Effect of Smear in Sand Drains

by

D. Babu Rao*
D. Bhaskara Rao**

Introduction

SAND drains used to accelerate compression of fine-grained soils permit consolidation due to radial drainage. Since most deposits are formed by the transportation and deposition of soil sediments in water, they usually exhibit an approximately horizontal stratification wherein the permeability in the plane parallel to the stratification tends to be considerably higher than that in the perpendicular direction. Therefore, flow in such soils is mostly horizontal as compared to stricktly vertical flow, and consolidation of these deposits takes place largely due to radial dissipation of excess pore water.

The operation of sand drains consists of forcing water to be squeezed towards the drain wells, backfilled with sand. This is a fairly well established practice today to hasten the consolidation of marshy and other highly compressible soils. The drain wells are generally installed by driving and pulling of the casing which distorts and remoulds the adjacent soil. In layered deposits, the finer layers will be dragged down and smeared over coarser layers. If the casing is thick, the wiping action is much more. The distortion is similar to that which takes place during soil sampling operation and may be compared to that around the cylindrical surface of the triaxial specimen. Thus a smeared zone is formed around the well. Smear created by drilling operation is a function of the diameter of the drill hole, type of drilling equipment, method of drilling and nature of soil. Smear zones of varying permeabilities might develop in the same soil when drilled by different methods. Due to remoulding action the permeability of the smeared zone is much smaller than that of the undisturbed soil. The remoulded or smear zone creates additional resistance which must be overcome by the excess water, being expelled from the saturated zone. The barrier slows down considerably the process of consolidation. The present work studies the retardation aspect of radial consolidation by the use of numerical techniques. A new concept of impeded drainage is introduced to simulate the boundary conditions of the smeared zone.

Previous Work

The problem of sand drains was analysed in a systematic manner by Barron (1948) and later discussed by Richart (1959). Although variations

*Reader in Civil Engineering, University College of Engineering, Osmania University, Hyderabad.

**Executive Engineer, Zilla Parishad, Amalapuram, Andhra Pradesh.

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in design procedures have been suggested in literature from time to time, the two works still remain competent. Barron recognised the existence of peripheral smear in drain wells and developed a rather involved solution to the boundary value problem. His results were interpreted by Richart, who suggested the use of an equivalent ideal well of reduced diameter to take the effect of smear into account. The closed form solution of Barron consists of Bessel functions of different kinds and orders and is, therefore, less tractable. With the advent of modern high speed computer, it is now possible to solve the governing field equations by numerical procedures and study the problem in an amenable way.

Theoretical Formulation

Although the smeared zone will not be of constant thickness nor will it be homogeneous with regard to soil properties, it will be assumed in this study that the smeared zone is of constant thickness and homogeneous. Admittedly, the amount of disturbance decreases with distance away from the well periphery. It will be further assumed that the smeared zone is incompressible. That is, its coefficient of volume change is zero and the flow therein is in steady state. Thus the excess pore pressure at one boundary of the smeared region is time dependant and is zero at the other boundary (well face). Considering the zone of influence of each well as a circle, Figure 1 shows the different zones of a soil deposit, radially consolidating by means of sand drains, including peripheral smear. H_1 and H_2 are thicknesses of smeared and undisturbed zones, respectively. The soil properties, including the coefficient of permeability, are different in the smeared zone from those in the surrounding soil.

The boundary conditions for the radial drainage problem are well-known and will not be stated here. Additional boundary conditions due

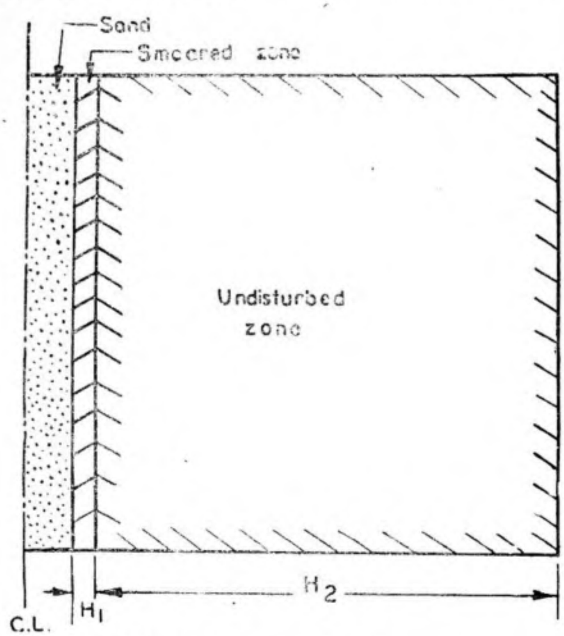

FIGURE 1 : Sand drain with smear.

to presence of smear are :

(1) The excess pore water pressure at the boundary of the smeared zone is the same in the undisturbed zone as in the smeared zone.

