The Energy Function of the Penetration Process in Heavy Clay (C.H.)

by

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Introduction

SOIL systems are normally composed of solid, liquid and gaseous phases. The composition and character of these phases vary from soil to soil. In this paper, the discussion and the analysis are limited to Heavy Clay or pure cohesive soils.

The penetration process such as the pile driving or soil cutting or soil loosening for soil reconditioning purpose affects the soil properties by remoulding soft and fine grained soils. It produces a basic type of deformation, intrinsic only in multi-dispersed systems. Deformation of soil and ground is accompanied by changes of structure and porosity, movement of individual particles, and flow of water and gas.

The first part of the treatment of the penetration process terminates with the development of the penetration function that is, the law of soil resistance to penetration. The second and properly thermodynamic part is concerned with the energies involved. An important portion of these is the external work required for the penetration and this will be very helpful in estimating the energy required or the horsepower needed in driving a pile or in soil-cutting or loosening process.

With respect to the type of energy exchange involved, the penetration process of a soil system involves in the conversion of mechanical energy to heat energy : Mechanical work is applied to the system and heat is created ; but the efficiency of this operation is affected by further densification : only a portion of the total work is changed into internal energy and heat, while another noteworthy portion is used up in the fragmentation of component particles of the system.

Analysis and Development of Theories

Considerable work, both past and present, has been devoted to the study of soil wheel interactions as related to sinkage or penetration process in soil. Since theory has proved inadequate to describe the sinkage of footings, various empirical relations have been developed, the most often used for vehicle flotation being that of Bekker (1956), which can be written in the form, $p = k\left(\frac{z}{a}\right)^N$ ...(1)

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Or more generally,

$$p = (ak_{\phi} + k_c)\left(\frac{z}{a}\right)^N \qquad ...(2)$$

where, p = Load per unit area of the plate or area of the penetrating plunger,

z = Sinkage of plate or plunger below the surface,

k = Constant,

N = Dimensionless coefficient.

k_c, k_{ϕ} are constants and functions of cohesion and friction angle of soil respectively. "a" is an approximate length dimension for the loading being considered. *The pertinent length is usually the smallest one* associated with the loaded area. In the case of tire, it appears that the tire width should be used for "a", and this has been the usual practice in the early soil-penetration analysis. The first phase of a penetration diagram of soil or ground being penetrated with a plunger is sometimes observed as a straight line section, which is an approximation with N equal to "one" in Equations (1) and (2).

A more general relationship between load and deformation of soil is based on the so-called contact theory of soil strength. In accordance with this theory, M.N. Troitskaya (1968) proposed the following equations for determining the stress "p" in relation to the magnitude of relative deformation, $\lambda = z/l$ (l=Equivalent height of deformed layer), during penetration in a closed volume,

$$p = p_c\,(e^{L\lambda} - 1) \qquad ...(3)$$

During shear, $p = p_s\,(1 - e^{L\lambda})$ ...(4)

During simultaneous penetration and shear,

$$p = p_s\,\frac{p_c\,(e^{L\lambda} - 1)}{p_s + p_c\,e^{L\lambda}} \qquad ...(5)$$

where, p_s = Limit of bearing capacity of the soil in psi, or kilos/cm²

p_c = The initial hardness in psi, or kilos/cm²

L = Relative coefficient of stiffness (Dimensionless value).

Y.V. Katsygin (1968) showed by experimental verification that all the above equations had limited application. However, with the help of examination and experimental results, he established the following hyperbolic function to govern the law of soil resistance to penetration :

$$p = p_o \tanh\left(\frac{k}{p_o}\,z\right) \qquad ...(6)$$

where, p_o = Limit of bearing capacity of soil psi, or kilos/cm²,

k = Coefficient of volume compaction of soil, pci, or kilos/cm³,

z = Penetration in inches, or centimetres.

