Discussion on Papers

Design Procedure for Sand Drains for Time Dependent Loading*

by

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FORMULATIONS and solutions of differential equations governing vertical consolidation due to vertical and radial drainages of a compressible stratum for a loading showing linear dependence on time are already available in the literature (Schiffman 1958-59). The solution for the case of one-dimensional consolidation, corresponding to a gradually applied load increasing linearly with time, reproduced by the authors from Lumb (1963), is in fact the same as Equation (49), p. 594 of Schiffman (1958). The solution is due neither to Lumb nor to Schiffman and the credit for this should indeed go to Terzaghi & Frohlich (1936) and Wilson & Grace (1942). The solutions under similar condition of loading but for radial flow, reproduced as Equations (3*a*) and (3*b*) of the paper, were published by Schiffman.

The above solutions which have been simplified through charts and graphs, Schiffman (*loc. cit*) straightaway lead to the computation of total percent consolidation (combined effect of vertical and radial drainages) by mere substitution into the following famous equation due to Carilio (1942):

$$(1-U) = (1-U_v)(1-U_R) \qquad ...(1)$$

Where, U = degree of consolidation for three-dimensional flow (total consolidation)

U_v = degree of consolidation for one-dimensional flow

U_R = degree of consolidation for radial flow.

Could the authors then explain the need for this exercise ? The equation

$$U_v = \frac{1}{T_{vo}} \left\{ T_v - \tfrac{1}{3} + \left(\frac{32}{\pi^4}\right) e^{-\frac{\pi^2 T_v}{4}} \right\} \qquad T_v \leqslant T_{vo}$$

is only valid for $T_v > 0{\cdot}3$ and therefore its use for values of $T_v < 0{\cdot}3$ is incorrect.

Equations (6) through (10) of the paper (on the basis of which Figure 2 through Figure 8 are drawn corresponding to 90 percent

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consolidation) are valid only for $T_v = T_{vo}$ case. It is surprising that the authors should have used these charts for a loading stipulated in the example (Figure 1) such that for $0< t <5$ months $T_v < T_{vo}$ and for $5< t <8$ months $T_v > T_{vo}$.

Design worked out by the authors indicate that sand drains 0·3 m in diameter when place @ 4·64 m centres in a square pattern will achieve 90 percent consolidation in a period of 5·5 months for the conditions set out in the illustration. Calculations made by the writer show that the above figure for the degree of consolidation is grossly in error. If the real pattern of loading, Figure 1, stipulated in the illustration is considered, one would obtain total percent consolidation of 69·5 percent. It is interesting to note that even if the loading was to come suddenly only 81 percent of the total consolidation will take place in 8 months time.

The authors have made mistakes in computing values of V on which their charts are based. For example the value of V corresponding to T_{vo} of 0·01 and 0·02 would be 1·23 and 1·04 and not 0·939 and 0·885 as given by the authors.

It is clear that the authors have taken the values of V as $(1-U)$ for $T_v=T_{vo}$ case in Table I of Lumb (1963). It must however be remembered that $V\neq(1-U)$ for values of $T_v<0{\cdot}3$ as would be clear from the conditions set out for the validity of Equation (7*a*), p. 315 of Lumb (*loc. cit*). Corresponding to $T_{vo}=0{\cdot}0226$ of the example, the value of 'V' should be 0·999 rather than 0·8803 as obtained by interpolation from Table III. If the correct value of V is taken, theory would require that 0·3 m diameter sand drains be placed 2 m c/c in a square pattern as against the spacing of 4·64 m worked out in the given example, if total consolidation of 90 percent is to be achieved at the end of construction for the case of a loading increasing linearly with time, the period of construction being 5·5 months.

The Central Road Research Institute, has acted as consultants on several sand drain projects in this country and the experience gathered has shown that the real problems in the design of sand drain installation relate to the variation of the coefficient of volume compressibility and the coefficient of permeability with depth of the compressible stratum as well as that with the progress of consolidation.

Finally, the following errors should be corrected :

Errors	*Incorrect*	*Correct*
(1) Eq. (3*i*) page 67	$\frac{C_v t}{4d^2}$	$\frac{C_{vR} t}{4d^2}$
(2) page 69	$V=\left(\frac{1}{3\,T_{vo}}-\frac{32}{T_{vo}}\cdot e^{-\frac{\pi^2 T_{vo}}{4}}\right)$	$V=\left(\frac{1}{3T_{vo}}-\frac{32}{\pi^4 T_{vo}}\ e^{-\frac{\pi^2 T_{vo}}{4}}\right)$

(3) Equation (8) does not appear in the paper. In fact, substituting (1*d*) and (3*j*) in Equation (7), Equation (9) is readily obtained.

