Short Communication

Stability of Soil Slopes with Pore Pressure

by

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Introduction

STABILITY of soil slopes has been the subject of numerous publications in the literature of soil mechanics. In most of these papers, the soil is usually treated as a homogeneous isotropic material with constant strength throughout the slope under consideration. The method described herein is an adaptation of Taylor's method (Taylor, 1937) which is quicker and easier to use than other slip-circle methods for preliminary use in slope designs.

The presently available stability charts based on the ϕ-circle method such as those formulated by Taylor (1937), are not readily adaptable to situations where pore pressures are involved. It is the purpose of this paper to show how one can use Taylor's Stability Charts (1937 and 1948) even when pore pressures due to sudden drawdown, study seepage, or construction must be considered. For this purpose, the writer has developed the sudden drawdown factor, β, seepage factor, δ, and construction pore pressure factor, ϵ.

Bishop (1952 and 1955) showed that, for a slope in which the ratio of the pore pressure, u to the vertical head of soil, γh above the element considered was a constant, the value of the factor of safety, F decreased almost linearly with increase in pore pressure ratio, $u/\gamma h$. Subsequent work by Bishop and Morgenstern (1960) has shown that, both for pore pressures obtained from flow patterns and for those obtained as a function of stress, the average value of the pore pressure ratio, $u/\gamma h$ is the most convenient dimensionless parameter by which to express the influence of pore pressure stability. This ratio is denoted by r_u. This r_u is similar to β in Equation (1), δ in Equation (2), and ϵ in Equation (3) of the writer's analysis. The unit weight of soil, γ becomes γ_s in Equation (1), combination of γ_t and γ_s in Equation (2), and γ_t in Equation (3).

Since the preliminary function of any stability chart is to provide the designer with a means of obtaining a quick estimate of slope requirements for prelimenary purposes, it is not intended that the procedures outlined in this paper should be considered as a substitute for the more rigorous analyses needed for the final design.

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Analysis of the Problem

(a) PORE PRESSRUE DUE TO SUDDEN DRAWDOWN

The writer adopts the following graphical method to calculate the pore pressure due to sudden drawdown.

In Figure 1(*b*) $\overline{AC}=W_b$ (Buoyant weight). This W_b is balanced by $\overline{CC_1}$ and $\overline{C_1A}$ (cohesion required for W_b). $\overline{CB}=W_o$ (weight of a mass of water of the same volume as the sliding mass).

$$\overline{AC}+\overline{CB}=W_b+W_o=W_s$$

From B, BK is drawn parallel to 0′0. From C, C_T is drawn parallel to C_1A to meet BK at T. Then C_T is the cohesion required to give resisting moment to overcome the overturning moment due to W_o. Then $\overline{TC}$ ($\overline{C_2C_1}$) is added to $\overline{C_1A}$. So, $C_T=\overline{C_2C_1}+\overline{C_1A}=C_o+C_b$. This C_T is the total cohesion to overcome the effect of total weight, $W_s=W_b+W_o$. Then BC_2 gives rise to ϕ_{mP}, the new friction angle obtained from pore pressure due to sudden drawdown. From point O', $O'S$ is drawn parallel to BC_2. Then a small circle is drawn such that $O'S$ is a tangent to this circle. This small circle gives rise to ϕ_{mP} (modified friction angle under pore pressure due to sudden drawdown). In Figure 1(*b*),

$$\frac{\overline{CB}}{\overline{AB}}=\frac{W_o}{W_s}=\frac{\gamma_w}{\gamma_s}=\beta \qquad ...(1)$$

(sudden drawdown factor), as defined by the writer. So, for different values of β, one will get different values of ϕ_{mP}/ϕ_i, and it is this value of ϕ_{mP} one uses in Taylor's chart. Modified slip circle corresponding to ϕ_{mP} can be used in analysis. And, the unit weight to be used in stability number is the saturated unit weight, γ_s.