Determination of "k" and "p_o" in Equation (6) (Katsygin, 1968) If p_1 is the stress corresponding to a penetration H_1, and p_2 is the stress corresponding to a penetration H_2, and if $H_2=2H_1$, then

$$p_o = \frac{p_1}{\sqrt{2p_1/p_2-1}} \quad \text{and} \quad k = \frac{p_o}{H_1} \tanh^{-1}\left(\frac{p_1}{p_o}\right).$$

This p_o is plotted against corresponding CBR in Figure 1 for the heavy clay at different water contents.

Development of Energy Function of the Penetration Process

Employing the original definition of mechanical work, the work performed is the product of the force acting at the system boundaries and the translation of the boundaries in the direction of the force. The differential of the work is then :

$$dw = pdv = pAdz$$

or

$$dw = A\left[p_o \tanh\left(\frac{k}{p_o} z\right) dz\right] \quad ...(7)$$

where, A = Area of the penetrating plunger. If a boundary translation process moves the system from state 1 (Penetration is "zero") to state 2 (Penetration is "z") then, Equation (7) gives :

$$w = A. \frac{p_o^2}{k} . \log_e \cosh\left(\frac{k}{p_o} z\right)$$

So, penetration energy per unit area of penetration is :

$$E = \frac{w}{A} = \frac{p_o^2}{k} . \log_e \cosh\left(\frac{k}{p_o} z\right) \quad ...(8)$$

Here, E is in kilos-cm/cm² or, E is in pound-inch per square inch. This E is plotted against z in Figure 2 for heavy clay at different water contents.

In accordance with the energy principle, all energies involved in a process are conserved. Converting the mechanical energy to heat energy (Alfred Holl, 1969),

$$w/J = Mc \triangle t \quad ...(9)$$

where, $M = \rho V = \rho(Az)$,

ρ = Mass density (specific gravity),

c = Specific heat capacity,

$\triangle t$ = Rise of temperature,

J = Joule's constant,

and $1/J = 2{\cdot}34 \times 10^{-3}$ that is, one kilogram-metre mechanical energy will produce $2{\cdot}34 \times 10^{-3}$ kilo-calori of heat.

From Equations (8) and (9) $E/J = \rho zc \triangle t$ ...(10)

If E is in pound-inch per square inch, then, $E' = E/J = 4{\cdot}15 \times 10^{-6}\, E$ ...(11)
= Kilo-calori per sq centimetre.

