The dynamic conditions described by the writers are transitional by nature as far as the present problem is concerned and may be more relevant to a different context, namely, when the influence of seepage is to be considered on tail erosion.

C.S. Martin (1970), in his investigation on the 'effect of a porous sand bed on incipient sediment motion' simulates the simultaneous seepage and surface flow conditions on a permeable bed. One of his findings is that seepage out of a bed does not affect incipient motion of bed particles measurably because the seepage force is lost once a sediment particle rocks.

The writers' interest in the subject matter is highly appreciated.

Reference

MARTIN, C.S. : "Effect of a Porous Sand Bed on Incipient Sediment Motion." Water Resources Research, Vol. 6, No. 4, August 1970.

* * *

Hvorslev Parameters*

by

R.P. Kulkarni

S. NARASIMHA RAO** AND A. SRIDHARAN†

The writers would agree with the author that the Hvorslev's hypothesis was an oversimplification and the Hvorslev parameters are dependent on soil fabric to a significant extent. The work of the writers (Sridharan and Narasimha Rao, 1973) and that of Narasimha Rao (1973), clearly reveals that the Hvorslev parameters can have a wide range of values depending upon the method by which they are determined, the initial conditions (stress history and water content of the sample) and the stress level.

From the author's results (Figure 1), it can be seen that the strength results plotted for both normally and overconsolidated samples fall essentially on the same line even though their water content V_s. σ'_{3c} plots are distinctly different. The writers' results presented in Figure 1 clearly show that the Hvorslev parameters of normally and overconsolidated samples are distinctly different. These results have been obtained from consolidated undrained triaxial tests on remoulded montmorillonite clay. It can also be seen that the Hvorslev parameters are affected by the initial water content.

It can also be seen that the parameters obtained for test series 1 and 2 (of the author's Table III), bring out some interesting observations. The

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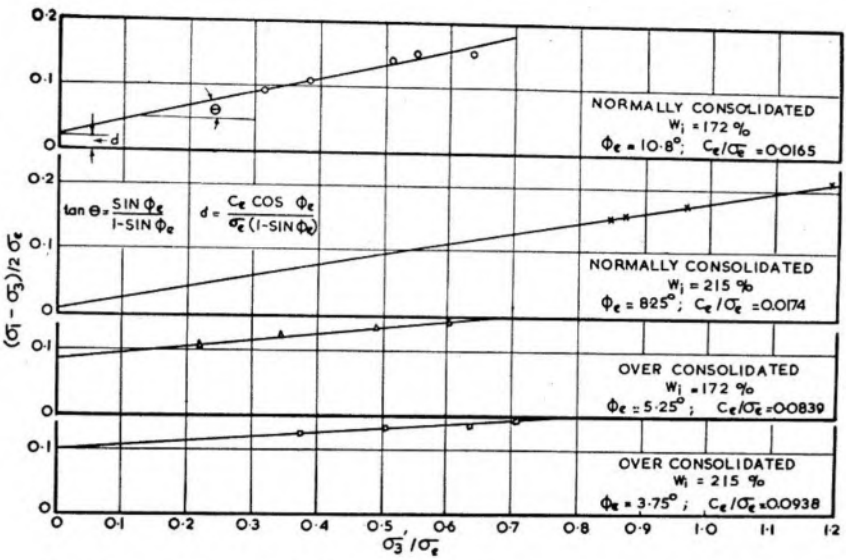

FIGURE 1 : Bishop and Henkel method of detrmining Hvorslev parameters.

samples in the series 1 are obtained by hand remoulding and the initial moulding water content is also less. In addition to this, the samples were isotropically consolidated. The lower moulding water content and the subsequent isotropic consolidation are expected to result in a relatively random oriented fabric when compared with that of series 2. This would result in higher value of ϕ_e. The author's results show just the opposite trend. The values of ϕ_e obtained from the series 3 and 6 are in the order of 14° which is incidentally higher than the value obtained in the series 1. The author's contention is that almost the same value of $\phi_e = 14°$ is obtained in both the series 3 and 6 because the particles become parallelly oriented to the failure plane. If this is true, the writers feel whether a greater degree of orientation is reached in series 1 (in which ϕ_e obtained was 10°—06′). Again when a comparison is made between the values obtained from series 3*D* and series 3 and 6, series 3*D* gives higher value of ϕ_e. As already brought out by the author, a higher degree of particle orientation is likely to reach in a sample tested under drained conditions. However, the ϕ_e obtained in the series 3*D* is slightly more. If a greater degree of orientation is reached, in the writers' opinion, it should have resulted in a lower value of ϕ_e. It should be noted that the values of ϕ_e, computed by making use of Bjerrum's method for the series 3 and 6 are very much low. It is the writers' experience that the parameters computed by making use of the procedure suggested by Bishop and Henkel (1962) for normally consolidated samples alone and the values obtained by Bjerrum's procedure should not differ considerably. The difference lies only in the method of plotting. The writers' wish to draw the attention of the author with regard to the parameters computed by making use of Bjerrum's (1954) method. While computing the parameters, the author has quite often made use of the test results obtained from samples whose initial mode of consolidation is different. If the reading of the writers is correct, it must be mentioned that only the samples whose initial moulding water contents are different, but their mode of consolidation is the same should be made use of.

