Non-Dimensional Analysis of Leakage through Layered River Beds

by

G.V. Inamdar*
S.K. Parikh**

Introduction

THE article deals with the comprehensive analysis of leakage around single wall sheet pile coffer dams resting on two layered river bed as shown in Figure 1. The tips of the sheet piles rest on the junction of the two layers.

With a view to cover the complete range of the factors affecting the leakage, thirty six combinations of the non-dimensional factors such as B/D, d/D and K_1/K_2 have been analysed by means of the well-known relaxation technique. Thus the cases having B/D equal to 1, 2 and 4 are studied, wherein for each of the values of B/D, the value of d/D is kept equal to 0·25, 0·50 and 0·75; and in turn for each of the values of d/D the ratio K_1/K_2 is ascribed the values of 1, 2, 4 and 10.

Description of Problem

Even though the river bed is layered each of the layers is isotropic and homogeneous. Assuming the length of the coffer dams to be large in comparison with their cross-sections, Equation (1) the Standard Laplace equation would govern the seepage phenomenon :

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \qquad \ldots(1)$$

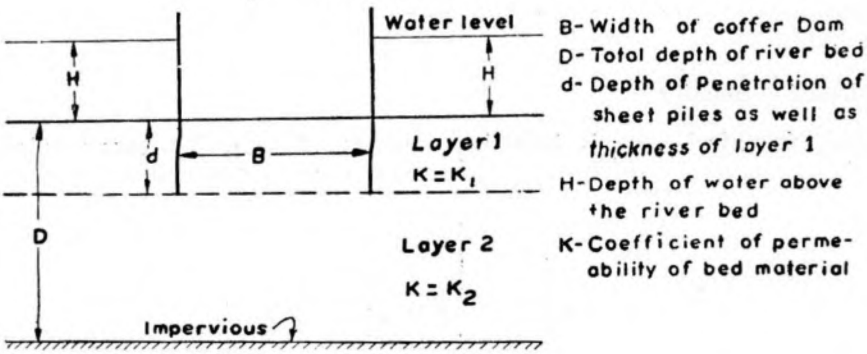


FIGURE 1.

*Lecturer in Civil Engineering
**Assistant Professor of Civil Engineering
} College of Engineering, Poona-5.

This paper was received on 15 March 1972. It is open for discussion up to September 1973.

Where 'ϕ' represents the flow potential and 'x' and 'y' are the reference axes as shown in Figure 2.

For estimating 'Q' the leakage per metre length of the coffer dam Equation (1) should be solved with respect to the following boundary conditions (Figure 2) :

(*a*) The layers extend laterally up to infinity.

(*b*) '*PA*' and '*QF*' being submerged under a constant head '*H*' of water constitute equipotential surfaces with the value of the potential $\phi = H$.

(*c*) Assuming that the space within the coffer dam being maintained dry by pumping out all the leakage received, the surface '*CD*' constitutes another equipotential boundary with the value of potential $\phi = 0$.

(*d*) The junctions '*ABC*' and '*DEF*' between sheet piles and the river bed layer are streamlines. Hence, the condition $\frac{\partial \phi}{\partial x} = 0$ should be satisfied along the portions '*AB*', '*BC*', '*DE*' and '*EF*'. Similarly at the points '*B*' and '*E*', representing the tip of the piles, the condition $\frac{\partial \phi}{\partial y} = 0$ should prevail.

(*e*) The x-axis coincides with the impervious surface, thus it is a streamline along which the condition $\frac{\partial \phi}{\partial y} = 0$ should be satisfied.

(*f*) At any point along '*RS*' the junction between the two layers, the flow velocity at the entrance should be equal to the flow velocity at the exit.

The solution so obtained would furnish the distribution of 'ϕ' within the seepage medium, which can in turn be utilised for the estimate of the leakage 'Q'.

