Effect of Skin Friction on the Buckling Resistance of an Axially Loaded Pile Embedded in Cohesive Soil

by

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Introduction

PILED foundations *of heavy structures in cohesive soil* overlying deep seated stratum of rock, face many problems of their design. Such foundations often require the use of long piles extended up to the hard rock to support the load. The conventional designs at present consider that the load is fully transferred to the rock through piles. In such cases, the pile is designed as an idealised column, laterally supported by an elastic medium, provided the deflection of its centre line is small. One of the approaches to the solution of the problem is by energy concepts which is due to Timoshenko (1961). The approach was subsequently adopted by Mazindrani and Sastry (1972). In the previous studies, the skin friction offered by the surrounding elastic medium to the surface of the embedded length of the pile was ignored. However, Siva Reddy and Valsangkar (1969) considered the effect of skin friction, using variational approach for an axially and laterally loaded pile and reported that conservative values of deflections and bending moments are obtained if the skin friction is not accounted in the analysis. The Authors therefore, present their study on the effect of skin friction on the buckling resistance of an axially loaded pile in soft cohesive soil, using energy method.

Some Considerations

In the analysis, some considerations in end conditions, skin friction and foundation modulus have been employed.

End Conditions

The end conditions of the pile in practice, range from both ends fixed to both ends hinged. It is possible to compute a factor which can convert the pile with any end conditions into an enquivalent pile with hinged ends. Therefore, in the analysis presented here ; the pile is considered with hinged ends.

Skin Friction

Skin friction along the embedded length of the pile is influenced by many factors, surface area of the pile being one of them. In this paper,

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uniform cross-section of the pile is considered. Hence, for other factors remaining unaltered with depth, the skin friction for the unit embedded length of the pile is constant. The force due to skin friction for the embedded length x of the pile, can then be expressed by Equation (1):

$$F(x) = \frac{P\psi x}{L} \qquad \ldots(1)$$

where, $\frac{P\psi}{L}$ is the force due to skin friction per unit embedded length of the pile.

This equation is represented by Figure 1.

Foundation Modulus

An experimental study made by McClelland and Focht (1956) revealed the fact that the foundation modulus is not constant with depth in cohesive soil. But, the available literature is not enough to arrive at a definite relation of its variation with depth. Hence, on the basis of the work reported by Terzaghi (1955), the foundation modulus is considered to be constant with depth.

Mathematical Analysis

A vertical pile embedded in homogeneous, elastic and cohesive soil is subjected to an axial load P (Figure 2). The deflection curve of the buckled pile satisfying the boundary conditions can be expressed in the form of a trigonometric series as

$$y = \sum_{n=1}^{n=\infty} a_n \sin \frac{n\pi x}{L} \qquad \ldots(2)$$

The work done by an axial load P when the pile buckles, is

$$\triangle T = \frac{P}{2} \int_0^L \left(\frac{dy}{dx}\right)^2 dx$$

$$= \frac{\pi^2 P}{4L} \sum_{n=1}^{n=\infty} n^2 a^2_n \qquad \ldots(3)$$

The strain energy of bending of the buckled pile is

$$\triangle U_1 = \frac{EI}{2} \int_0^L \left(\frac{d^2y}{dx^2}\right)^2 dx$$

$$= \frac{\pi^4 EI}{4L^3} \sum_{n=1}^{n=\infty} n^4 a^2_n \qquad \ldots(4)$$

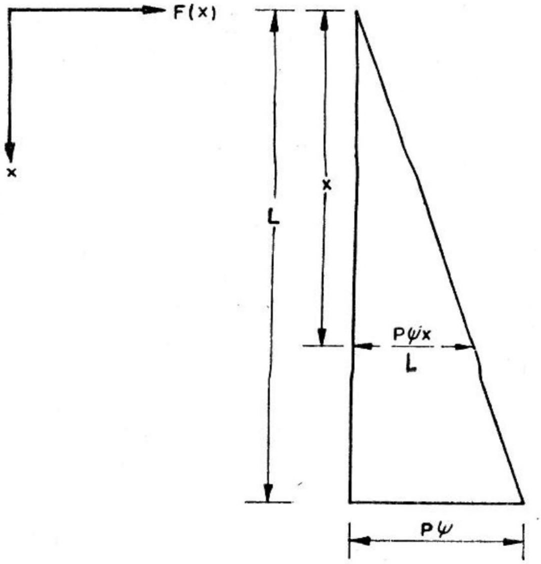

FIGURE 1 : Variation of skin friction with depth.

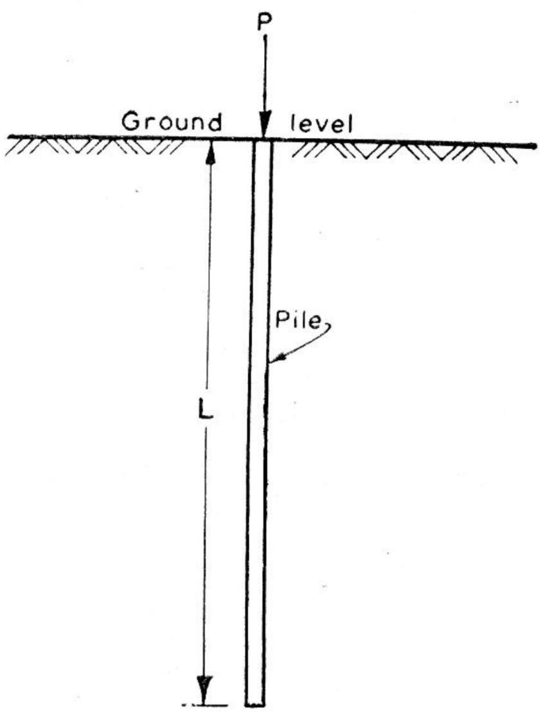

FIGURE 2 : Vertical pile carrying axial load.