Another aspect the writers wish to bring out is that when soil exhibits such a pronounced variation in Hvorslev parameters, it is but natural to expect that these parameters vary with stress level. The results of the writers are presented in Figure 2 to bring out the effect of stress level. This is essentially the Bjerrum's method in principle. The strength variation with water content is plotted towards the right hand side of the figure and the modified Mohr-Coulomb strength lines are plotted towards the left hand side. From the water content *Vs.* $(\sigma_1 - \sigma_3)/2$ plot, it is quite possible to locate the points on both the strength envelopes having the same failure water content. From the line joining these points, C_e and ϕ_e can be calculated. The values of C_e/σ_e and ϕ_e calculated at a number of failure water contents are presented in Table I. The values obtained clearly show that the parameters vary significantly with failure water content. It must be mentioned here in that the changes in failure water content are brought about by the changes in the stress level.

TABLE I

Calculation of Hvorslev Parameters using Bjerrum's method (Computed from Figure 2).

Soil	Water content at failure (%)	ϕ_e	C_e/σ_e
Montmorillonite	180	7·0°	0·033
(initial water	170	6·7°	0·042
contents, 172% and 215%)	160	6·4°	0·050
	150	5·6°	0·069
	140	4·2°	0·095

Figures 10 through 13 (of the author) show the variation of strength parameters with strain. The author's results show that both the strength parameters, C_e and ϕ_e assume zero values at zero axial strain. They are shown to increase with strain and the rates of variation are different for C_e and ϕ_e. The basic question arises whether the strength parameters can assume zero values in unstrained condition. C_e and ϕ_e, according to Hvorslev (1937), are the intrinsic parameters. But the subsequent works of several authors (Lo, 1962, Skempton, 1964, Kenney, 1967,

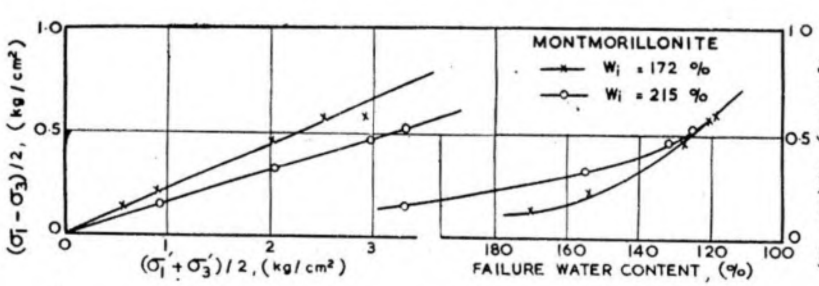

FIGURE 2 : Bjerrum's (modified) method of determining Hvorslev parameters.

Chandler, 1967) show that they are not intrinsic parameters, but vary between wide limits depending upon the fabric. The shearing strains are known to induce a better orientation of particles. The ordered orientation only brings in a reduction in the value of ϕ_e. In the opinion of the writers, what the author is trying to bring out in Figures 10 through 13, is only the mobilization of these parameters with axial strain. Only the values at peak stress difference should be taken as Hvorslev parameters. As such the authors' conclusions regarding the variation of Hvorslev parameters with strain cannot be evaluated. If the effects of mobilized values of cohesion and friction are treated in a cumulative way with strain, only a stress-strain curve is constructed. As such nothing can be said of the dependency of these parameters from the results presented through Figures 10 through 13. If there is a procedure to reach failure at different values of axial strain in different samples of the same soils, it is possible to trace out the influence of strain.

In Tables III and IV, only the values of C_e and ϕ_e are presented. According to Hvorslev's hypothesis, the true cohesion, C_e is dependent on failure water content at which it is computed. The water content is related to the equivalent consolidation pressure, σ_e on the virgin consolidation curve and C_e/σ_e is supposed to be a constant for a particular soil. As such the writers feel that the authors could have given the failure water contents at which these parameters are computed in Tables III and IV.