(2) Rate of flow into smeared zone is equal to that of undisturbed zone.

Considering radial flow only, consolidation equation in cylindrical co-ordinates may be written as

$$\frac{\partial u}{\partial t}=c_v\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) \qquad ..(1)$$

where, the excess pore water pressure, u, is a function of radial distance, r and time, t. The coefficient of consolidation, c_v, is assumed constant for undisturbed soil.

Classical consolidation theory, as expressed in Equation (1) is generally used to analyse situations, where the boundaries are either fully draining or entirely impervious. Such boundaries are likely to be extreme conditions and are not always realised in the field. The present problem is an example of impeded drainage. A typical impeded drainage boundary is one in which the zone adjacent to the boundary is incompressible, but has a finite permeability (Schiffman, 1970). Such a boundary is referred to as radiation boundary in the literature of heat conduction. Bishop and Gibson (1963) have considered impeded drainage in determining the effects of porous stones and filter drains in triaxial apparatus. The following material will develop an additional boundary condition to be used alongwith Equation (1), while considering the effect of smear.

Since the smeared zone is in steady state of flow,

$$\frac{\partial^2 u'}{\partial r^2}+\frac{1}{r}\frac{\partial u'}{\partial r}=0 \qquad ...(2)$$

where u' is the excess pore water pressure in the smeared zone. Also, the sand surface in the well is free draining. Considering the origin at the centre of well in Figure 1,

$$u'=0 \quad \text{at} \quad r=r_w \qquad ...(3)$$

Integrating Equation (2),

$$u'=A\log r+B \qquad ...(4)$$

where A and B are constants to be evaluated by the use of boundary conditions. From Equation (3),

$$B=-A\log r_w \qquad ...(5)$$

Thus,

$$u'=A\log\frac{r}{r_w} \qquad ...(6)$$

The conditions of interface between smeared and undisturbed zones are,

$$u'=u \text{ at } r=r_w+H_1 \qquad ...(7)$$

$$k_1\frac{\partial u'}{\partial r}=k_2\frac{\partial u}{\partial r} \qquad ...(8)$$

where, k_1 and k_2 are the coefficients of premeability of the smeared and undisturbed zones, respectively. From Equations (6) and (8),

$$A = r\frac{k_2}{k_1}\frac{\partial u}{\partial r} \qquad \ldots(9)$$

Therefore,

$$u' = r\frac{k_2}{k_1}\frac{\partial u}{\partial r}\log\left(\frac{r}{r_w}\right) \qquad \ldots(10)$$

From Equation (7),

$$u = \frac{k_2}{k_1}(r_w+H_1)\frac{\partial u}{\partial r}\log\left(\frac{H_1}{r_w}\right) \text{ at } r=r_w+H_1 \qquad \ldots(11)$$

Thus the boundary condition for the undisturbed soil is,

$$H_2\frac{\partial u}{\partial r} - \lambda u = 0 \qquad \ldots(12)$$

where, λ is the impedance factor, given by,

$$\lambda = \frac{k_1 H_2}{k_2(r_w+H_1)}\log\left(\frac{r_w}{H_1}\right) \qquad \ldots(13)$$

Equation (12) may be rewritten as,

$$\frac{\partial u}{\partial r} - \frac{k_1}{k_2(r_w+H_1)}\log\left(\frac{r_w}{H_1}\right)u = 0 \qquad \ldots(14)$$

Equation (14) is the impeding boundary condition to be used at the boundary between smeared and undisturbed zones.