Method of Solution

Numerical analysis involving concepts of finite difference coupled with the well-known relaxation technique was employed to solve Equation

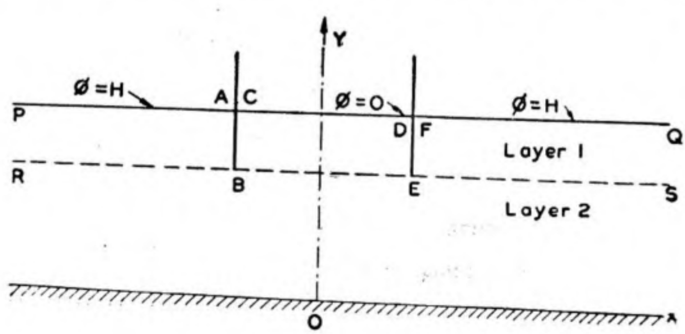


FIGURE 2.

(1). To facilitate such computations the boundary conditions as described below were adopted (Figure 3).

(a) LATERAL EXTENT

In any study except mathematical the infinite lateral extent of the seepage zone cannot be and need not be undertaken. By now it is well established that to obtain the results of acceptable accuracy, the lateral extent of up to two times the depth of the pervious strata may be considered to be equivalent of the infinite lateral extent. Thus it is in order to treat the vertical line '*PQ*' at a distance of '2*D*' from the sheet pile as an impervious boundary. Naturally the condition $\frac{\partial \phi}{\partial x}=0$ should be satisfied along '*PQ*'.

(b) LINE OF SYMMETRY

As is seen from Figure 2, the centre line '*Oy*' of the coffer dam happens to be a line of symmetry. Consequently the distribution of 'ϕ' on either side of this line would be identical. Due to this there is no flow across this line. Thus the vertical line '*SR*' passing through the centre of the coffer dam can be treated as an impervious boundary over which the condition $\frac{\partial \phi}{\partial x}=0$ should be satisfied.

(c) OTHER BOUNDARY CONDITIONS

The other boundary conditions remain same as already described. The region shown in Figure 3, was covered with a square grid. The effect of the grid size on the accuracy of solution was studied to a great extent for an arbitrary but similar problem and finally a grid size of *D*/4 by *D*/4 was adopted as the common basis for the entire investigation.

Adopting the nodal point notations as shown in Figure 4 (*a*) and applying the Taylor's expansion theorem, Equation (1) could be transformed into equivalent finite difference equations in terms of the nodal 'ϕ' values as given below (Scott, 1963) :

$$\phi_1+\phi_2+\phi_3+\phi_4-4\phi_o=0 \qquad \text{...(2)}$$

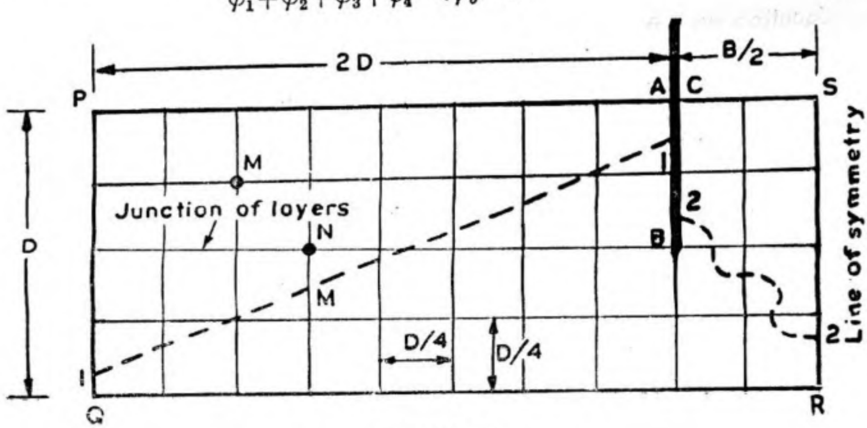


FIGURE 3.

$$\phi_1+\frac{2K_1}{K_1+K_2}\cdot\phi_2+\phi_3+\frac{2K_2}{K_1+K_2}\cdot\phi_4-4\phi_o=0 \qquad ...(3)$$

With reference to Figure 3, it may be noted that Equation (2) holds for the nodal points within the layer such as '*M*', whereas Equation (3) is applicable to the nodal points lying on the junction such as '*N*'. Following scheme was adopted for obtaining the nodal 'ϕ' values.

(1) The nodal points on the equipotential boundary $\phi=H$ (*viz.*, '*PA*') were ascribed an arbitrary value of 1,000.

