Charts for Moments and Shears in Retaining Walls

by

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Introduction

EVALUATION of moments and shears due to horizontal component of back-fill pressure constitutes an important part in retaining wall calculations. Moments and shears at the base are required for stability analysis. Moments along the stem height are required to proportion the section with due curtailment of reinforcement. Shears along stem height may have to be checked particularly in the case of the counterfort type. These computations, *though not involved, may become tedious if the effects of the* variations in the level of water-table and magnitudes of surcharge are to be investigated at a number of points. Any time saving design aids, therefore, should be welcome.

Two charts presented here can save considerable computation time reducing the numerical work to the minimum. This has been demonstrated by simple examples. Derivations of the basic expressions are included so that the range of graphs and the assumptions involved may be appreciated. Metric units are employed.

Derivation of Expressions

Referring to the loading diagram in Figure 1 or Figure 2, if H= height in metres, K=coefficient of horizontal earth pressure, M=bending moment in tonne metres, S=equivalent height of surcharge in metres, V= shear force in tonnes, Y_b=bulk density of back-fill (=1 tonne per cubic metre for water), Y_s=submerged density of back-fill, and x=a fraction representing height of water-table, then considering one metre length of retaining wall, and assuming triangular load distribution,

$$V = KSHY_b+\tfrac{1}{2}KH^2Y_b\,(1-x^2)+\tfrac{1}{2}KH^2X^2\,(Y_s-1)+\tfrac{1}{2}H^2X^2$$

$$\text{or } V = KHY_b\,(S+\tfrac{1}{2}H)+\tfrac{1}{2}H^2X^2\,[1-K\,(1+Y_b-Y_s)] \qquad \ldots(1)$$

$$M = \tfrac{1}{2}KSH^2Y_b+\tfrac{1}{6}KH^3Y_b\,(1-X^3)+\tfrac{1}{6}KH^3X^3\,(Y_s-1)+\tfrac{1}{6}H^3X^3$$

$$\text{or } M = \tfrac{1}{2}KH^2Y_b\,(S+\tfrac{1}{3}H)+\tfrac{1}{6}H^3X^3[1-K(1+Y_b-Y_s)] \qquad \ldots(2)$$

Expressions (1) and (2) are plotted in Figures 1 and 2 respectively to read V/H and M/H^2 directly. In Figures 1 and 2 K' equals $K(1+Y_b-Y_s)$.

Example 1 : The back-fill of a 10 m high retaining wall has Y_s= 2·0 t/m³, Y_b=1·8 t/m³ and K=0·5. Find total sliding force and over turning moment if (*a*) S=1 m and x=0, (*b*) S=0 and x=0·8.

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Solution : Here $H=10$ m, $KY_b=0{\cdot}9$ t/m³, and $K'=0{\cdot}4$.

(*a*) In Figure 1, from $H=10$, go vertically to $S=1$, thence horizontally to $KY_b=0{\cdot}9$, thence vertically down to read $\frac{V}{H}=5{\cdot}4$. Thus, $V=54$ t. In Figure 2, following the same procedure, read $\frac{M}{H^2}=1{\cdot}95$. Thus, $M=195$ tm.

(*b*) In Figure 1, from $H=10$, go vertically up to $S=0$, thence horizontally to $KY_b=0{\cdot}9$, thence vertically down to read $\frac{V}{H}=4{\cdot}5$. Again, from $Hx=8$, go vertically down to $K'=0{\cdot}4$, thence horizontally to $x=0{\cdot}8$, thence vertically up to read $\frac{V}{H}=1{\cdot}9$. Total of $\frac{V}{H}=4{\cdot}5+1{\cdot}9=6{\cdot}4$. Thus, $V=64$ t. In Figure 2, following the same procedure, read $\frac{M}{H^2}=1{\cdot}5+0{\cdot}5=2{\cdot}0$. Thus, $M=200$ tm.

Example 2 : In example 1 (*b*), stem at 6 m from top has a moment capacity 39·2 tm. Check if the stem is safe.

Solution : In this case, $H=6$ m, $Hx=4$ m, $x^2=0{\cdot}44$, $S=0$, $K'=0{\cdot}4$, $KY_o=0{\cdot}9$.

In Figure 2, following the procedure indicated by arrows in upper and lower parts, $\frac{M}{H^2}=0{\cdot}9+0{\cdot}18=1{\cdot}08$. Thus, $M=1{\cdot}08\times6\times6=38{\cdot}9$ tm. Therefore, the stem is safe.

