Short Communication

A Minor Modification in Bishop's Expression on Stability of Slopes

by

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BISHOP'S (1955) method of the stability analysis of slopes raises a point.

Assuming a circular failure surface and considering the stability of the mass when rupture has taken place, he arrived at the following expression for the factor of safety :

$$F=\frac{1}{\Sigma W \sin\alpha}\sum\left[\left\{c'\bar{b}+\left(W(1-\bar{B})+\overline{Xn-Xn+1}\right)\tan\phi'\right\}\frac{\mathrm{Sec}\,\alpha}{1+\dfrac{\tan\alpha\tan\phi'}{F}}\right]$$

Here all the notations have the same meaning as in the original article and also given in the figure.

It is clear that when a slice on left of the vertical line passing through the centre of circular are 0 (such as slice number 1 in Figure 1) is considered, weight component W Sin α acts as disturbing force. S is the shear force which acts in clockwise direction and is restoring force. For all such slices, the expression for F is correct.

Now we can consider a slice on the other side of the vertical line, such as slice number 2 of the figure. Here the manner in which the forces will affect the stability of the slope is different. The weight component W Sin α of all such slices will act as a restoring force, whereas the force S will still be acting in the same direction, i.e., clockwise direction.

Angle α is defined as the angle between BC and the horizontal and which is equal to the angle made at the centre 0 by the centre of any slice with the vertical line passing through centre of circular arc 0. The author has not given any consideration to the sign of this angle. One is, thus, forced to take care of the sign of the term W Sin α, judging the position of the slice. In other words, one just can't use the expression mechanically, during the analysis of slope stability.

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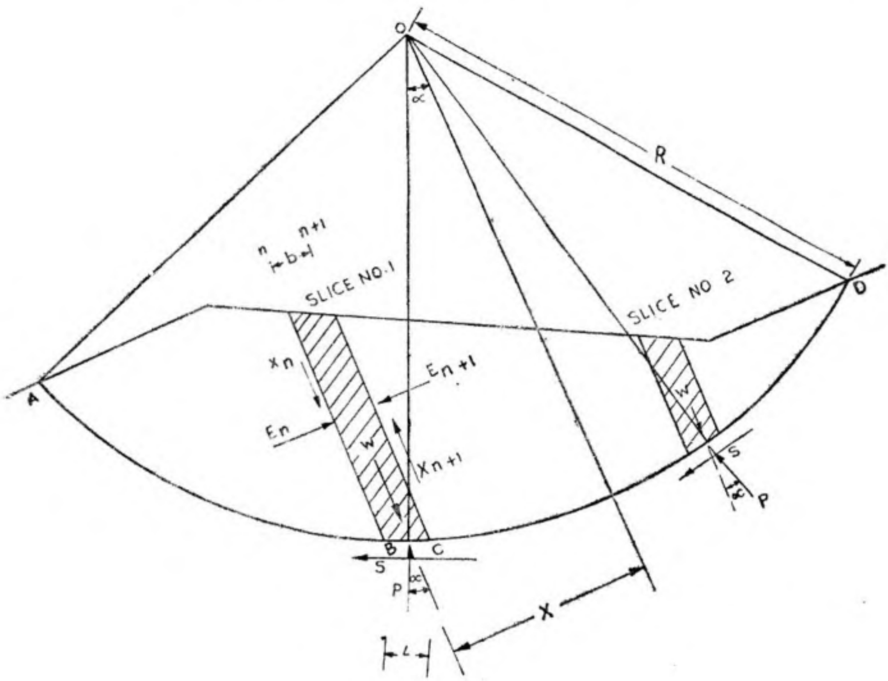

FIGURE 1.

However, if we treat all angles to the left of vertical line to be positive and on right side to be negative, then another problem will arise. Of course, this sign convention will automatically take care of the sign of $W \operatorname{Sin} \alpha$. When the slice is on left, α will be positive and $W \operatorname{Sin} \alpha$ too, will be positive. And when slice is on right, α will be negative and as $\operatorname{Sin}(-\alpha) = -\operatorname{Sin} \alpha$, $W \operatorname{Sin} \alpha$ will be negative. Thus one need not care about position of slice, and the denominator will be all right for all the slices.

Now coming to numerator, we shall consider the term

$$\frac{\operatorname{Sec} \alpha}{1 + \dfrac{\tan \alpha \tan \phi'}{F}}.$$

As force S does not change sign throughout the arc, the numerator should be correct, irrespective of situation of slice. But in the expression we see that for slice on left, the numerator will have the term

$\dfrac{\operatorname{Sec} \alpha}{1 + \dfrac{\tan \alpha \tan \phi'}{F}}$ (α being positive) and for slice on right it will be

$\dfrac{\operatorname{Sec} \alpha}{1 - \dfrac{\tan \alpha \tan \phi'}{F}}$ α being negative and as $\operatorname{Sec}(-\alpha) = +\operatorname{Sec} \alpha$ and $\tan(-\alpha)$

$= -\tan \alpha$ change in sign of term $\dfrac{\tan \alpha \tan \phi'}{F}$ will change the value of numerator considerably, and ultimately it will result in wrong value of F.

If such a problem is solved on a computer, using this expression, then computer will consider the sign of angle α and will give a wrong result unless some special instructions are given to treat the term

$\frac{\tan \alpha \tan \phi'}{F}$ always positive.

This shows that though Bishop's expression for F may be correct, yet it is not foolproof. One has to give certain conditions or instructions along with the expression for getting correct results.

However, if a small modification is applied to the expression, so as to take this into consideration, the expression will be perfect and any one can use it without any special instructions.

In this connection it is suggested that instead of simp'e "tan α", if we take *"modulus of tan α" (i.e., $|\tan \alpha|$) then the expression will be* all right for all slices irrespective of their position. This will change the expression to the following form :

$$F=\frac{1}{\Sigma W \operatorname{Sin} \alpha} \sum\left[\left\{ \overline{b}c' + \left(W(1-\overline{B}) + \overline{Xn-Xn+1}\right) \tan \phi' \right\} \frac{\operatorname{Sec} \alpha}{1+\frac{|\tan \alpha| \tan \phi'}{F}}\right]$$

In the modified expression α will be assigned with a proper sign (+) positive or negative (−) depending on the location of slice in relation to the vertical line through centre of the slip circle.

Acknowledgement

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Reference

BISHOP, A.W. (1955) : "The use of the Slip Circle in the Stability Analysis of Slopes". *Geotechnique*, Vol. 5, pp. 7-17.