(2) The nodal points on the equipotential boundary $\phi=0$ (*viz.*, '*CS*') were ascribed value of zero.

(3) The conditions such as $\frac{\partial\phi}{\partial x}=0$, $\frac{\partial\phi}{\partial y}=0$ ...... etc., along the impervious or stream boundaries were fulfilled by creating the necessary mirror images such as shown in Figures 4(*b*) and 4(*c*).

(4) A set of linear algebraic equations in terms of the unknown nodal 'ϕ' values could be written with the help of Equation (2) or (3).

(5) The set of equations so established were solved by applying the standard point, line and block relaxation technique.

A typical numerical solution is presented in Figure 5.

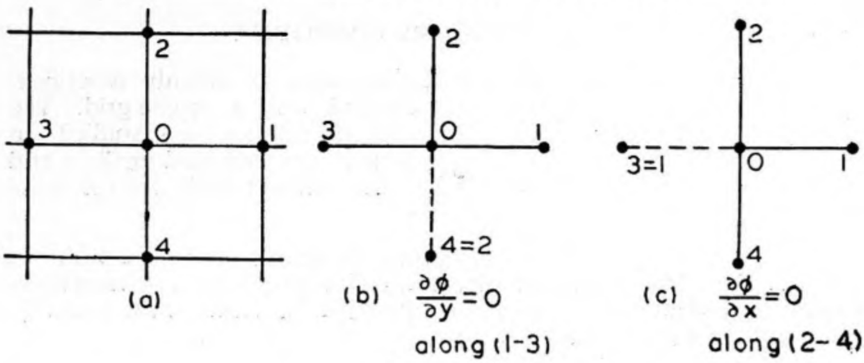


FIGURE 4.

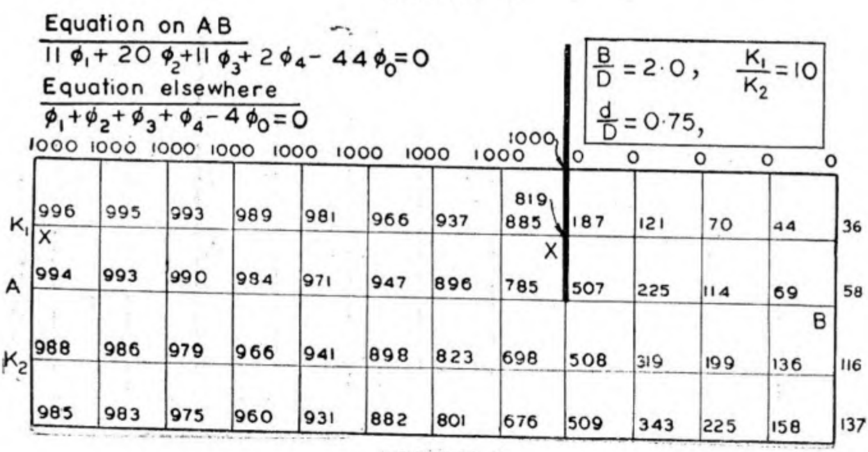


FIGURE 5.

Computation of Leakage

In Figure 3, it may be noted that any section, either straight or curved such as the Sections (1-1) or (2-2), spanning the stream boundaries '*ABC*' and '*PQRS*' intercepts half the leakage ($Q/2$), the remaining half being contributed from the region to the right of the line of symmetry. So any such section can be employed for the numerical estimate of 'Q'.

For example, consider the Section (1-1). Let the horizontal and vertical components of the flow through an elementary area A_n be governed by the gradients i_{x_n}, i_{y_n} and the cross-sectional areas A_{x_n}, A_{y_n} as shown in Figure 6. Then :

$$Q/2 = \sum_{n=1}^{n=N} (K_x \cdot A_{x_n} \cdot i_{x_n} + K_y \cdot A_{y_n} \cdot i_{y_n})$$

$$\therefore \quad Q = 2\sum_{n=1}^{n=N} (K_x \cdot A_{x_n} \cdot i_{x_n} + K_y \cdot A_{y_n} \cdot i_{y_n}) \qquad \text{...(4)}$$

Where, K_x and K_y are the coefficients of permeability in 'x' and 'y' directions respectively.