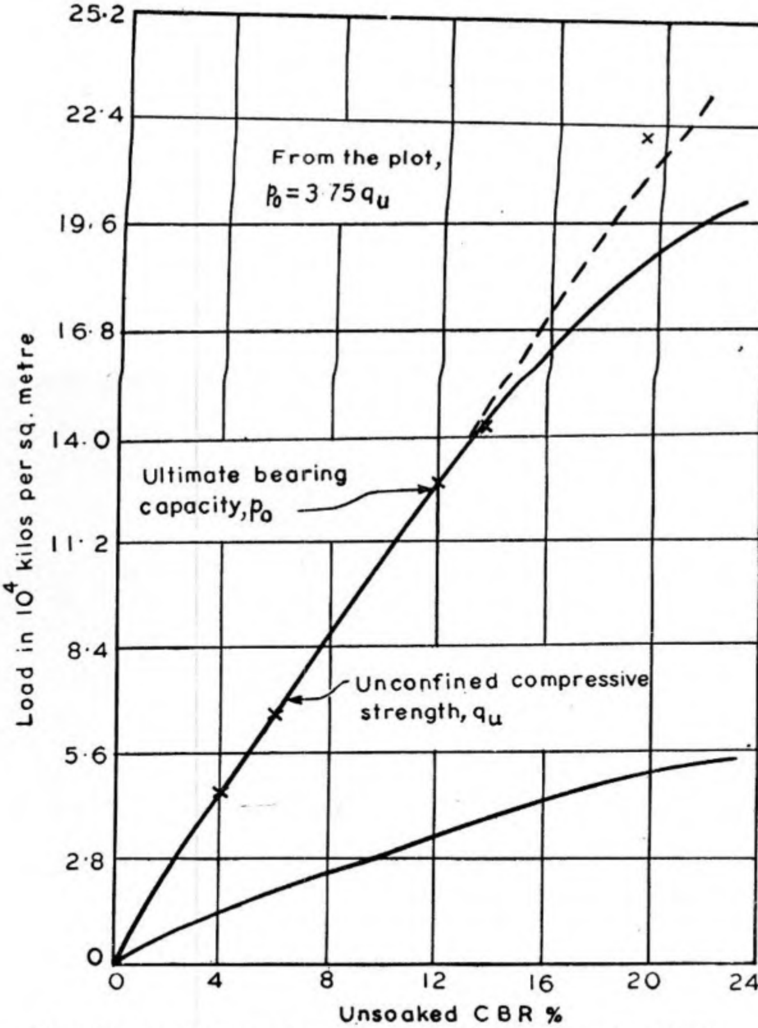

FIGURE 1 : CBR versus p_o and CBR versus q_u for heavy clay (C.H.).

If ρ is in gram per cubic centimetre,

$= 2{\cdot}70$ gm/cc for heavy clay,

$c = 0{\cdot}20$ kilo-calori/kilogram, °*c* for heavy clay,

Then from Equations (10) and (11),

$$\triangle t\ (^{o}c) = \frac{E'}{1{\cdot}37 \times 10^{-3}\, z} \qquad \ldots(12)$$

Figure 2 gives the relationship between $\triangle t$ and z for the heavy clay at different water contents.

Test Data

Heavy clay : $LL = 55$

$PI = 28$

$SL = 10$; $SI = 16{\cdot}8$

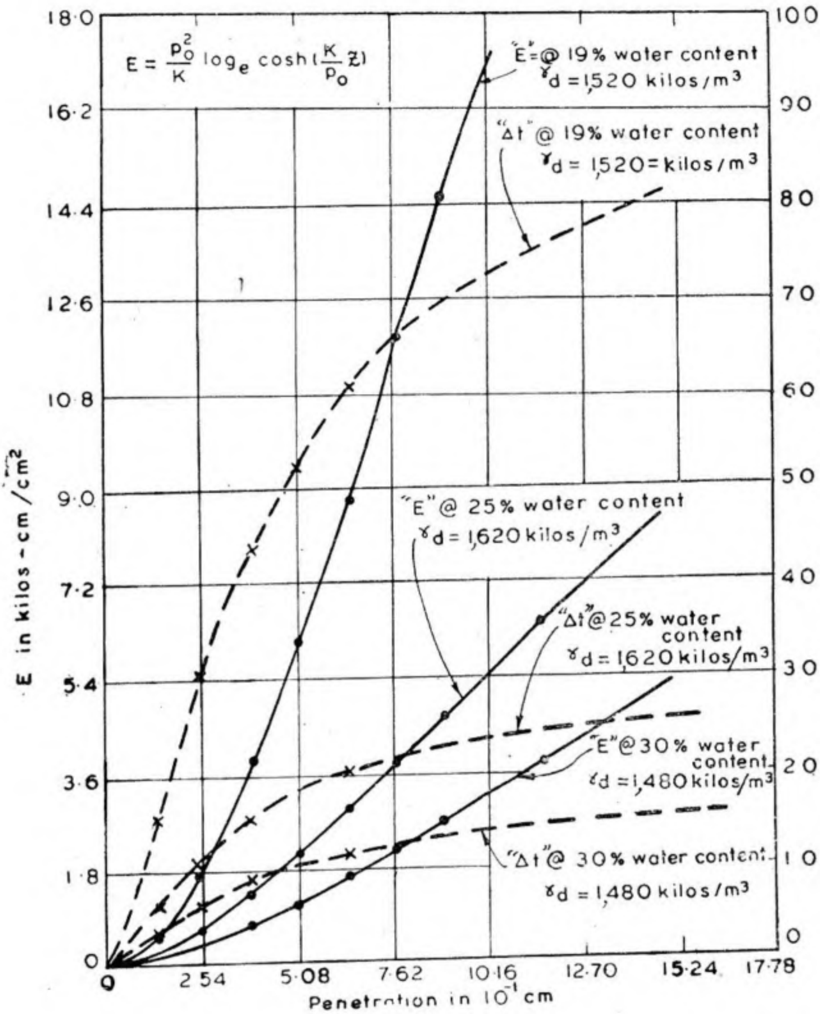

FIGURE 2 : Energy-temperature-penetration relationship of heavy clay (C.H.).

