Short Communication

Tail Erosion and Piping Problem

by

D. Veerabhadra Rao*

1. Introduction

ONE of the criteria for the design of weirs and barrages on permeable foundations is the safety against piping. The factor of safety against piping is obtained by dividing the critical gradient which is a function of properties of the porous media, by the exit gradient which is the hydraulic gradient of percolating water at the downstream end of the seepage trajectory.

For the computation of exit and critictal gradients it is assumed that the downstream river bed is horizonal. But this may not be always realised in practice. If the rear face of the structure is not protected well against scour or the existing protection work gets damaged, the bed may be scoured away during floods and assume a sloping surface near the toe (Figure 1). The changed conditions are likely to affect the values of both critical gradient and exit gradient.

2. Critical Gradients

Equations for critical gradients can be derived by equating the seepage force to the opposing forces at the exit for equilibrium conditions. It may be noted that the seepage trajectory meets the downstream bed surface at right angles since the latter is an equipotential (0 percent) line (Figure 1). The seepage force at the exit, *Fse*, acts normal to the surface and submerged weight of soil, *W* acts vertically downwards. Friction and cohesion are neglected. From Figure 1 it may be seen that only a component of *W* (along *X—X'*) opposes the seepage force.

Failure of the downstream bed may occur in two ways.

Case 1—Failure by lifting or piping of the particles : This will occur if the seepage force is greater than the opposing force due to submerged weight of soil.

At the equilibrium conditions, (considering a unit volume of material).

$$F_{se} = W \cos \alpha \qquad \ldots(1)$$

where W=Unit weight of soil in submerged condition

α=The angle made by the bed surface with the horizontal

Also, $W = \omega(\rho - 1)(1 - e)$

---

* Head of the Department of the Applied Mechanics, Government Polytechnic, Khamgaon (Maharashtra).

This paper in the present form was received on 21 August 1972. It is open for discussion up to March 1973.

where ω=Unit weight of water
ρ=Special gravity of sub-soil
e=Porosity of sub-soil.

Denoting critical gradient as Scr 1 and taking seepage force per unit volume as the product of unit weight of water, ω, and hydraulic gradient, we obtain

$$\omega.\ \text{Scr}\ 1=\omega(\rho-1)\ (1-e)\ \text{Cos}\ \alpha$$

or $$\text{Scr}\ 1\ =(\rho-1)\ (1-e)\ \text{Cos}\ \alpha \qquad ...(2)$$

Case (2) Failure by sliding or 'slip' of the granular particles down the slope : This will occur if frictional resistance acting up the slope is overcome by the component of submerged weight of soil acting down the slope (Figure 1).

The disturbing force= W Sin α

The stabilising force due to frictional resistance } = (W Cos α $-Fse$) tan ϕ where ϕ=angle of internal friction of sub-soil.

At equilibrium,

$$W\ \text{Sin}\ \alpha= W\ \text{Cos}\ \alpha\ -Fse)\ \tan\ \phi \qquad ...(3)$$

Denoting critical gradient as Scr 2 for this case and substituting for W and Fse, we obtain after rearranging,

$$\text{Scr}\ 2=(\rho-1)\ (1-e)\ [\text{Cos}\ \alpha-\text{Sin}\ \alpha\ \cot\ \phi] \qquad ...(4)$$

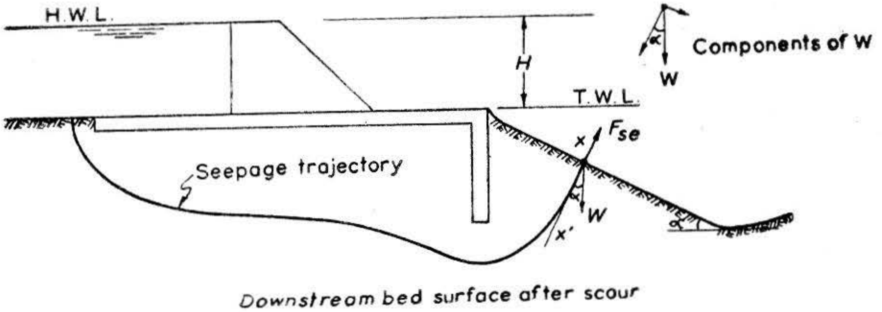

FIGURE 1.