Finite Difference Schemes

Equation (1) alongwilh the usual boundary conditions and that given by Equation (14) can be solved by a numerical procedure, such as finite difference scheme. Using non-dimensional variables such that,

$$U = u/u_i$$
$$T_2 = t/\tau$$

where, $1/\tau = c_v/H$, u_i and H being arbitrary reference values of pore water pressure and distance, respectively, and letting $R=r/H$, Equation (1) is transformed to

$$\frac{\partial U}{\partial T} = \frac{\partial^2 U}{\partial R^2} + \frac{1}{R}\frac{\partial U}{\partial R} \qquad \ldots(15)$$

Also,

$$\frac{1}{r}\frac{\partial u}{\partial r} \rightarrow \frac{\partial^2 u}{\partial r^2}$$

Adopting the notation in Figure 2, partial derivatives can be written in the form of finite differences as

$$\frac{\partial U}{\partial T} = \frac{1}{\triangle T}(U_{o,\,T+\triangle T} - U_{o,\,T})$$

$$\frac{\partial^2 U}{\partial R^2} = \frac{1}{(\triangle R)^2}(U_{2,\,T} + U_{4,\,T} - 2U_{o,\,T})$$

$$\frac{1}{R}\frac{\partial U}{\partial R} = \frac{1}{R}\left(\frac{U_{2,\,T} - U_{4,\,T}}{2\triangle R}\right)$$

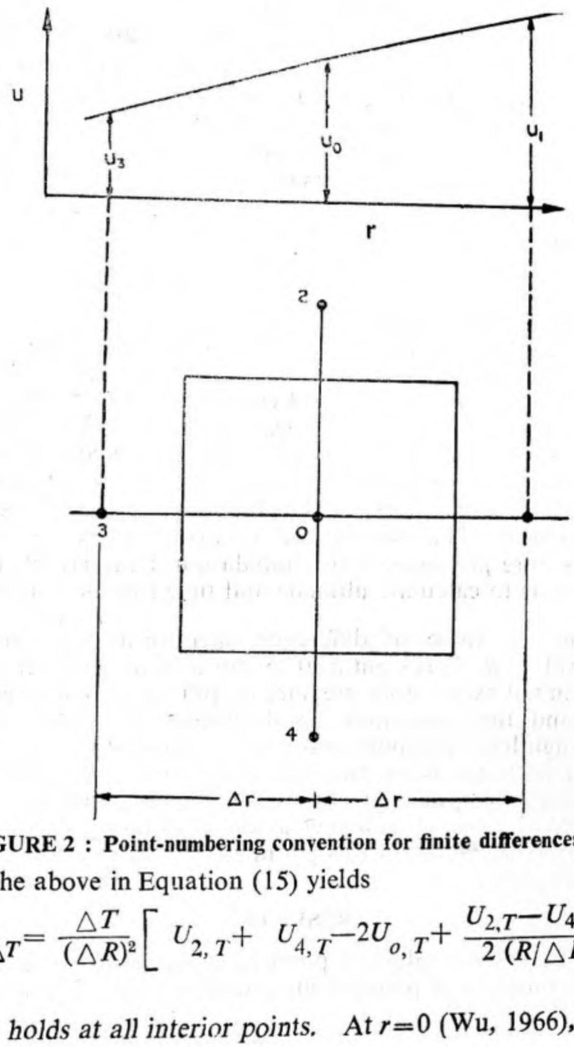

FIGURE 2 : Point-numbering convention for finite differences.

Substituting the above in Equation (15) yields

$$U_{o,\,T+\triangle T}=\frac{\triangle T}{(\triangle R)^2}\left[U_{2,\,T}+U_{4,\,T}-2U_{o,\,T}+\frac{U_{2,T}-U_{4,T}}{2\,(R/\triangle R)}\right]+U_{o,\,T} \quad \text{...(16)}$$

Equation (16) holds at all interior points. At $r=0$ (Wu, 1966), Equation (16) takes the form

$$U_{o,\,T+\triangle T}=\frac{2\triangle T}{(\triangle R)^2}\left(U_{2,\,T}+U_{4,\,T}-2U_{o,\,T}\right)+U_{o,\,T} \quad \text{...(17)}$$

Similarly, Equation (14) can be written in the terms of finite differences as,

$$\frac{U_{1,\,T}-U_{o,\,T}}{\triangle R}=\frac{k_1}{k_2}\;\frac{H}{(r_w+H_1)}\;\log\left(\frac{r_w}{H_1}\right)U_{1,\,T}$$

Or,

$$U_{o,\,T}=\frac{1}{1+\dfrac{k_1}{k_2}\,\dfrac{\triangle R}{\left(\dfrac{r_w}{H}+R_1\right)}\log\dfrac{r_w}{H_1}}\,U_{1,\,T} \quad \text{...(18)}$$

where, $R_1=H_1/H$.