100 percent finer than 0·1 mm and 43 percent finer than 5 μ

CBR plunger : (for penetration test)	1·98 inch diameter, and 3 sq cm area or, 5 cm diameter, and 19·2 cm² area
CBR mould : (Preparation of specimen)	6 in. diameter, or 15·24 cm diameter 5 layers, 25 blows/layer, 10 lbs Hammer @ 18 in. drop, or 4·54 kilos @ 45·72 cm drop

TABLE I

Heavy clay at 30 percent water content $\gamma_d = 91$ pcf, or 1,456 kilos/m³

Unconfined compressive strength $q_u = 17$ psi, or 1·19 kilos/cm²

Unsoaked CBR = 3·8 percent

ρ (mass density) = 2·7 grams/cc

c (specific heat capacity) = 0·2 kilo-calori kg °C

Penetration "z" (inch)	Load p^i (psi)	$\frac{k}{p_o} z$	$\tanh\left(\frac{k}{p_o} z\right)$	$p^i_c = p_o \tanh\left(\frac{k}{p_o} z\right)$ (calculated) (psi)
0·05	23	0·375	0·358	21·5
0·10	38	0·750	0·630	37·8
0·15	47	1·125	0·810	48·6
0·20	54	1·500	0·905	54·3
0·25	60	1·875	0·954	57·2
0·30	64	2·250	0·978	58·7
0·35	67	2·625	0·990	59·4
0·40	70	3·000	0·995	59·7

1 in. = 2·54 cm ; 1 psi = 0·07 kilos/cm².

Calculation

$p_1 = 38$ psi, $z_1 = 0{\cdot}10$ in. or 0·254 cm
or 2·66 kilos/cm²

$p_2 = 54$ psi, $z_2 = 0{\cdot}20$ in. or 0·508 cm
or 3·78 kilos/cm²

$$p_o = \frac{p_1}{\sqrt{\frac{2\,p_1}{p_2} - 1}} = 4.2 \text{ kilos/cm}^2 = 60 \text{ psi (ultimate bearing capacity)}$$

$$x = \tanh^{-1}\sqrt{\frac{2p_1}{p_2} - 1} = 0.75$$

$$k = \frac{p_o}{z_1}\,x = \frac{60}{0.10} \times 0.75 = 450 \text{ lb/cu in.}$$

or 12·6 kilos/cm³.

TABLE IA

Energy and temperature rise calculated from the experimental data in Table I.

Penetration "z" (inch)	$\frac{k}{p_0} z$	$\frac{p_o^2}{k}$ lb/in.	$\text{Cosh}\left(\frac{k}{p_o} z\right)$	$\log_e \cosh \left(\frac{k}{p_o} z\right)$	$E = \frac{p_o^2}{k} \log_e \cosh \frac{k}{p_o} z$ pound inch per in.^{2}	$E' = \frac{E}{J} = 4{\cdot}15 \times 10^{-6} E$ k. cal/cm^2	$\Delta t\,(^\circ\text{C}) = \frac{E'}{1{\cdot}37 \times 10^{-3} z}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0·05	0·375	8·0	1·072	0·070	0·560	$2{\cdot}3 \times 10^{-6}$	$34\ 0 \times 10^{-3}$
0·10	0·750	8·0	1·296	0·262	2·096	$8{\cdot}6 \times 10^{-6}$	$63{\cdot}0 \times 10^{-3}$
0·15	1·125	8·0	1·740	0·560	4·480	$18{\cdot}5 \times 10^{-6}$	90.0×10^{-3}
0·20	1·500	8·0	2·352	0·850	6·800	$28{\cdot}1 \times 10^{-6}$	$102{\cdot}5 \times 10^{-3}$
0·25	1·875	8·0	3·340	1·200	9·600	$39{\cdot}7 \times 10^{-6}$	$116{\cdot}0 \times 10^{-3}$
0·30	2·250	8·0	4·685	1·540	12·320	$51{\cdot}0 \times 10^{-6}$	$123{\cdot}3 \times 10^{-3}$
0·35	2·625	8·0	6·945	1·940	15·520	$64\ 2 \times 10^{-6}$	$134{\cdot}3 \times 10^{-3}$
0·40	3·000	8·0	10·068	2·310	18·480	$76{\cdot}2 \times 10^{-6}$	$140{\cdot}0 \times 10^{-3}$

