developed in this case also probably because the lateral strains caused due to the movement of the wall along with the strains connected with compaction of the backfill were insufficient for the formation of a wedge.

Effect of Acceleration on Dynamic Earth Pressures

The observed values of total dynamic increment, the ratio of dynamic increment and static pressure, and the point of application of dynamic increment are plotted against peak acceleration employed in different tests in Figure 9. The total dynamic increment and the ratio of dynamic increment and the static pressure have been found to have a straight line relation with acceleration as the curve has to pass through the origin, while the point of application of dynamic increment is independent of acceleration in the tests. It may be pointed out that these relations are purely based on the experimental data as the best possible interpretation. These findings justify to some extent the use of equivalent static methods for determining dynamic earth pressures. The results from this study cannot be directly compared with the data from previous investigations because of two reasons :

(*i*) No data is available on the dynamic earth pressures on flexible retaining walls.

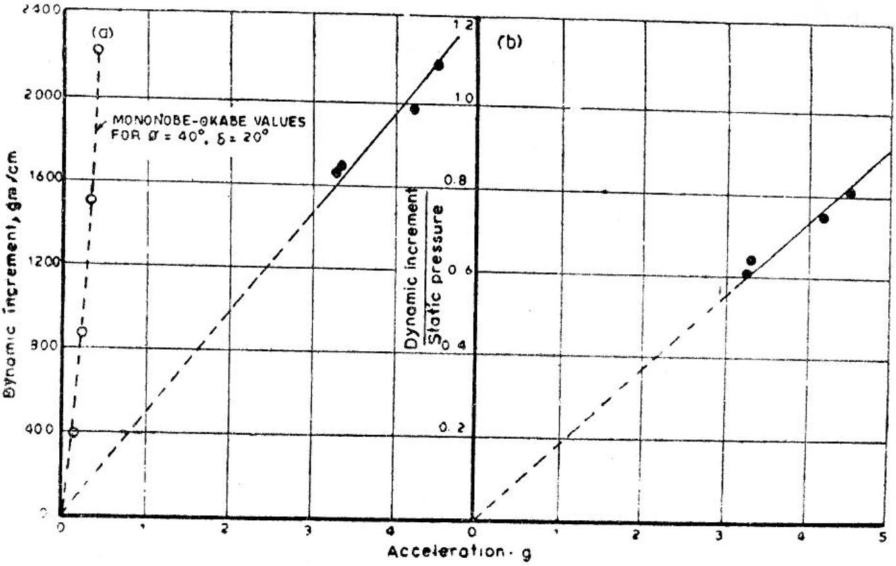
(*ii*) The acceleration level used in this study is too large compared to the acceleration employed by other workers.

However, the interpolated data from the present study in the usual working range indicates that the magnitude of dynamic increment is smaller than those computed from Mononobe's formula [Figure 9 (*b*)].

Conclusions

(1) The use of Coulomb's theory for the design cf cantilever walls resting on rigid foundations is questionable because the lack of rotation of the wall prevents the backfill from attaining active equilibrium conditions. The high factor of safety used in the design may prevent any cracking of the wall with the result that the earth pressure acting on the wall is always likely to be equal to the at-rest pressures. Therefore a more rational method of the design of such walls will be to use 'at-rest' pressures.

(2) The dynamic increment in earth pressures can be taken as having a point of application of 0.55 times the height of wall above the base, for such walls. This value is independent of the magnitude of the acceleration applied.



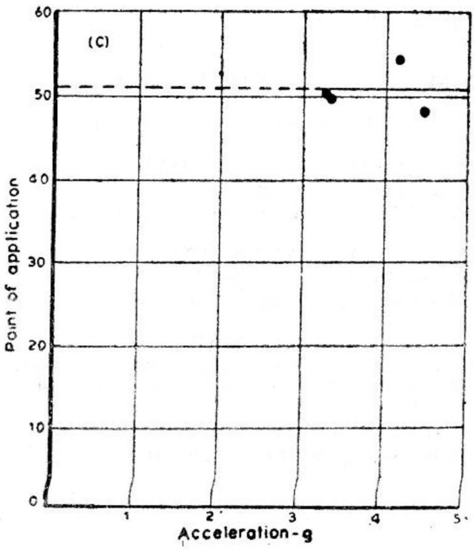


FIGURE 9 : Effect of acceleration on dynamic earth pressure.

(3) No rupture surface develops in the backfill behind a cantilever wall on rigid foundations even during dynamic loading though the wall moves outward and the backfill settles. This means that the soil does not get into an active state and remains 'at-rest'.

Acknowledgements

The investigation reported here was sponsored by Beas Dam Project. The wall and pressure cells were fabricated under the supervision of Shri H.C. Dhiman. All help is acknowledged.

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