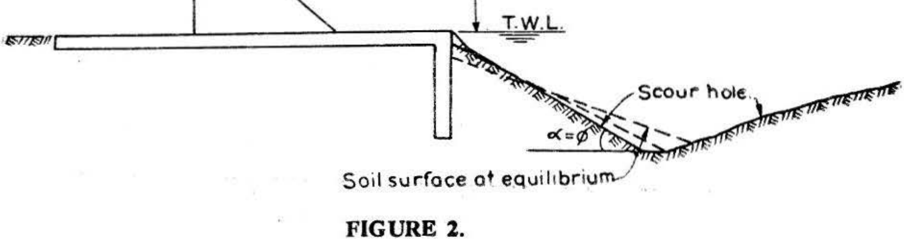

FIGURE 2.

This aspect of piping problem was first concieved by S. Leliavsky (1948) and extensive work on critical gradients was done by J.V. Rao and S.K. Sharma (1958 and 1969).

A comparison of Equations (2) & (4) shows that sliding of particles takes place even earlier to piping.

3. Comments on Scr 1 and Scr 2

(*a*) Scr 1, the critical gradient for piping to take place : From Equation (2) it may be seen that Scr 1 decreases as per Cosine law. The maximum value of α can be taken as ϕ for a cohesionless soil. If ϕ is taken as 30° approximately, the maximum reduction in Scr 1 is 13.4 per cent of the value for an unscoured bed.

(*b*) Scr 2, critical gradient for *'slip' to occur : A study of Equation* (4) reveals that as α increases, the value of Scr 2 decreases rapidly and approaches zero when $\alpha=\phi$. So for slopes nearer to ϕ slip of granular material is bound to occur. However, since the sloping bed surface does not extend to an infinite extent, the sliding of particles takes place only until the bed readjusts itself to a safer slope where equilibrium conditions are obtained (Figure 2).

4. Exit Gradient

In order to compute the factor of safety, it is also necessary to calculate the value of exit gradient for the changed conditions. Exit gradients for weirs with horizontal downstream bed can be calculated by Khosla's method. For weirs with complicated boundary conditions these can be computed by flownet, electrical analogy method, etc.

TABLE I

The effect of scour on Exit Gradients and Critical Gradients.

Head acting on the structure=4 m.

Sl. No.	Bed Slope	Exit Gradient (maximum)	Critical Gradients		Factor of Safety	
			Scr 1	Scr 2	Case 1	Case 2
1	2	3	4	5	6	7
1.	Horizontal	0.13	1.0000	1.0000	7.69	7.69
2.	6 : 1	0.12	0.9866	0.7036	8.22	5.86
3.	5 : 1	0.13	0.9805	0.6405	7.54	4.93
4.	4 : 1	0.12	0.9702	0.5502	8.08	4.59
5.	3 : 1	0.13	0.9489	0.4029	7.30	3.10
6.	2 : 1	0.14	0.8944	0.1184	6.39	0.85

($\phi=30°$; $\rho=2.65$, and $e=0.4$ for sub-soil).

The author's study (1960) on a typical weir profile has yielded the results given in Table I. As indicated by the results, the head acting on the structure remaining same, a maximum deviation of 8 percent from that of horizontal (unscoured) bed case is observed in the values of exit gradients. Hence, for all practical purposes, the maximum value of exit gradient may be taken approximately constant for all bed slopes and equal to that when the bed is not scoured (Scour of bed should not be construed as retrogression of bed level).

5. Conclusions

When tail erosion leaves the bed at a slope near the toe of weirs and barrages the values of exit gradient and critical gradient are likely to be affected. Failure of a sloping bed made of granular material and subjected to efflux of seepage may occur due to slipping of material along the slope and also due to piping. Equations for critical gradient are available for both the cases.

The maximum value of exit gradient for scoured beds is found to be approximately same as that for an unscoured bed by experimental investigation. Hence this value can be computed by well known Khosla's method.

After obtaining critical gradient and exit gradient the factor of safety against failure can be computed.

6. References

LILIAVSKY, S. and HASAN ZAKI BEY : "Tail Erosion as a Factor affecting the Safety Coefficient against Piping." Question No. 10, R 4, *Third International Congress on Large Dams*, Stockholm, 1948.

RAO, J.V. : "An Anaytlical Approach for Testing the Stability of Slopes subjected to Efflux of Seepage." *Journal of Science and Engineering Research*, I.I.T. Kharagpur, Vol. II, Part 2 (1958).

SHARMA, S.K. and RAO, J.V. : "Generalised Theory of Exit Gradients for the Stability of Sloping Surfaces in Porous Media against Efflux of Seepage." *Irrigation and Power, Journal of CBI & P*, Vol. 26, No. 3, July 1969.

VEERABHADRA, RAO, D. : "The Effect of Scour Downstream on the Security of Weirs and Barrages against Exit Gradients". *Thesis submitted in partial fulfillment of the requirements for the Degree of Master of Technology*, I.I.T Kharagpur, 1960.