A Method for the Evaluation of the Shear Modulus of Soil for Machine Foundations

by
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Introduction

FOR many years foundations for machines have been sized using long standing rules of thumb and personal experience of the designer, usually resulting in satisfactory installations. But the need for a rational method of designing machine foundations has existed for a long time. The recent soil dynamics approaches permit the execution of such analysis economically and without recourse to computers and give more confidence to the engineer.

Numerous experiments have shown that the behaviour of a vibrating soil-mass system is dependent on the soil characteristics and that the resonant frequency and the amplitude of motion are the factors required for the satisfactory evaluation of design. Briefly, the analysis requires the machine foundation and soil profile information shown in Figure 1.

Engineering solutions for the dynamically loaded foundations are based on the half-space theory which can be represented by the lumped parameter system, Richart et al (1968). For a single degree of freedom system, the lumped parameters can be described by the equation of motion.

$$m\ddot{z}+C\dot{z}+Kz=Q(t) \quad \ldots(1)$$

where,
m = the mass
C = the damping constant
K = the spring constant
$Q(t)$ = the time dependent exciting force and z, $\dot{z}$ and $\ddot{z}$ are displacement, velocity and acceleration of the mass.

The Spring Constant of the Soil

The spring constant is the most important parameter involved in the lumped parameter system. Its value affects the resonance and other frequencies. It is generally said that the estimation of a dynamics response can be no better than the estimation of the spring constant.

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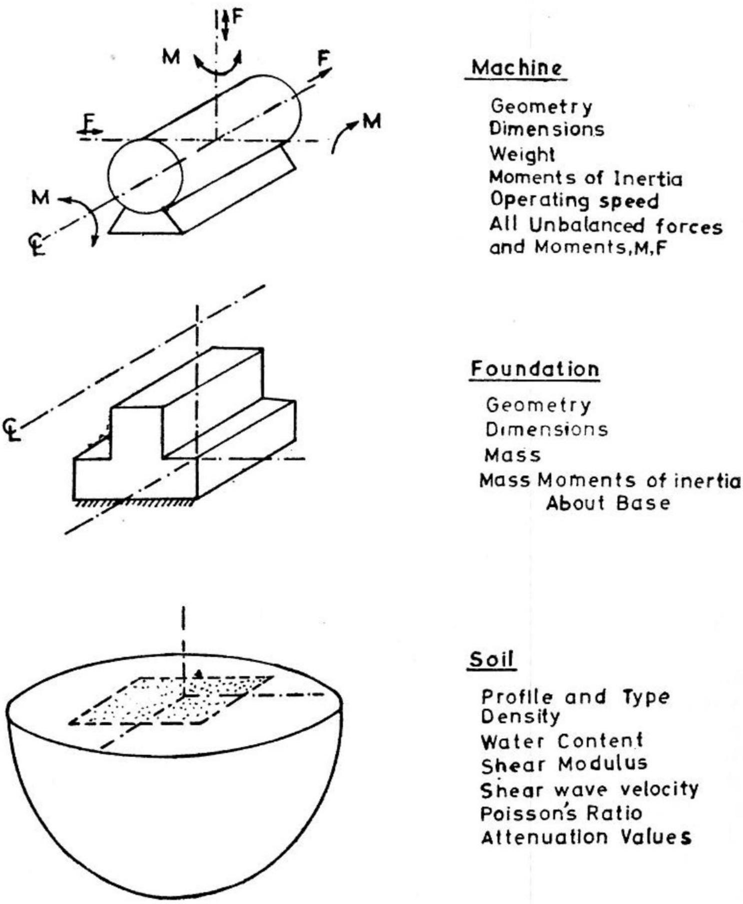

FIGURE 1.

The spring constant represents a linear relation between applied load and displacement of the foundation which in turn implies a linear stress-strain relation for the soil. The evaluation of spring constant is, therefore, based on the theory of elasticity relating the spring constant to the basic stress-strain behaviour of the soil.

The spring constant can be computed using the resonant frequency of the soil obtained by placing a small vibrator on the surface of the soil and observing resonance. This method, however, involves certain uncertainties. First is the conversion of resonant frequency to the spring constant. Second is the non-linear effect of the soil on the resonant frequency itself. Also, as small plates are usually used for the tests, it is difficult to keep the acceleration to less than 0.5 g and the correlation of the spring constant so determined for the model to that for the prototype conditions becomes rather complicated.