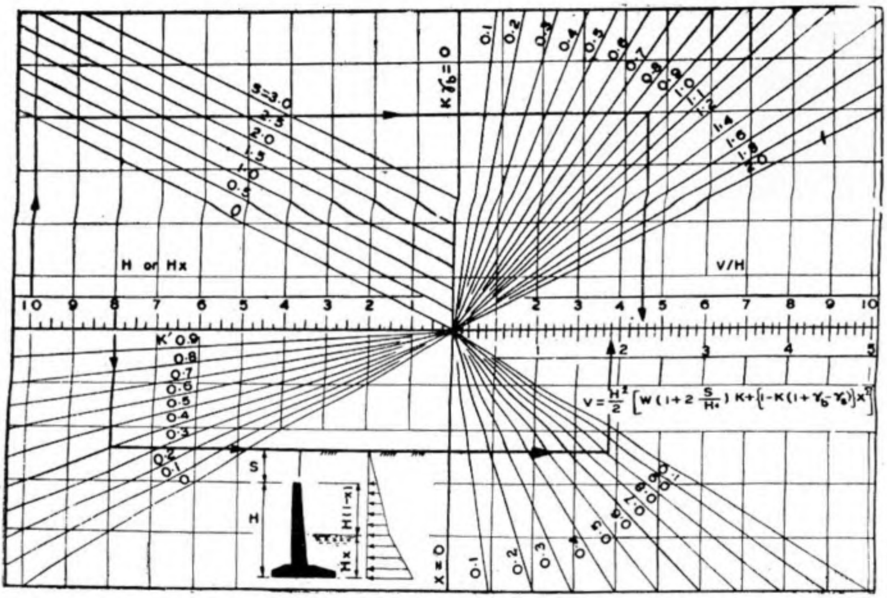

FIGURE 1 : Chart for retaining wall shears.

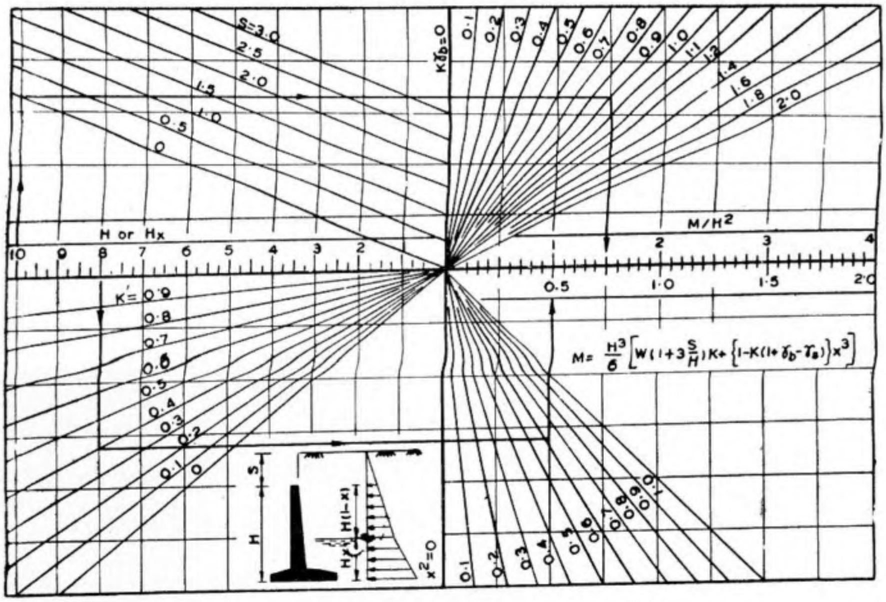

FIGURE 2 : Chart for retaining wall moments.

Example 3 : For a retaining wall with $H=7{\cdot}5$ m, $K=0{\cdot}61$, $S=1{\cdot}5$ m, $Y_s=2{\cdot}0$ t/m^3, $Y_b=1{\cdot}8$ t/m^3 and $x=0{\cdot}8$, draw a bending moment and shear force diagrams for the counterforts spaced 3 m apart.

Solution : This problem can be neatly handled in a tabular form. Noting that $S=1{\cdot}5$, $KY_b=1{\cdot}1$ and $K'=0{\cdot}488$ and measuring H from top, tabulate as follows :

H	H^2	x	x^2	H_x	V/H	M/H^2	$3V$	$3M$
7·5	56 25	0·80	0·640	6·0	7·00	2·54	158·0	414·0
6·0	36·00	0·75	0·563	4·5	5 83	2·1.5	105·0	232·0
4·5	20·25	0·675	0·456	3 0	4·63	1·77	62·5	108·0
3·0	9·00	0·50	0·250	1·5	3 50	1 44	31·5	39·0
1·5	2·25	0·00	0·00	0·00	2·50	1·10	11 3	7·45

Accuracy and Saving in Time

Any chart, graph or nomogram, especially if interpolated, cannot give the accuracy in numerical work comparable to hand calculation. Figures 1 and 2 do have this limitations, especially for smaller values of H, X, K and K'. However, for small values of these variables (say for instance $H=1{\cdot}5$), the minimum practical thickness of stem would govern and great accuracy in numerical work is of little use. Again, any attempt to obtain finer figures may be inconsistent with the accuracy with which the soil parameters may be known. Considering the uncertain and variable nature of soil parameters and assumptions involved in loading pattern, the charts presented here can be safely followed. Enlarging the graphs for office use would add to the convenience of the designer. For preliminary analysis, and if soil properties are not fully known, K' may be taken equal to 0.8 K.

No attempt is made here to compare the time taken in hand calculations and that taken in reading charts. It is obvious, however, that reading charts is much faster than making calculations from first principles, especially if moments and shears are required at various sections under various values of surcharge and water-table.

Conclusion

There is little originality in this paper except the charts. The charts facilitate rapid calculations of moments and shears in retaining walls with accuracy consistent with the assumptions involved in designs of retaining walls and with the uncertainty and variability of soil parameters. It is hoped that these charts would provide useful time saving design aid to the designers.