1 in. = 2·54 cm ; 1 lb/in. = 0·178 kilos/cm ; 1 lbs in./in^2 = 0·178 kilos/cm^2

TABLE II

Heavy clay at 28 percent water content: γ_d = 95·5 pcf; unsoaked CBR = 5·7 percent
= 1,528 kilos/m³

Unconfined compressive strength, q_u = 24 psi,
= 1·68 kilos/cm²

Penetration "z" (inch)	Load, p_i (psi)	$p_o = \frac{p_1}{\sqrt{2p_1/p_2 - 1}}$ (psi)	Remarks
0·10	57	92	Here, p_1 = 57 psi = 4 kilos/cm² p_2 = 82 psi = 5·7 kilos/cm² z_1 = 0·10 *in.* = 0·254 cm z_2 = 0·20 in. = 0·508 cm
0·20	82		

1 in. = 2·54 cm ; 1 psi = 0·07 kilos/cm²

TABLE III

Heavy clay at 25 percent water content : γ_d = 100 pcf, unconfined compressive strength = 1,600 kilos/m³

(q_u) = 27 psi = 1·89 kilos/cm²

ρ = 2·7 gm/cc; Unsoaked CBR = 6·6 per cent
c = 0·2 k. cal/kg °C

Penetration "z" (in.)	Load, p_i (psi)	$\frac{k}{p_o} z$	$\tanh\left(\frac{k}{p_o} z\right)$	$p_{i_c} = p_0 \tanh\left(\frac{k}{p_o} z\right)$ (calculated) (psi)
0·05	45	0·420	0·396	38·5
0·10	66	0·840	0·676	65·5
0·15	80	1·260	0·850	82·5
0·20	90	1·680	0·933	90·5
0·25	98	1·100	0·970	94·0
0·30	103	2·520	0·987	95·5
0·35	108	2·940	0·995	96·4
0·40	112	3·360	0·997	96·6

1 in. = 2·54 cm ; 1 psi = 0·07 kilos/cm²

Calculation

p_1 = 66 psi, = 4·62 kilos/cm² z_1 = 0·10 in. = 0·254 cm
p_2 = 90 psi, = 6·3 kilos/cm² z_2 = 0·20 in. = 0·508 cm

$$p_o = \frac{p_1}{\sqrt{\frac{2\,p_1}{p_2} - 1}} = 97 \text{ psi} = 6{\cdot}79 \text{ kilos/cm}^2$$

$$x = \tanh^{-1}\sqrt{\frac{2\,p_1}{p_2} - 1} = \tanh^{-1}(0{\cdot}68) = 0{\cdot}84\,;$$

$$k = \frac{p_o}{z_1}\,x = \frac{66}{0{\cdot}1} \times 0{\cdot}84 = 815 \text{ lb/cu in.} = 23 \text{ kilos/cm}^3.$$

TABLE IIIA

Energy and temperature rise calculated from the experimental data in Table III.