Equations (16), (17) and (18) permit the computation of pore pressures at all points in the consolidating soil mass by iterative process. Explicit method employed in the present work is shown schematically in Figure 3. The value of difference operator, $\triangle T/(\triangle R)^2$, is kept less than 1/4 (Scott, 1963) to obtain a stable solution. A detailed study was undertaken to check the stability of the solution : a value of the operator of more than 1/2 resulted in oscillation and divergence and the solution was stably oscillating for a value of 1/2. Degree of consolidation is computed by the use of trapezoidal rule.

Example

To illustrate the procedure developed above, consider a sand drain of 45 cm radius with uniform peripheral smear zone of thickness 23 cm. The radius of influence of the drain may be taken as 2.75 m. Coefficient of consolidation of undisturbed soil is 94 cm²/day. It is required to assess the effect of various degrees of smear on the progress of consolidation. In a typical field problem such as this one, one may like to know the point pore pressures for a period of, say, 90 days at 10 days interval to control the stabilisation process so that the construction programme can be suitably planned. This information is also required to evaluate the residual excess pore pressures in the foundation material at the start of construction so as to calculate ultimate and time rate of settlement.

Choosing the value of difference operator as 0.2, non-dimensional distance interval, $\triangle R$, works out as 0.25 for a time interval of 10 days. Assuming an initial excess pore pressure of 100 units, pore pressure at any given point and time can now be determined using the finite difference mesh. Although hand computations can be performed, a digital computer was employed here to avoid the tedium of repetitive calculations. The permeability parameter, defined as the ratio of permeabilities of undisturbed and smeared zones of soil, was assumed to take values of 2, 5, 10, 15 and 20. The higher values of this parameter show increasing disturbance of this smeared zone.

RESULTS

Figure 4 plots the values of point pore pressures in the consolidating medium, as a function of permeability parameter, after a period of 90 days.

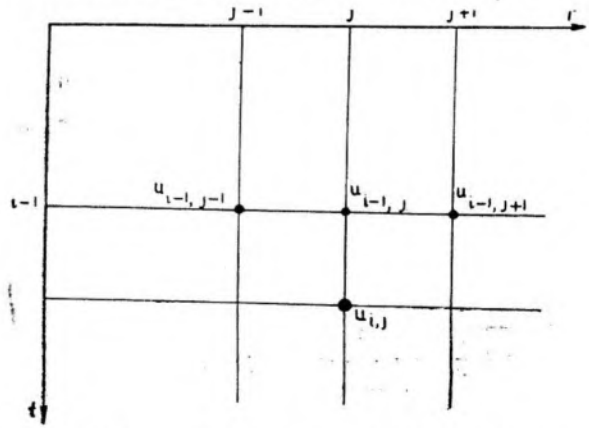

FIGURE 3 : Finite difference mesh—Explicit scheme.

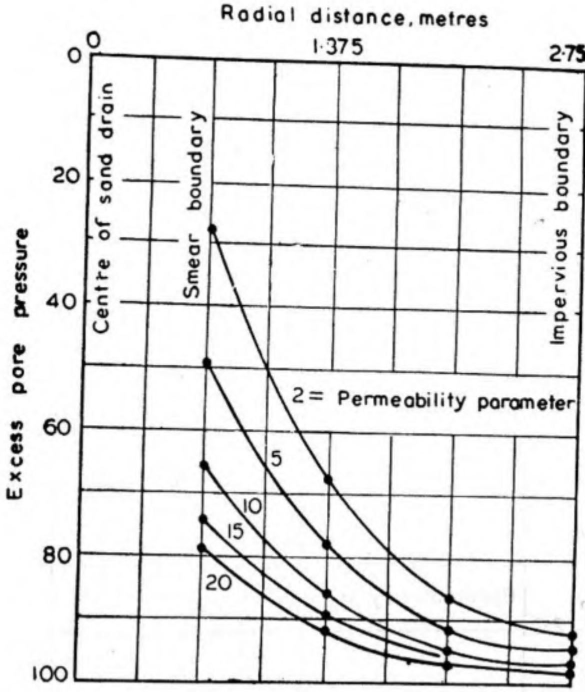

FIGURE 4 : Excess pore pressure as a function of permeability parameter.