In addition to the methods attempted, the procedure suggested by Crawford (1961) offers a potential method of obtaining the Hvorslev parameters. According to Crawford, the shearing resistance at stresses less than failure may be represented by

$$S=\frac{\sigma' \tan \phi'}{F'} \qquad \ldots(1)$$

where,

$$F=\frac{(\sigma_1-\sigma_3)_f}{(\sigma_1-\sigma_3)_\varepsilon}$$

$(\sigma_1-\sigma_3)_f$ is the deviator stress at failure and $(\sigma_1-\sigma_3)_\varepsilon$ is the deviator stress at any strain, ϵ and $\tan \phi'=F \tan \alpha_m$ ...(2)

$\alpha_m = \sin\left(\frac{\sigma_1-\sigma_3}{\sigma_1'+\sigma_3'}\right)$ and it can be shown that the values of ϕ' mobilized at any strain, ϵ *is given by*

$$\tan \phi'=F/2\left[\frac{\sigma_1-\sigma_3}{{}_{\prime}\sigma_1 \cdot \sigma_3'}\right]_\varepsilon \qquad \ldots(3)$$

when ϕ' was computed in this way at various degrees of maximum stress, it was found to be fairly constant up to about half the maximum deviator stress. This constant value of ϕ' is believed to be the true angle of internal friction. It has been found that the value of ϕ_e computed as per Crawford's procedure agrees with the values obtained by other procedures computed for the same stress levels and failure water contents under the same consolidation stress histories (Narasimha Rao, 1967 and 1973, Sridharan and Narasimha Rao, 1973). Figure 3 presents a typical example of calculating Hvorslev parameters by Crawford's method. Figure 4 presents the variation of ϕ_e with consolidation pressure, thus bringing in the influence of stress level. This procedure has a distinct advantage in that, only one sample is sufficient to get the parameters.

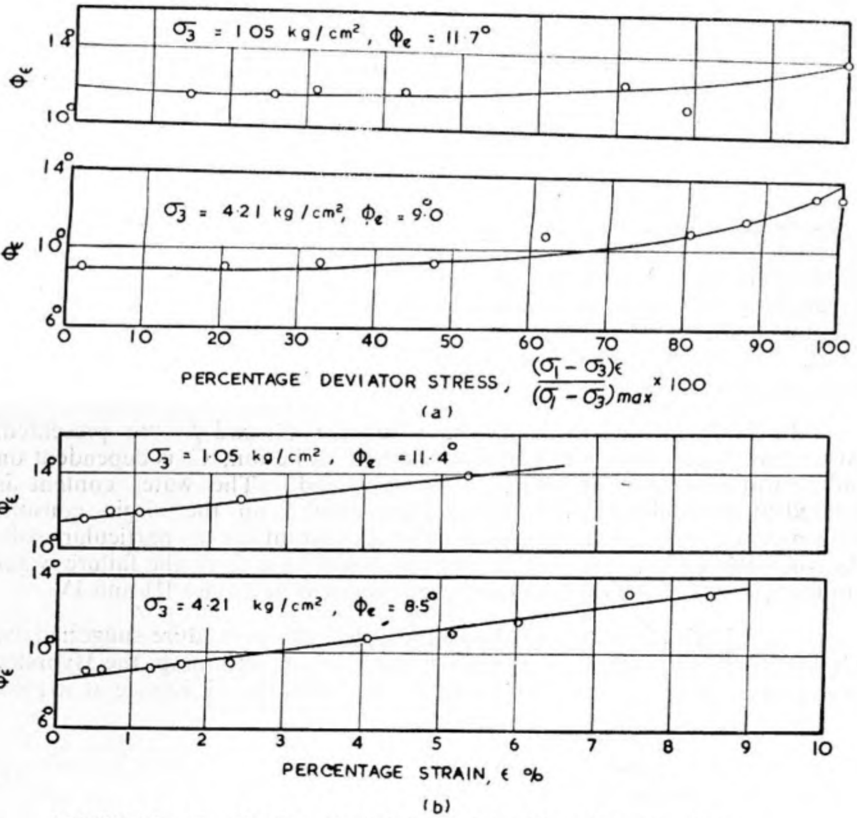

FIGURE 3 : Crawford's method of determining Hvorslev parameters.