Another reasonable approach is by using the shear modulus of soil which can be determined by employing the velocity of propagation of surface waves in the soil (IS : 5249—1969). Much care is, however, required in the determination of the soil shear modulus for the response frequency is proportional to the square root of the shear modulus and the amplitude is closely related to the shear modulus (Margason et al, 1968).

Auxiliary Equipment

The auxiliary equipment used consisted of an electromagnetic vibrator, input and output amplifiers, an oscillator, a vibration pick-up, an accelerometer and a dual channel oscilloscope.

The circuit set-up of the equipment is shown in Figure 2.

Vibratory Test Technique

The vibrator was placed directly on the soil and set into vertical sinusoidal motion at a selected frequency. Care was taken to ensure that the contact between the base of the vibrator and the soil was perfect as any mismatch will lead to contact compliance resulting in an increase in static stress even for the same plate. Robsen (1956) and others used a thin layer of Plaster of Paris between the base plate of the vibrator and the soil. Here in the question arises as to whether the layer of Plaster of Paris acts with the soil or with the vibrator. In the present work, however, for simplicity, Plaster of Paris was omitted.

A pit was excavated to seat the vibrator and a horizontal trench leading away from the vibrator was dug for about 12 m to locate in it a tape measure for wave length measurements. The vibrator was set in operation and a measuring tape was extended in the trench leading away from the vibrator. The pick-up was then pushed into the ground surface as near the vibrator as possible and the peaks of the sine waves from the pick-up and the vibration generator were superimposed on a dual beam oscilloscope. The pick-up was then moved further until peaks coincided again. The distance between these points was taken as equal to one wave length. This procedure was continued until three to five wave lengths had been traversed. Maxwell and Fry (1968) recommend four to eight wave lengths as sufficient for computations. The product of the wave length times the frequency of excitation is equal to the velocity of Rayleigh wave, or

$$V_R = \lambda . f. \qquad \ldots(2)$$

where, V_R = the velocity of Rayleigh wave

λ=the wave length

f=frequency of excitation.

The velocity of the Rayleigh wave can be taken equal to the velocity of the shear wave and hence the velocity so computed gives the velocity of the shear wave from which the shear modulus of the soil can be computed using the relationship :

$$G=\rho . V_s^2 \qquad \ldots(3)$$

where,

G=the shear modulus of soil

ρ=the density of soil

V_s=the shear wave velocity.

The frequency of the vibrator was then changed to another selected frequency and the procedure repeated.

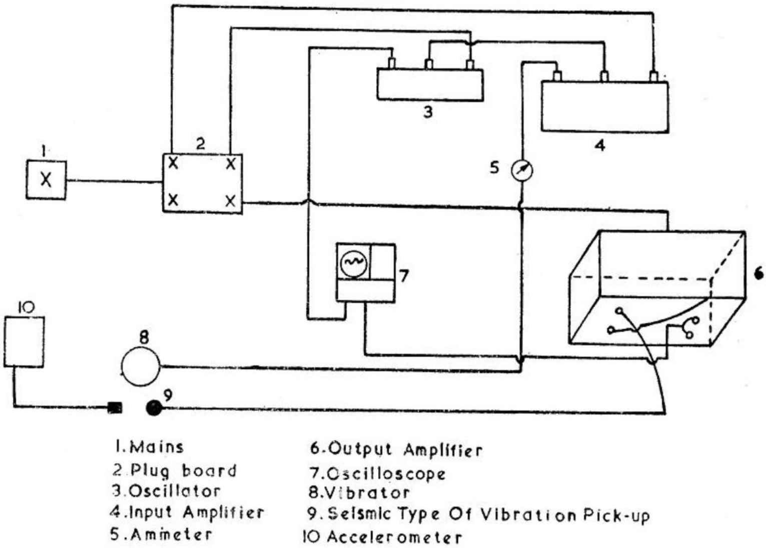

FIGURE 2.

The choice of range of frequency depends upon the information required. The asymptotic velocity given by vibrations of relatively short length, i.e., at high frequencies is identified as the Rayleigh wave velocity in the top layer of soil and it is from this velocity that the shear modulus of the soil is calculated (Jones, 1958). Average results at a number of frequencies provide the dispersion curve, i.e., the relation between velocity and wave length from which the elastic properties of the soil are computed.