Penetration "z" (inch)	$\frac{k}{p_o} z$	$\frac{p_o^2}{k}$ lb/in.	$\cosh\left(\frac{k}{p_o} z\right)$	$\log_e \cosh\left(\frac{k}{p_o} z\right)$	$E = \frac{p_o^2}{k} \log_e \cosh\left(\frac{k}{p_o} z\right)$ (lb-in./in².)	$E' = \frac{E}{J} = 4{\cdot}15 \times 10^{-6} E$ (K. cal/cm²)	$\triangle t$ (°C) $= \frac{E'}{1{\cdot}37 \times 10^{-3} z}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0·05	0·420	11·6	1·090	0·090	1·04	4×10^{-6}	58×10^{-3}
0·10	0·840	11·6	1·382	0·325	3·75	15×10^{-6}	110×10^{-3}
0·15	1·260	11·6	1·907	0·685	7·94	33×10^{-6}	160×10^{-3}
0 20	1·680	11·6	2·777	1·020	11·80	49×10^{-6}	178×10^{-3}
0·25	2·100	11·6	4·144	1·425	16·50	68×10^{-6}	198×10^{-3}
0 30	2·520	11·6	6·269	1·835	21·30	88×10^{-6}	215×10^{-3}
0·35	2·940	11·6	9·506	2·255	26·00	108×10^{-6}	225×10^{-3}
0·40	3·360	11·6	14·430	2·680	31·00	128×10^{-6}	235×10^{-3}

1 in. = 2·54 cm ; 1 lb/in. = 0·178 kilos/cm ; 1 lb-in./in.² = 0·178 kilos-cm/cm²

TABLE IV

Heavy clay at 23 percent water content: γ_d = 100·5 pcf ; Unsoaked CBR = 12·5 percent
= 1,608 kilos/m³

Unconfined compressive strength, q_u = 50 psi
= 3·5 kilos/cm²

Penetration "z" (in.)	Load, p_i (psi)	$p_o = \frac{p_1}{\sqrt{2\, p_1/p_2 - 1}}$ (psi)	Remarks
0·10	125	183	Here, p_1 = 125 psi = 8·75 kilos/cm²
0·20	170		p_2 = 1·70, psi = 11·9 kilos/cm² z_1 = 0·10 in. = 0·254 cm z_2 = 0·20 in. = 0·508 cm

1 in. = 2·54 cm ; 1 psi = 0·07 kilos/cm²

TABLE V

Heavy clay at 21 percent water content : γ_d = 98 pcf ; unsoaked CBR = 14 percent
= 1568 kilos/m³

Unconfined compressive strength, q_u = 55 psi
= 3·85 kilos/cm²

Penetration "z" (in.)	Load, p_i (psi)	$p_o = \frac{p_1}{\sqrt{2\, p_1/p_2 - 1}}$ (psi)	Remarks
0·10	140	206	Here, p_1 = 1·40 psi = 9·8 kilos/cm²
0·20	192		p_2 = 192 psi = 13·44 kilos/cm² z_1 = 0·10 in. ; = 0·254 cm z_2 = 0·20 in. = 0·508 cm

1 in. = 2·54 cm ; 1 psi = 0·07 kilos/cm²

TABLE VI

Heavy clay at 19 percent water content : $\gamma_d = 94$ pcf; Unsoaked CBR=18·5 percent

$= 1{,}504$ kilos/m^3

Unconfined compressive strength, $q_u = 67$ psi

$= 4{\cdot}69$ kilos/cm^2

$\rho = 2{\cdot}7$ gm/cc

$c = 0{\cdot}2$ k. cal/kg°C

Penetration "z" (in.)	Load, p_t (psi)	$\frac{k}{p_o}. z$	$\tanh\left(\frac{k}{p_o} z\right)$	$p_{t_c}=p_o \tanh\left(\frac{k}{p_0} z\right)$ (calculated) (psi)
0·05	115	0·345	0·335	106·0
0·10	185	0·690	0·600	190·0
0·15	235	1·035	0·765	243·0
0·20	275	1·380	0·880	280·0
0·25	300	1·725	0·938	298·0
0·30	320	2·070	0·968	307·0
0·35	340	2·415	0·984	314·0
0·40	350	2·760	0·992	316·0

1 in. = 2·54 cm ; 1 psi = 0·07 kilos/cm^2

Calculation

$$p_1 = 185 \text{ psi}, \qquad z_1 = 0{\cdot}10 \text{ in.} = 0{\cdot}254 \text{ cm}$$

$$= 12{\cdot}95 \text{ kilos/cm}^2$$

$$p_2 = 54 \text{ psi}, \qquad z_2 = 0{\cdot}20 \text{ in.} = 0{\cdot}508 \text{ cm}$$

$$= 3{\cdot}78 \text{ kilos/cm}^2$$

$$p_o = \frac{p_1}{\sqrt{\frac{2\,p_1}{p_2} - 1}} = 317 \text{ psi}, = 22{\cdot}19 \text{ kilos/cm}^2$$

$$x = \tanh^{-1}\sqrt{\frac{2\,p_1}{p_2} - 1} = 0{\cdot}69\,;$$

$$k = \frac{p_o}{z_1} x = 2{,}190 \text{ lb cu in.}$$

$$= 61 \text{ kilos/cm}^3$$

TABLE VI(A)

Energy and temperature rise calculated from the experimental data in Table VI.