(b) PORE PRESSURE DUE TO SEEPAGE

Its effect is similar to that of sudden drawdown. Here, in place of

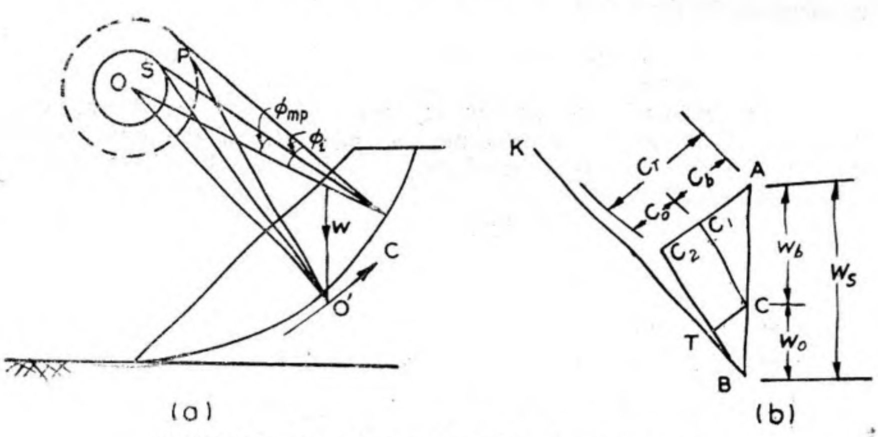


FIGURE 1 : Force diagram for sudden drawdown case.

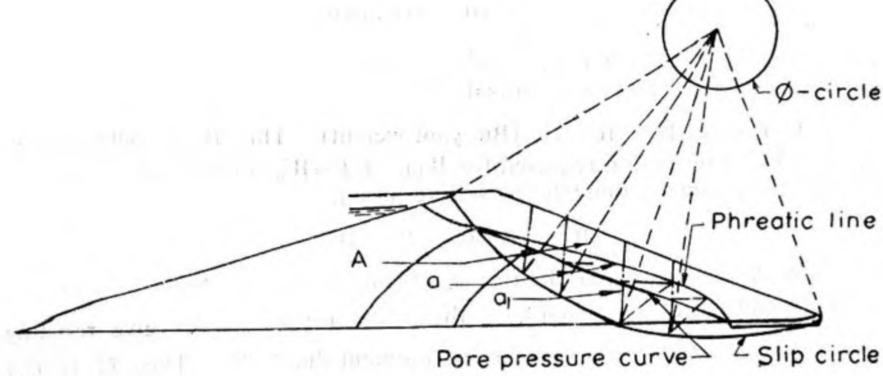


FIGURE 2 : Effect of seepage.

β, one can use seepage factor as defined by the writer :

$$\therefore \quad \delta=\frac{\gamma_w a}{\gamma_t(A-a)+\gamma_s a}=\text{seepage factor} \qquad \ldots(2)$$

γ_w=Unit weight of water

γ_t=Unit weight of soil at any moisture content

γ_s=Saturated unit weight of soil

a_1=The area between the pore pressure curve and slip circle obtained by ϕ-circle method

a=The area between the phreatic line and the slip circle

A=Total area of sliding mass.

In a similar manner to that of sudden drawdown, a chart of δ versus ϕ_{mP}/ϕ_i can be obtained for the case of pore pressure due to seepage. It is this value of ϕ_{mP} that one will use in Taylor's chart. And, the unit weight to be used in stability number is $\gamma_E=\frac{\gamma_t(A-a)+\gamma_s a}{A}$ ...(2a) as suggested by the writer.

(c) PORE PRESSURE DUE TO CONSTRUCTION

Its procedure is also similar to that of sudden drawdown. Only instead of β, one uses ϵ which is expressed as a ratio of the pore pressure force to the total weight on the rupture plane. If the height of the soil above any point on the rupture plane is h, and the height due to pore water pressure is h_1 above the same point, then as defined by the writer

$$\epsilon=\frac{\gamma_w h_1}{\gamma_t h} \qquad \ldots(3)$$

Where,

γ_w=Unit weight of water

γ_t=Unit weight of soil at any moisture content

ϵ=Construction pore pressure factor.

If the effect of pore pressure due to construction is 10 percent of the full hydrostatic pressure then

$$\epsilon = 0.10 \frac{\gamma_w}{\gamma_t} \qquad \ldots(3a)$$

So then, like those of cases (*a*) and (*b*) a chart of ϵ versus ϕ_{mP}/ϕ_i can be obtained. This chart will give the friction angle modified due to construction pore pressure. And the unit weight of soil in this case is γ_t.