As an illustration the leakage 'Q' for the case reproduced in Figure 5, is estimated below. The horizontal section 'XX' is considered for this purpose. For such a section 'A_{x_n}' the cross-sectional area normal to the x-axis at various nodal points becomes equal to zero. Also $K_y = K_1$, because all the nodal points are in the layer 'I'. Substituting these in Equation (4), the Equation (5) is obtained.

$$Q = 2K_1 \sum_{n=1}^{n=N} i_{y_n} \cdot A_{y_n} \qquad \text{...(5)}$$

The value of 'i_{y_n}' at the nodal points can be obtained by means of Equation (6), which could easily be derived with the help of the Taylor's expansion theorem (Scott, 1963) :

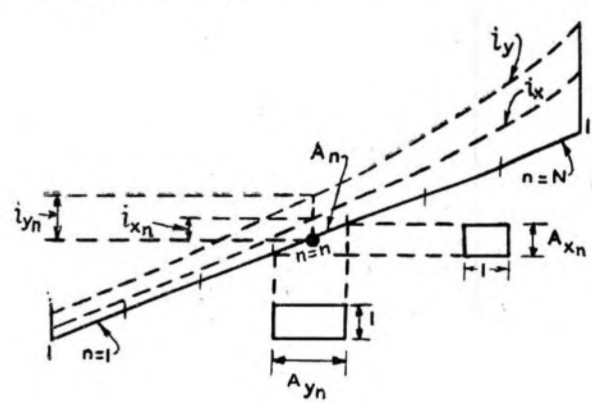


FIGURE 6.

$$i_{y_n}=\frac{\phi_{2n}-\phi_{4n}}{2\times D/4}=\frac{2(\phi_{2n}-\phi_{4n})}{D} \qquad ...(6)$$

Substituting Equation (6) into Equation (5), Equation (7) is obtained:

$$Q=\frac{4K_1}{D}\sum_{n=1}^{n=N}(\phi_{2n}-\phi_{4n})\times A_{y_n} \qquad ...(7)$$

For the end nodal points 'X' and 'X' the area 'A_{y_n}' is given by $D/4\times 1/2=D/8$, whereas for the remaining nodal points it is equal to $D/4\times 1=D/4$. Thus:

$$Q=\frac{4K_1}{D}\Big[(1000-994)\times\frac{D}{8}+(1000-993)\times\frac{D}{4}+(1000-990)\times\frac{D}{4}$$

$$+(1000-984)\times\frac{D}{4}+(1000-971)\times\frac{D}{4}+(1000-947)\times\frac{D}{4}$$

$$+(1000-896)\times\frac{D}{4}+(1000-785)\times\frac{D}{4}+(1000-507)\times\frac{D}{8}\Big]$$

$$Q=683{\cdot}5\ K_1. \qquad ...(8)$$

As 'H' was equated arbitrarily to 1000, the coefficient 683·5 in Equation (8) is in fact equal to $0{\cdot}6835\ K_1H$. Hence :

$$Q=0{\cdot}6835\ K_1H \qquad ...(9)$$

Non-Dimensional Parameters

To enhance the utility of the present investigation a pseudo-discharge term 'Q_o' and certain non-dimensional parameters were introduced as follows.

(a) Q_o PSEUDO-DISCHARGE TERM

Simple Bligh's concepts have been employed to define 'Q_o' the pseudo-discharge term. The value of 'Q_o' is estimated by the scheme given below.

Consider BB' the vertical cross-section below the tip of one of the sheet piles (Figure 7). The section provides a flow area of $(D—d)$, having