Penetration "z" (in.)	$\frac{k}{p_o} z$	$\frac{p_o^2}{k}$ lb/cm	$\cosh \frac{k}{p_o} z$	$\log \cosh \left(\frac{k}{p_o} z\right)$	$E = \frac{p_o^2}{k} \log_e \cosh\left(\frac{k}{p_o} z\right)$ (lb-in./in^2).	$E' = \frac{E}{J} = 4{\cdot}15 \times 10^{-6} E$ k. cal/cm^2	$\triangle t\ (^{\circ}C) = \frac{E'}{1{\cdot}37 \times 10^{-3} z}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0·05	0·345	46·2	1·063	0·050	2·31	10×10^{-6}	145×10^{-3}
0·10	0·690	46·2	1·248	0·220	10·20	43×10^{-6}	315×10^{-3}
0·15	1·035	46·2	1·585	0·460	21·30	88×10^{-6}	426×10^{-3}
0·20	1·380	46·2	2·115	0·750	34·70	144×10^{-6}	525×10^{-3}
0·25	1·725	46·2	2·898	1·060	49·00	204×10^{-6}	595×10^{-3}
0·30	2·070	46·2	4 042	1·400	65·00	270×10^{-6}	655×10^{-3}
0·35	2·415	46.2	5·641	1·730	80·00	332×10^{-6}	690×10^{-3}
0·40	2·760	46·2	7·900	2·067	96·00	398×10^{-6}	725×10^{-3}

1 in. = 2·54 cm ; 1 lb/in. = 0·178 kilos/cm ; 1 lb-in./in^2. = 0·178 kilos-cm/cm^2.

Summary and Conclusion

The basic assumption of the analysis of transfer of mechanical energy into heat energy is that the mechanical energy is converted directly into heat energy. The energy lost due to friction is not taken into account in this analysis. Only a portion of the total work is changed into internal energy and heat, while another noteworthy portion is used up in the fragmentation of component particles of the system. The thermodynamic part of the analysis is very helpful in estimating the energy required or the horsepower needed in driving a pile or in soil-cutting or loosening process.

In case of heavy clay in saturated condition, there is a tendency of the soil mass to flow out besides getting compressed and thus the mass of soil computed in Equation (9), has to be corrected or can be taken as an approximate value. Also, since the soil mass contains solid particles as well as pore water and gaseous phases, the mass quantity M calculated on the basis of solid phase will be an approximate value. However, this can be accurately calculated by considering the mass specific gravity of the three-phase system.

During an increase of deformation of soil by a plunger, the penetration stress p converges to the determined limit of bearing capacity of soil, p_o as noted in Equation (6). From the plot of Figure 1, it can be concluded that the ultimate bearing capacity, p_o is equal to $3{\cdot}75\ q_u$. For purely cohesive soil, the unconfined compressive strength $q_u=2c=2$ times shear strength. So, $p_o=7{\cdot}5c$, which is greater than Prandtl's value, $p_o=5{\cdot}14c$, but close to Golder's value (1941) developed from experimental results, $p=6{\cdot}7c$ (on a square footing). Golder's value also increases on a strip footing.

It is easy to show in Equation (6) expanded as a power series that the known relation $p=kz$ is the first term ; and in the same way in Equation (8), $E=kz^2/2$ is the first term. The energy function developed in this paper is limited to the type of soil whose penetration resistance function is hyperbolic in nature. In Figure 2, one can observe the Energy-Temperature-Penetration relationship of heavy clay at different water contents.

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