Results

From the graphical analyses as shown in Figure 3, the values of ϕ_{mP}/ϕ_i are obtained for different values of β, or δ, or ϵ. These values are plotted in Figure 4. Now from this figure, ϕ_{mP} can be obtained. And it is with this value of ϕ_{mP} one will look into Taylor's chart when pore pressure is to be considered.

Summary and Conclusions

The ϕ-circle method employed to obtain the relation between the stability number, $c/F\gamma H$ and slope angle, i for various angles of internal friction, ϕ, makes the problem of stability analysis of earth slopes simpler compared to many other methods so far used in design. When the slope angle and the friction angle are known, $c/F\gamma H$ may be obtained directly from Taylor's charts. In case of pore pressures, the modified friction angle is obtained from Figure 4 and then the Taylor's charts are used to get the stability number, $c/F\gamma H$. The unit cohesion, c and the initial friction angle, ϕ_i in all three cases involving pore pressures are the effective values and are generally obtained from undrained triaxial tests with pore pressure measurements or drained direct shear tests.

For the analysis of the condition of long-term stability under steady seepage and for the case of sudden drawdown, it is necessary to consider the effect of saturation on the values of c and ϕ_i. The principal factors controlling the pore pressure set-up during construction are:

(1) The placement moisture content and amount of compaction,

(2) The state of stress in the zone of the fill considered,

(3) The rate of dissipation of pore pressure during construction.

The average pore pressure coefficient along a potential slip surface at the end of construction may be kept within safe limits either by restricting the size of the impervious zone, by controlling the placement water content or by special drainage control measures. The most economical method depends on the climatic condition and the fill materials available.

As shown by Bishop and Bjerrum (1960), the lowest values of factor of safety for the upstream slope are usually reached at the end of construction and upon sudden drawdown. For the downstream slope, the end of construction and the steady seepage case are the two critical stages. However, during steady seepage the danger is generally not so much from the pore pressures which are easily controlled by drainage measures, but from the possibility of piping and from crack formation in the fill.

$\beta = \frac{\gamma_\omega}{\gamma_s}$ = Sudden drawdown factor (as in Eq.1)

$\delta = \frac{\gamma_\omega a_1}{\gamma_t (A-a) + \gamma_s a}$ = Seepage factor (as in Eq.2)

$\varepsilon = \frac{\gamma_\omega h_1}{\gamma_t h}$ = Construction pore pressure coefficient (as in Eq.3)

Information

$i = 45°$

$\phi = 20°$

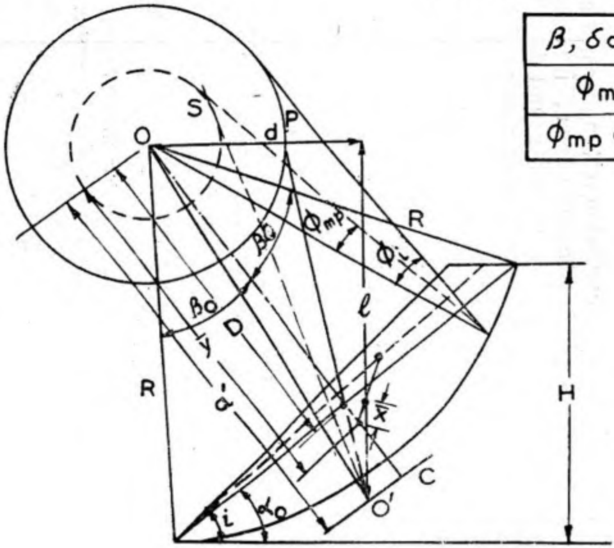
α_0	β_0
38°	34.5°

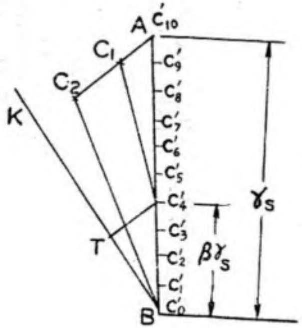
H = (50′) 15·2m

R = (71′) 21·6m D = (59′) 18 m

$\bar{y}$ = (63·5′) 19·3m; $\bar{x}$ = (4·05′) 1·2 m; a′ = (75·1′) 22·9m

β, δ or ε	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
ϕ_{mp}	20°	18.1°	16.5°	14.3°	12.4°	10.8°	8.0°	6.0°	4.0°	1.6°	—
$\phi_{mp}\ \phi_i$	1.0	0.91	0.83	0.72	0.62	0.54	0.40	0.30	0.20	0.08	—