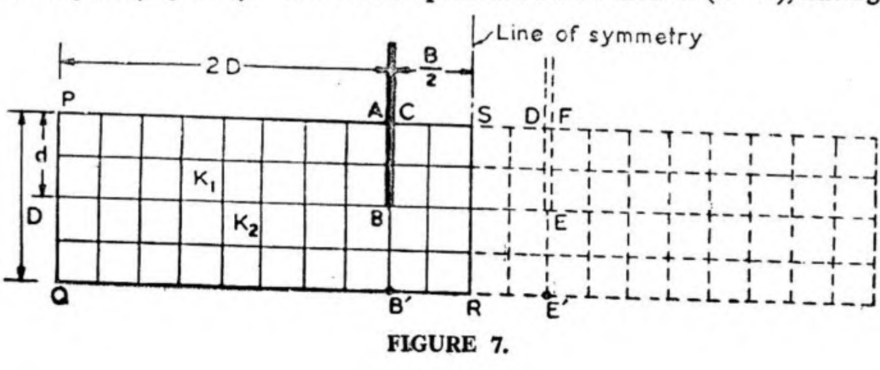


FIGURE 7.

the coefficient of permeability of K_2. If the average hydraulic gradient for the flow through the section BB' is 'i' then Equation (10) follows from the Darcy's law :

$$\frac{Q_o}{2}=K_2(D-d)\times i \qquad ...(10)$$

$Q_o/2$ is also contributed from the vertical cross-section EE' making the total Q_o for the coffer dam.

The average hydraulic gradient 'i' is determined in the following manner.

According to the Bligh's creep concepts, it may be argued that the stream boundary 'ABC' has a creep length L_1 given by Equation (11) :

$$L_1=AB+BC \qquad ...(11)$$

On the similar basis the impervious boundary '$PQRS$' has the creep length L_2 defined by Equation (12) :

$$L_2=PQ+QR+RS \qquad ...(12)$$

The gradients of flow along these creep lengths are 'i_1' and 'i_2' as defined in Equation (13):

$$\left.\begin{array}{ll}(a) & i_1=H/L_1 \\ (b) & i_2=H/L_2\end{array}\right\} \qquad ...(13)$$

It may be argued that 'i_1' and 'i_2' are the gradients at the points B and B' respectively, and further that the variation of the gradient along BB' is linear. Thus 'i' the average gradient is given by Equation (14) :

$$i=\frac{i_1+i_2}{2}=\frac{H}{2}\left(\frac{1}{L_1}+\frac{1}{L_2}\right) \qquad ...(14)$$

Substituting Equation (14) in Equation (10), Equation (15) is obtained:

$$Q_o=HK_2(D-d)\left[\frac{1}{L_1}+\frac{1}{L_2}\right] \qquad ...(15)$$

Let $n=\frac{K_2}{K_1}$, hence;

$$Q_0=nK_1H(D-d)\left[\frac{1}{L_1}+\frac{1}{L_2}\right] \qquad ...(16)$$

(b) NON-DIMENSIONAL PARAMETERS

The non-dimensional parameters N_Q, N_{Q_o}, and R are defined as shown in Equation (17) :

$$\begin{array}{ll}(a) & N_Q=\frac{Q}{K_1.H} \\ (b) & N_{Q_o}=\frac{Q_o}{K_1.H} \\ (c) & R=\frac{N_Q}{N_{Q_o}}=\frac{Q}{Q_o}\end{array} \qquad ...(17)$$

It may be noted that the parameter 'R' co-relates actual discharge 'Q' with the easily calculable pseudo-discharge Q_0.

As an illustration, the value of 'R' for the case shown in Figure 5, is computed below :

(*i*) N_Q —From Equations (9) & (17*a*), it follows that $N_Q = 0{\cdot}6835$.

(*ii*) N_{Q_0}—Following informations are obtained from Figure 5.

$$n = \frac{K_2}{K_1} = \frac{1}{10}\ ;\ (D-d) = \frac{D}{2}$$

$$L_1 = 2\times\frac{D}{4} + 2\times\frac{D}{4} = D\ ;\quad L_2 = 4\times\frac{D}{4} + 12\times\frac{D}{4} + 4\times\frac{D}{4} = 5D$$

Substituting these informations in Equations (16) & (17*b*) :

$$N_{Q_0} = 0{\cdot}06.$$

(*iii*) Using the values of N_Q and N_{Q_0} calculated above :

$$R = \frac{N_Q}{N_{Q_0}} = \frac{0{\cdot}6835}{0{\cdot}06} = 11{\cdot}4.$$

In the same manner the values of these parameters have been computed for all the cases and they are shown in Table I.