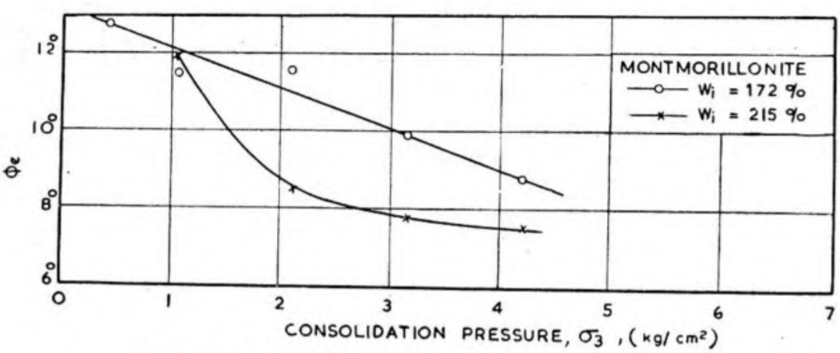

FIGURE 4 : Variation of ϕ_e with consolidation pressure.

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AUTHOR'S REPLY

The author wishes to thank the discussors for their interest in the paper and additional data they have given on Hvorslev parameters from their study.

Figure 1 is drawn by the author by plotting all the points given in Figure 1 of discussors on one plot. This figure includes points giving relationship between $\frac{1}{2}(\sigma_1-\sigma_3/\sigma_e)$ and σ_3'/σ_e for normally consolidated and overconsolidated montmorillonite clay with two different initial water contents namely, 215 percent and 172 percent. It appears that a common line can be drawn for all the points and this slight stagger of points is acceptable, being commensurate to the accuracy of tests conducted and the difference in Hvorslev parameters with stress history and initial water contents as shown by discussors seems to be uncalled for.

It appears that Hvorslev parameters probably, may not represent the structure of soil. Apart from the test results given by author, the test results the writers have given also support this presumption. In Figure 2 from discussion it is seen that for a given failure water content the failure

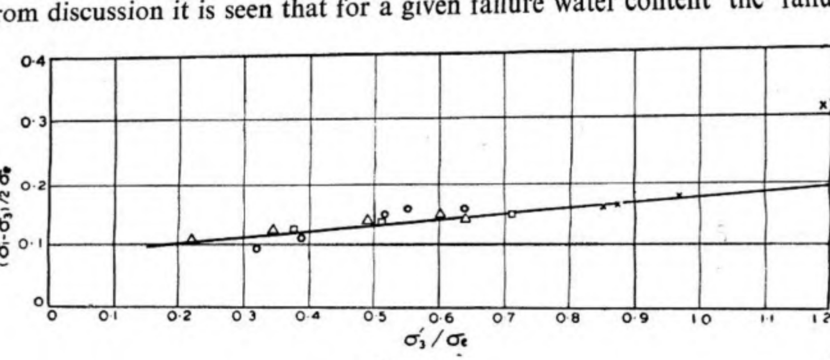

FIGURE 1.

shear stress $\frac{1}{2}(\sigma_1-\sigma_3)$ is more for soil with higher initial water content than for that with lower initial water content, which is logical. It is, however seen, that the magnitude of the angle of internal friction, ϕ_e for a given consolidation pressure σ_3 is less for soil samples with initial higher water content (215 percent) (Figure 4 from discussion), in comparison to the one with initial lower water content (174 percent), for all the values of consolidation pressures. Soil samples with initial higher water content would have flocculated structure in comparison to ones with initial lower water content. It is, therefore, expected that, for a given consolidation pressure, the magnitude of the angle of internal friction, ϕ_e, of soil samples with higher initial water content would be more than of those with lower initial water content. The test results plotted in Figure 4 on the other hand, do not indicate so.

Bjerrum (1954) in Table No. 5.11, P.59, mentioned that method *C* may be used to determine Hvorslev parameters by using either direct shear or triaxial apparatus. Soil sample is consolidated under K_o condition in a direct shear apparatus, whereas, soil sample is consolidated isotropically in triaxial apparatus (according to procedure described by Bjerrum, 1954). If the magnitude of Hvorslev parameters obtained, according to Bjerrum are same irrespective of the type of apparatus used, then it seems that according to him the magnitudes of Hvorslev parameters are independent of type of consolidation also. It is, therefore, inferred that while using method *C* to obtain Hvorslev parameters, one may use the test results of soil samples not only with different initial water contents but also consolidated with different methods of consolidation.

Regarding variation of Hvorslev parameters with strain, the discussors contention that the magnitude of Hvorslev parameters should decrease with increase in shearing strain instead of increasing, is already answered for while discussing the influence of structure of soil on Hvorslev parameters and need not be further elaborated.

The discussors stated that the values at peak stress difference should be taken as Hvorslev parameters. They are requested to refer Schmertmann and Osterberg (1960) who had shown that "the Coulomb-Hvorslev failure criteria equation" is valid at any strain and not only at failure strain.

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