The energy of lateral deformation of the soil is

$$\triangle U_2 = \frac{\beta}{2} \int_0^L y^2 \, dx$$

$$= \frac{\beta L}{4} \sum_{n=1}^{n=\infty} a^2{}_n \qquad ...(5)$$

The energy due to skin friction can be derived in the similar way as suggested by Timoshenko and Gere (1961):

$$\triangle U_3 = \frac{1}{2} \int_0^L F(x). \left(\frac{dy}{dx} \right)^2 dx$$

$$= \frac{\pi^2 \psi P}{8L} \sum_{n=1}^{n=\infty} n^2 \, a^2{}_n \qquad ...(6)$$

The condition when the equilibrium configuration changes from stable to unstable, is

$$\triangle T = \triangle U_1 + \triangle U_2 + \triangle U_3 \qquad ...(7)$$

Substituting Equations (3), (4), (5) and (6) in Equation (7), we obtain

$$P = \frac{1}{\left(1 - \frac{\psi}{2}\right)} \left\{ \frac{\pi^2 EI}{L^2} \, \frac{\sum_{n=1}^{n=\infty} n^4 \, a^2{}_n + \frac{\beta L^4}{\pi^4 EI} \sum_{n=1}^{n=\infty} a^2{}_n}{\sum_{n=1}^{n=\infty} n^2 \, a^2{}_n} \right\} \qquad ...(8)$$

The quantity into the brackets is the expression to obtain the theoretical buckling load, ignoring skin friction in case of a bearing pile, carrying axial load. Denoting the buckling load for an axially loaded bearing pile as P_b, Equation (8) will be

$$P = \frac{P_b}{\left(1 - \frac{\psi}{2}\right)} \qquad ...(9)$$

The fraction of the load P taken by skin friction along the embedded length L of the pile is

$$f = \frac{P\psi}{P} = \psi \qquad ...(10)$$

Substituting Equation (10) in Equation (9) and rearranging, we obtain Equation (11) for the ratio of increase in buckling load due to skin friction and buckling load for bearing pile as

$$\alpha = \frac{P-P_b}{P_b} = \frac{f}{2-f} \qquad ...(11)$$

Equation (11) is represented by Figure 3.

Discussions

Figure 3 shows that the buckling load increases more rapidly with the increase in force due to skin friction. In practice, 80 to 90 percent of the axial load is taken by skin friction (Siva Reddy and Valsangkar, 1969). In such cases, the actual buckling strength is 67 to 82 percent more than that of bearing pile. Further, in limiting conditions of the axial load fully resisted by skin friction, the buckling strength is 100 percent more than that of bearing pile. Present approach of design, ignoring skin friction results in a very conservative value of buckling strength. About 30 to 40 percent of the total cost of the structure is spent for the construction of the foundations. Hence, consideration of the skin friction while designing the piled foundation, may result in considerable economy.

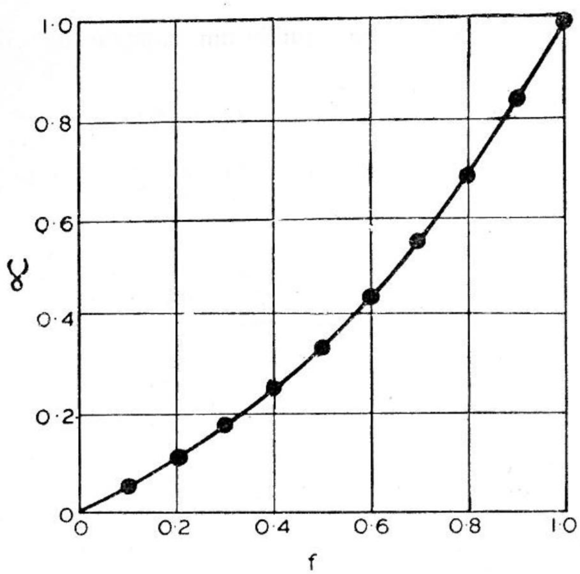

FIGURE 3 : Relation of ratio of increase in buckling load due to skin friction and buckling load of bearing pile with fraction of axial load taken by skin friction.

Conclusions

On the basis of the study, the following conclusions are drawn :

(1) Inclusion of the skin friction in the analysis considerably increases the buckling load of the pile.

(2) An economy can be effected if skin friction is given its due importance in the design.

(3) The theoretical buckling strength of a fully frictional pile is twice that of a bearing pile.

(4) Energy method can conveniently be applied to the buckling problems of the pile.

Notations

P = Axial load.

$\frac{P\psi}{L}$ = Force due to skin friction per unit embedded length of the pile.

y = Deflection of the centre line of the pile at a section at a distance x from the upper end.

L = Embedded length of the pile.

EI = Flexural rigidity of the pile.

β = Foundation modulus for the soil.

P_b = Buckling load for the bearing pile.

f = Fraction of load P taken by skin friction.

α = Ratio of increase in buckling load due to skin friction and buckling load of bearing pile.

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