Nature of Parameter 'R'

Figure 8 shows the plot of 'R' versus K_1/K_2. The linear relationship indicates the unifying character of 'R'. It is of interest to note that N_Q could not be represented linearly. In view of this the importance of 'R' in the present investigation becomes obvious.

It is of further interest to note that the slope angle $\theta°$ and the intercept C_1 on the vertical axis, of various straight lines in Figure 8 when plotted against non-dimensional factors d/D and B/d gave another set of straight lines as shown in Figures 9(*a*) & 9(*b*) respectively. The ratio B/d may be viewed as $\frac{B/D}{d/D}$.

The straight lines in Figure 9 in turn revealed following interesting characteristics :

(*a*) All the straight lines in Figure 9(*a*) were parallel having the slope angle of $80{\cdot}2°$.

(*b*) The intercept C_2 on the vertical axis of various straight lines in Figure 9(*a*) and the intercept C_3 on the vertical axis of various straight lines in Figure 9(*b*) as well as their slopes $m = \tan\phi$ when plotted against $\log_{10} B/D$ gave linear relationship as shown in Figure 10(*a*), (*b*) & (*c*) respectively.

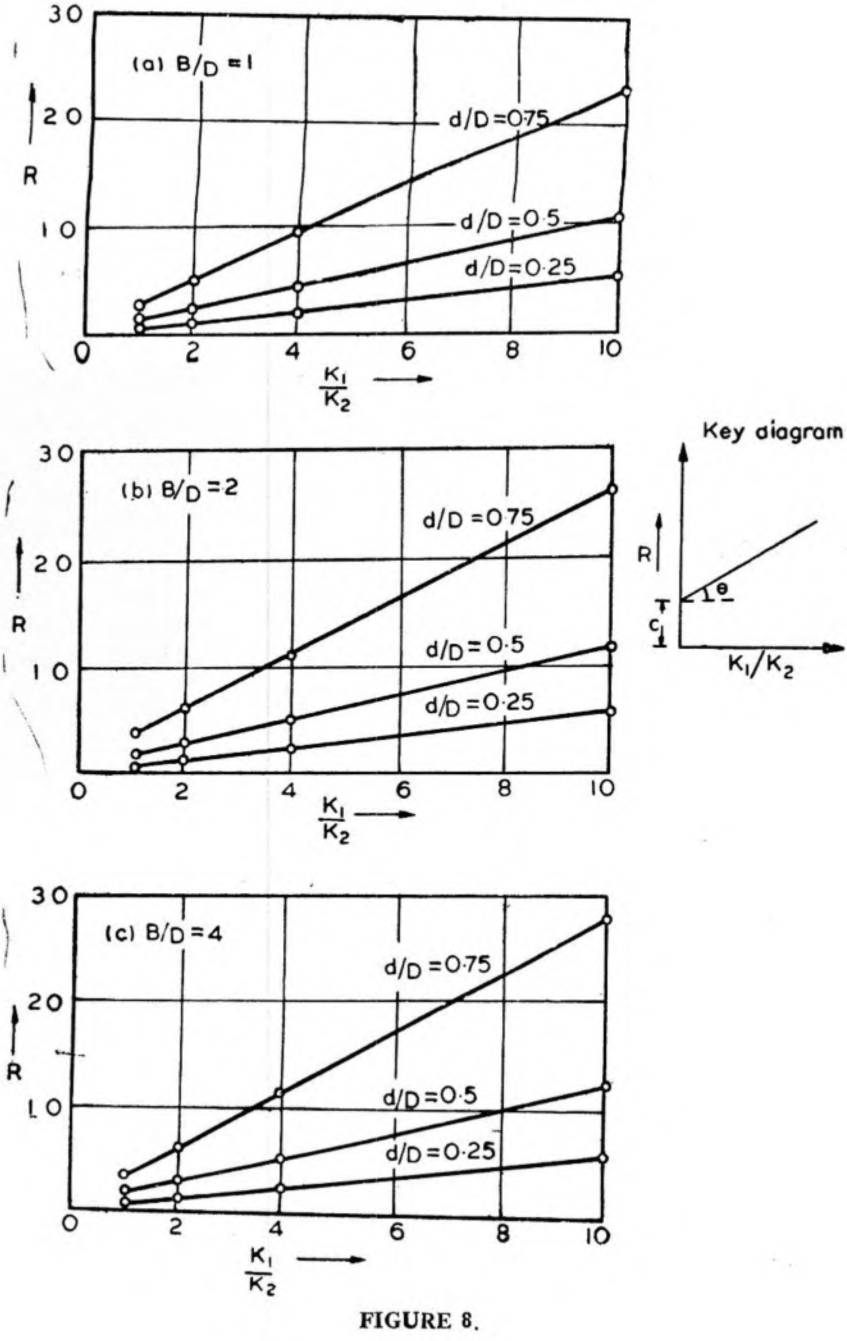
The method of least squares (Fair and Geyer, 1954) was employed to obtain the best fit for all the straight lines appearing in Figures 8, 9 & 10. Working upon the numerical values of their slopes,