It may be noted that higher degree of smear results in increasing pore pressures, which represent the slow down of the consolidation process. This would explain the field observation that consolidation occurs slower than can be accounted for on the basis of conventional theory. The variation in average degree of consolidation with time is shown in Figure 5. When the permeability parameter is increased $2\frac{1}{2}$ times, the value of average degree of consolidation falls by 32 percent at 90 days, The decrease in degree of consolidation due to smear is seen to be more pronounced at lower values of permeability parameter. The retardation effect points out that successful operation of a drain well requires an efficient drilling procedure.

The fact that installations of drain wells have met with varied success indicates the necessity of detailed investigation. Knowledge of point pore pressures, and comparison with piezometer readings in the field are required for the evaluation of success or failure of a given project. From this point of view, the results presented above are of practical utility. It should, however, be kept in mind that a reasonable determination of thickness of smeared zone and the coefficient of permeability of remoulded soil is required to assess the effects of smear.

SUMMARY

A new concept of impeded drainage is introduced in the analysis of radial consolidation due to sand drains, including smear. Numerical technique based on finite differences is employed to evaluate the point pore pressures in the consolidating medium. The retardation in the time rate of consolidation is studied with reference to permeability parameter. It is suggested

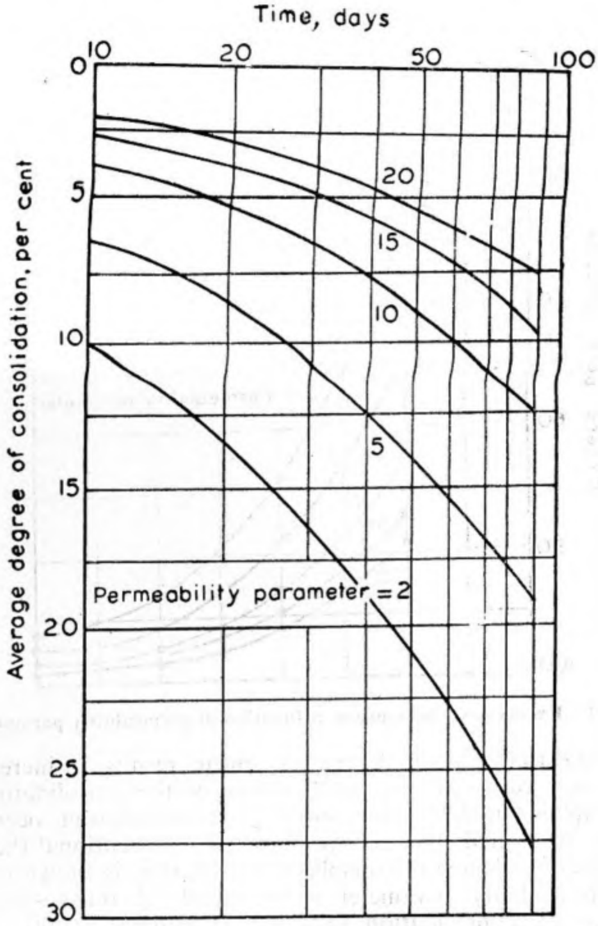

FIGURE 5 : Effect of permeability parameter on progress of consolidation.

that better agreement between theoretical and field behaviour would be obtained if smear effects are included.

Notations

A, B = constants ;

c_v = coefficient of consolidation ;

H = arbitrary distance ;

H_1, H_2 = thicknesses of smeared and undisturbed zones ;

R_1, R_2 = coefficients of permeability of smeared and undisturbed zones ;

r = radial distance ;

R = non-dimensional radial distance ;

$R_1 = H_1/H$;

t = time ;

T = non-dimensional time ;

u, u_i, u' = pore water pressure ;

U = non-dimensional pore water pressure ;

Δ = increment ;

λ = impedance factor ;

$\tau = H^2/c_v$; and

∂ = partial differentiation.

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