Computation of Shear Wave Velocity

The data from the vibratory test described above is plotted as distance between peaks against the number of wave lengths (Figure 3). Each of the lines on the figure represent the data for one frequency. The wave length for a particular frequency is determined as the reciprocal of the slope of the line through the plotted data points. The velocity of the shear wave is then determined by Equation (2).

By decreasing the frequency of the vibrator, the wave length increases and the R-wave effectively samples a greater depth. Conversely, by increasing the frequency, the wave length decreases and the sampled depth decreases. For an elastic half-space in which the elastic properties change with depth. V_R varies with the frequency of excitation, resulting in different wave lengths. Figure 4 shows the variation of wave length with frequency of excitation. The different wave lengths obtained for different elastic frequencies effectively sample material with different elastic properties. It is possible, therefore, to obtain valuable information on the elastic properties of a half-space from steady state vibrations at the surface.

The steady state vibration method requires plotting of the wave velocity as computed above at a depth corresponding to one-half wave length. This is equivalent to a shear wave velocity versus depth profile. This type of plot is shown in Figure 5. Such a plot provides a visual picture of the change in soil characteristics with increasing depth.

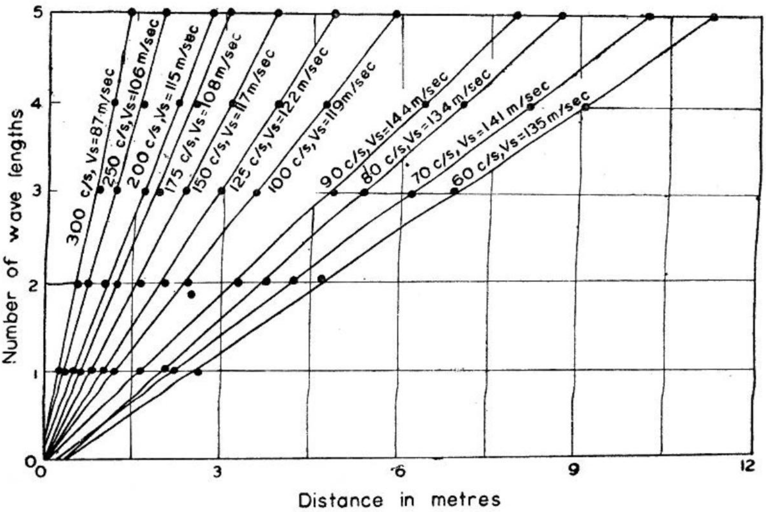

FIGURE 3.

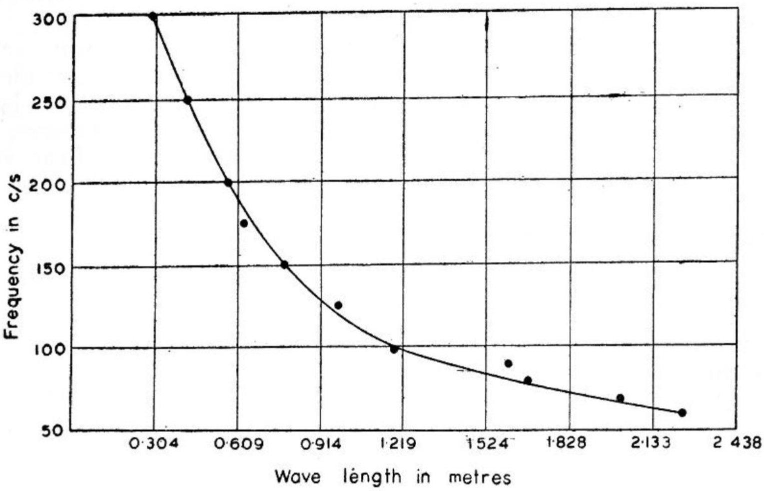

FIGURE 4.

The shear modulus plotted against frequency in Figure 6, shows that the moduli increases with decrease in frequency. In case of subsurface materials consisting only of soils, a radical or abrupt change in shear modulus with depth would not be expected although some general increase in modulus often occurs because of increasing density and stress level with increasing depth. The general shape of the modulus-depth curve shown in Figure 7 provides visual picture of the change in soil characteristics with increasing depth. The rise in phase velocities and hence the shear modulus at low frequencies indicate that below the upper layer of soil, there is a medium of relatively higher rigidity. This is also evident from depth versus wave velocity profile (Figure 5). This was confirmed at the site where rock was revealed by a nearby railway cutting.