(4) In $\rho_f=m_v\triangle\sigma' H$ (Equation 11, page 70), H stands for the total thickness of the compressible stratum rather than vertical drainage path distance, as defined on page 66 of the paper.

(5) In the numerical example solved, it is given that $C_{vv}=0{\cdot}2$ and $C_{vR}=0{\cdot}001$ cm²/sec. From the solution of the problem it appears that $C_{vv}=0{\cdot}001$ cm²/sec and the calculations have been done for $\epsilon=5$ giving $C_{vR}=0{\cdot}005$ cm²/sec.

References

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AUTHORS' REPLY

The authors thank Dr. R.K. Bhandari for the interest he has shown in their paper.

The discussor stated that formulations and solutions of differential equations governing vertical consolidation due to vertical and radial drainages of a compressible stratum for a loading showing linear dependence on time are already available in the literature. There cannot be two opinions regarding it. The authors have very clearly mentioned that point and the various contributions in this regard from which information has been obtained and used in the paper, have been acknowledged both in the text of the paper and in the reference. The need from the presentation of the various solutions is to make, the paper complete by itself instead of referring the reader to different courses.

It was clearly mentioned in the paper on page 66 that Equations (3*a*) and (3*b*) for radial flow, are from the work of Schiffman.

The total percentage of consolidation can no doubt be obtained by mere substitution of the individual percentage consolidations due to vertical and radial flows in the equation due to Carillo. In any design project connected with sand drains such a procedure leads to solutions by trial and error. With the help of the design charts the exact design can be done in minimum time and yet all the possibilities required to cover the wide range of practical interest can be examined.

The equation,

$$U_v=\frac{1}{T_{vo}}\left\{T_v-\tfrac{1}{3}+\frac{32}{\pi^4}\ e^{-\frac{\pi^2 T_v}{4}}\right\} \qquad \ldots T_v \leqslant T_{vo}$$

is only valid for $T_v > 0 \cdot 3$ as mentioned by the discussor. The authors also agree with him that the above equation should not be used for values of $T_v < 0 \cdot 3$. In fact it was never used by the authors in getting their general expression for design charts. Instead Equation (2) in the paper, i.e.,

$$U_v = 1 - \frac{1}{3T_{vo}} + \frac{32}{\pi^4 T_{vo}} \ e^{-\frac{\pi^2 T_{vo}}{4}}$$

is used in preparing the design charts. Equation (2) has no restrictions in its usage. Therefore his presumption is wrong.

Specifications in any sand drain project insist a certain percent of primary consolidation to be achieved at the end of construction period (page 69 lines 1 to 3). This can be obtained only when $T_v = T_{vo}$ and based on that, Equations (6) through (10) have been formulated. To have a further idea regarding the authors excercise, the discussor is requested to refer their earlier work (Jeebala Rao et al, 1971).

Regarding surcharge, in Figure 1 a general pattern is shown. Figure 1 is meant to show the various patterns of arranging sand drains. For getting exact loading the discussor is requested to refer Figures 2 through 8. Then he will find no cause for any surprise.

The authors still feel that the design example worked out is correct. All the steps are indicated in numericals in the paper. Unfortunately the discussor has not given any of his calculations to disprove it.

The value of V taken by the authors is perfectly correct. The discussor is once again reminded that $T_v < 0 \cdot 3$ case has not been used by the authors in the formulation of their equations or charts. Another fundamental aspect the discussor should know is that the value of V should never be greater than $1 \cdot 0$. If it is so, a correction for base pore pressures should obviously be made. Because of this the example cited has no meaning.

The authors appreciate the work that is being done by the Central Road Research Institute as consultants on several sand drain projects in this country. It will be of use if they publish their experiences. The discussor has mentioned that the real problems in the design of sand drain installation relate to the variation of the coefficient of volume compressibility and the variation of coefficient of permeability. Linear variations are less complicated to account for in any problem. Soil is such a heterogeneous material that is impossible to consider its true or exact variations in nature.

Finally the following points are given :

(1) and (2) The authors thank Dr. Bhandari for correcting the printing mistakes. The values of V, tabulated in Table I, for various values of T_{vo} are obtained from the correct equation. The discussor may calculate the value of V for $T_{vo} = 0 \cdot 1$ and $0 \cdot 2$ using the above equation and satisfy himself.

(3) Equation (8) can be obtained by substituting for T_{Ro} in Equation (7) :

$$U = 1 - v\left[\frac{F(n)}{8} \ \frac{C_{vv}}{C_{VR}} \cdot \frac{4d^2}{H^2 T_{vo}}\right]\left[1 - e^{-2\frac{C_{VR}}{C_{VV}} \frac{H^2}{d^2} \frac{T_{vo}}{F(n)}}\right]$$