In case of sudden drawdown,

$AB = C'_{10} C'_0 = \gamma_s$;

$C'_0 C'_1$, $C'_0 C'_2$ ... etc. = γ_w

In case of pore pressure due to seepage,

$C'_{10} C'_0 = \gamma_t (A-a) + \gamma_s a$;

$C'_0 C'_1$, $C'_0 C'_2$, ...etc. = $\gamma_w a_1$,

In case of pore pressure due to construction,

$C'_{10} C'_0 = \gamma_t h$ = Total weight of soil of the sliding mass

$C'_0 C'_1$, $C'_0 C'_2$, ...etc. = $\gamma_w h_1$

FIGURE 3 : Stability analysis due to pore pressure.

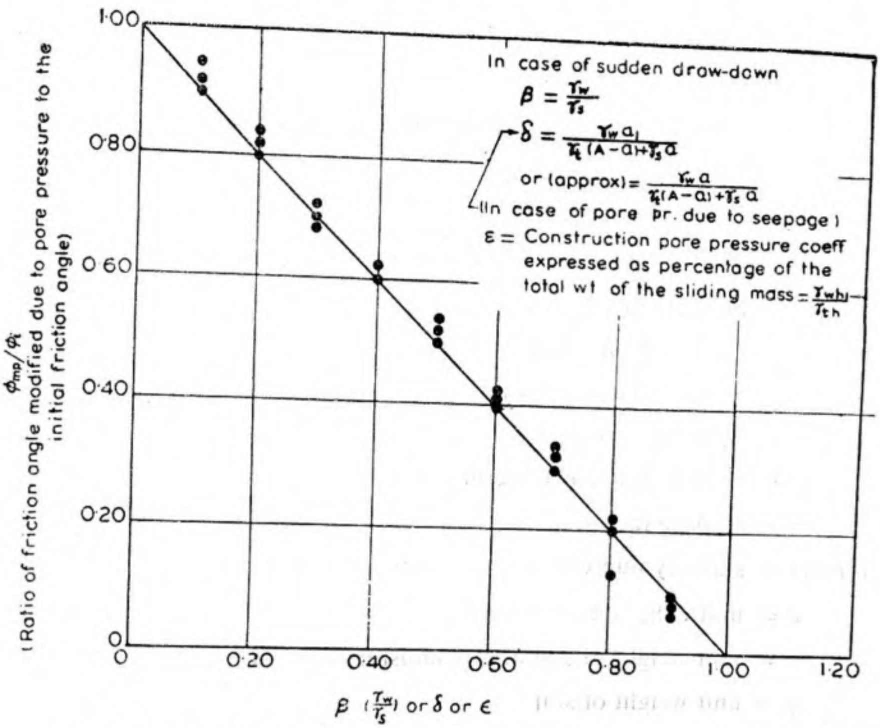


FIGURE 4 : **Ratio of modified friction angle (due to pore pressure) and initial friction angle versus sudden drawdown or construction pore pressure coefficient seepage factor.**

As previously mentioned, the procedures outlined in this paper are intended for use by the designer primarily to obtain a preliminary concept of slope stability. A more rigorous analysis which accounts for variations in the assumed homogeneous and isotropic properties of the soil is still required for the final design.

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Notation

The following symbols have been adopted for use in this paper :

ϕ = angle of internal friction

g = acceleration due to gravity

$\overline{L}$ = arc length

γ_b = bouyant unit weight of soil

ϵ = construction pore pressure factor

$\overline{L}$ = chord length

γ_E = equivalent unit weight

ϕ_{mp} = friction angle modified due to pore pressure

C = force due to cohesion

F = factor of safety

ϕ_i = initial angle of internal friction without pore pressure

l/d = moment arm ratio

γ_s = *saturated unit weight of soil*

G = specific gravity of soil

i = slope of embankment

β = sudden drawdown factor

δ = seepage factor

$c/F\gamma H$ = stability number

c = unit cohesion

γ_t = unit weight of soil at any moisture content

γ = unit weight of soil

γ_w = unit weight of water

e = void ratio

W = weight of sliding mass

u = pore pressure

h = vertical distance of the surface above the element.

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