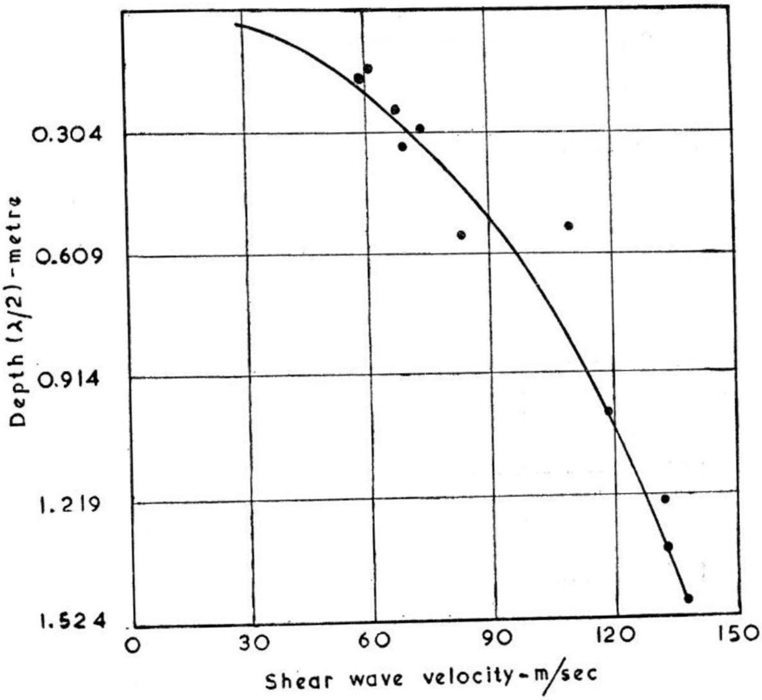

FIGURE 5.

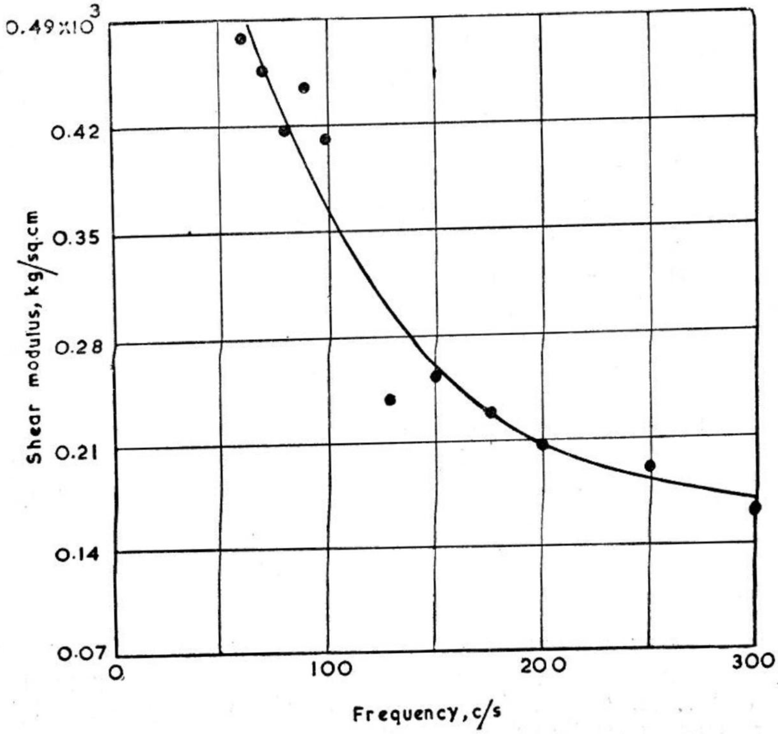

FIGURE 6.