TABLE I

B/D	d/D		K_1/K_2			
			1	2	4	10
1	0·25	N_Q	0·986	0·914	0·846	0·822
		N_{Q_0}	1·668	0·834	0·418	0·167
		R	0·590	1·100	2·020	4·930
	0·50	N_Q	0·908	0·776	0·706	0·640
		N_{Q_0}	0·612	0·306	0·153	0·061
		R	1·480	2·540	4·610	10·460
	0·75	N_Q	0·626	0·578	0·542	0·512
		N_{Q_0}	0·222	0·111	0·056	0·022
		R	2·820	5·210	9·760	23·060
2	0·25	N_Q	1·194	1·052	0·984	0·924
		N_{Q_0}	1·650	0·826	0·412	0·165
		R	0·720	1·270	2·390	5·600
	0·50	N_Q	1·092	0·880	0·770	0·688
		N_{Q_0}	0·600	0·300	0·150	0·060
		R	1·820	2·940	5·130	11·470
	0·75	N_Q	0·776	0·664	0·598	0·568
		N_{Q_0}	0·216	0·108	0·054	0·022
		R	3·590	6·130	11·030	26·300
4	0·25	N_Q	1·224	1·068	0·996	0·940
		N_{Q_0}	1·626	0·814	0·407	0·163
		R	0·750	1·310	2·450	5·780
	0·50	N_Q	1·122	0·896	0·776	0·692
		N_{Q_0}	0·584	0·292	0·146	0·058
		R	1·920	3·070	5·320	11·850
	0·75	N_Q	0·788	0·668	0·614	0·574
		N_{Q_0}	0·208	0·104	0·052	0·021
		R	3·790	6·410	11·760	27·600

intercepts, etc., Equation (18) the generalised equation for 'R' representing all the thirty-six cases could be derived:

$$R=\frac{1}{n}\,.\,\tan\left[80{\cdot}20\,p+5{\cdot}5\log_{10}10{\cdot}26\,q\right]$$
$$+0{\cdot}22\left[4{\cdot}22+\frac{q}{p}\right].\log_{10}q$$



30
20
10
R
(a) B/D = 1
d/D = 0.75
d/D = 0.5
d/D = 0.25
0
2
4
6
8
10
K1/K2
30
20
10
R
(b) B/D = 2
d/D = 0.75
d/D = 0.5
d/D = 0.25
K1/K2
Key diagram
R
θ
c
K1/K2
30
20
10
R
(c) B/D = 4
d/D = 0.75
d/D = 0.5
d/D = 0.25
K1/K2

FIGURE 8.