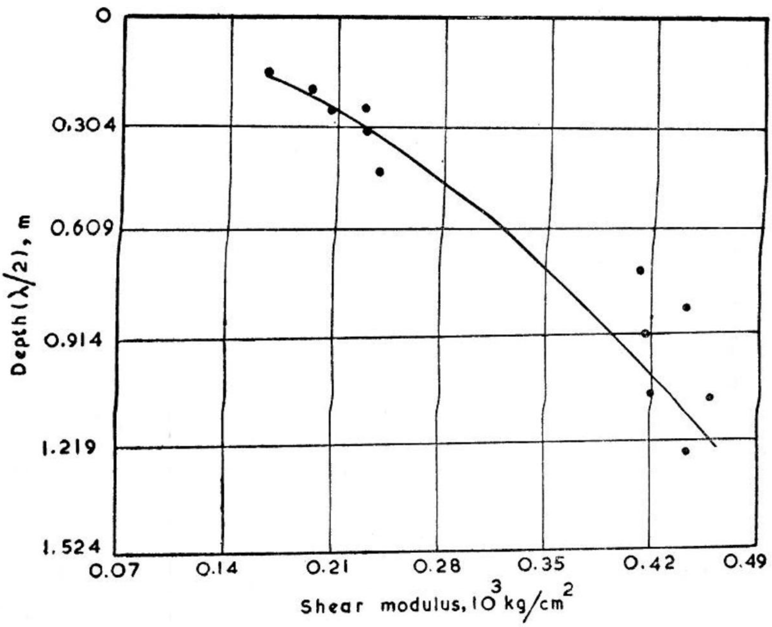

FIGURE 7.

Conclusion

The steady state vibration technique using an electro-magnetic vibrator enables an accurate measurement to be made of the shear modulus of the soil. The method is also applied in situ as the method prescribed by the Indian Standards but differs in the way the resulting wave lengths are measured. In the Indian Standards method, the lassajous figures from the two geophones are made circular on the oscilloscope whereas for the proposed method the sine curves from the vibration generator and the pick-up are superimposed. For the Indian Standards method of test, a concrete block is needed but for the proposed method no such block is necessary. However, steel plates of various sizes can be bolted to the vibration generator to increase the contact pressure on the soil. Effect of contact pressure on the shear wave velocity and the frequency response of the vibration generator are described elsewhere (Farooqui, 1969).

The dispersion curve obtained from the plot of the frequency versus wave length provides a visual picture of the changes in soil characteristics with increasing depth. It thus effectively samples a sufficient length of the material and gives an insight into the elastic properties of the half space.

Acknowledgement

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Notations

The following symbols are used in this paper :—

C = the damping constant,
f = frequency of excitation,
G = the shear modulus of soil,
g = acceleration due to gravity,
k = the spring constant,
m = the mass,
$Q(t)$ = the periodic force,
V_r = velocity of Rayleigh wave,
V_s = velocity of shear wave,
$\ddot{z}$ = the acceleration,
$\dot{z}$ = the velocity,
z = the displacement,
ρ = the density of soil, and
λ = wave length.

References

FAROOQUI, A.T. (1969): "A Study of Dynamic Characteristics of Soils for the Evaluation of Machine Foundations." *Thesis submitted to the University of Bradford*, England for the degree of M. Sc.

INDIAN STANDARDS : 5249-1969 : "*Method of Test for Determination of In Situ Dynamic Properties of Soil.*"

JONES, R. (1958): "In Situ Measurement of the Dynamic Properties of Soil by Vibration Method." *Geotechnique*, Volume 8, pp. 1—21.

MARGASON, B.E.; McNEILL, R.L. and BABCOCK, F.M. (1968): "Case Histories in Foundation Vibrations." *Paper presented at the 71st Annual Meeting of A.S.T.M.*, San Francisco, California, June 1968.

MAXWELL, A.A. and FRY, Z.B. (1967) : "A Procedure for Determining Elastic Moduli of In Situ Soil by Dynamic Techniques." *Proc. Int. Symposium on Wave Propagation and Dynamic Properties of Earth Materials.* University of New Maxico., June, 1967, pp. 913-919.

RICHART, F.E.; HALL, J.R. and WOOD, R.D. (1963): "*Vibrations of Soils and Foundations.*" Department of Civil Engineering, University of Michigan, USA, June 1963,

ROBSON, J.D. (1956): "Effect of Non-linearity on the Resonant Frequency of a Body on Soil." *9th International Congress on Applied Mechanics*, Brussels, pp. 344-349.