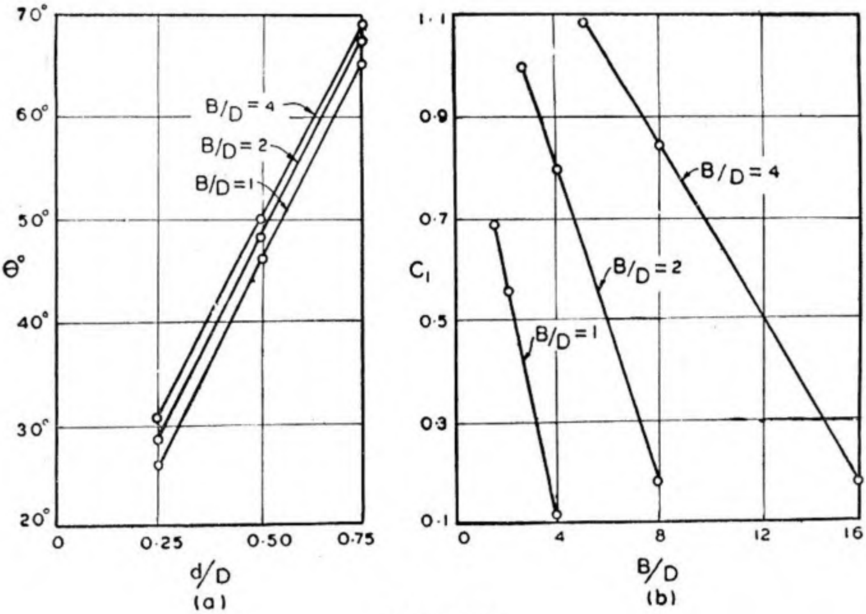


FIGURE 9.

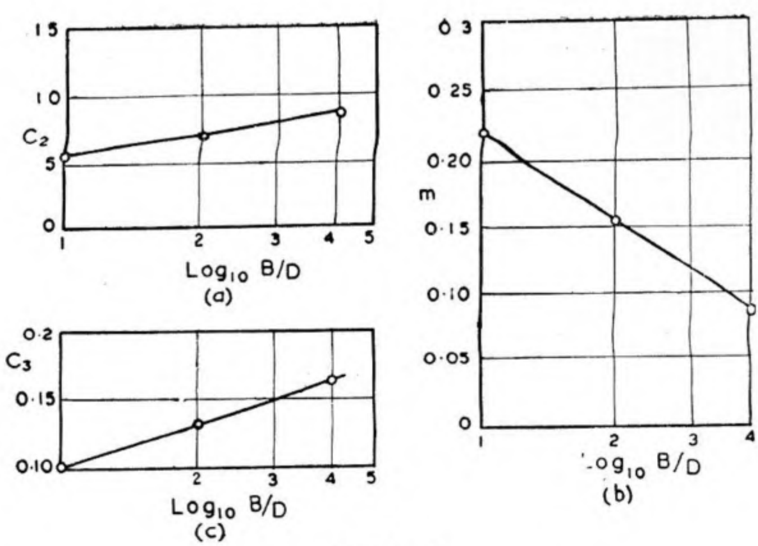


FIGURE 10.

$$+0{\cdot}22\left[4{\cdot}74-\frac{q}{p}\right] \qquad \ldots(18)$$

where, $n=\frac{K_2}{K_1}$; $p=\frac{d}{D}$; $q=\frac{B}{D}$.

Conclusions and Remarks

A systematic and reliable study of the leakage around various coffer dams resting on the layered river bed was made possible by relaxation technique. A novel non-dimensional parameter 'R' could be defined, so that the leakage for all the cases is represented by a single algebraic expression shown in Equation (18). Though this equation is not very attractive for the immediate practical use, it certainly reveals the scientific nature of the problem.

For higher ratio of K_1/K_2 a random case was studied with $K_1/K_2=100$. The results proved the validity of Equation (18). This indicates that the Equation (18) may be true, for all the possible ratios of K_1/K_2, even not covered in the investigation.

Acknowledgement

The article is based upon the thesis submitted by Shri G.V. Inamdar to the University of Poona, for partial fulfilment of M.E. (Civil) course.

The authors take the opportunity to express their heartfelt thanks to Prof. S.V. Kulkarni, the Head of Civil Engineering Department and Prof. Y.S. Sane of the Civil Engineering Department, College of Engineering, Poona, for their encouragement in preparing the article.

Thanks are also due to Shri N.B. Vadnere, who prepared the drawings appearing in the article.

References

FAIR, G.M. and GEYER, J.C. (1954) : "Water Supply and Waste Water Disposal". John Wiley and Sons.

SCOTT, R.F. (1963) : "Principles of Soil Mechanics". Addision-Wesely Publishing Company. Inc.