Study of Uplift Pressures below Apron with Downstream Cutoff Founded on Two-Layer Media

by

Alam Singh*

B.C. Punmia†

M.B. Phatak††

Introduction

AS a result of seepage, which is caused by the difference in water-level between upstream and downstream side of a hydraulic structure, built upon a pervious media, the foundation of structure is subjected to uplift pressure, causing lifting up of floor, and the soil below the foundation is subjected to seepage force acting in the direction of flow. The seepage force, which obviously acts vertically at the exit, causes the dislodging of the material. This further reduces the resistance to the upward force causing more damage. The process of undermining thus proceeds backwards towards the upstream end, forming a channel or a pipe underneath the foundation. Thus the two important factors which are to be considered are (*i*) uplift pressure and (*ii*) the seepage force or hydraulic gradient at the exit.

After conducting a series of experiments Col. Clibborn (1895-97) enunciated the 'Hydraulic Gradient Theory' which emphasised the importance of length of travel of seepage water below the weir before it rises up. "Bligh's Creep Theory" is in principle same as hydraulic gradient theory but he went one step ahead in stating that the length of path has the same effectiveness in reducing the uplift pressure whether it is horizontal or vertical. In 1912 Lane advanced his "Weighted Creep Theory". which proved the fallacy of Bligh's Creep Theory. He put the stress over a point that vertical Creep is more effective than horizontal one.

Prof N.N. Pavlovsky (1922) used the technique of Conformal mapping for the solution of problem of seepage. Weaver (1932) analysed mathematically the problem of uplift pressure. Dr. Khosla and his associates (1936) were inspired from Weaver's solution and studied variety of problems dealing with foundations on pervious media of infinite depth. The charts given by them are still taken as an authority for the solution of weir foundation problems.

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* Professor and Head of Civil Engineering Department, University of Jodhpur, Jodhpur.

† Reader in Civil Engineering, University of Jodhpur, Jodhpur.

†† Reader in Civil Engineering, University of Jodhpur, Jodhpur.

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Prof. N.N. Pavlovsky (1922) for the first time used the technique of electrical analogy method. Dr. V.L. Vaidhianathan (1935) found out the pressure observations in hydraulic and electric models on which close agreement was found between experimental and analytical results.

The Khosla's Curves are based on two assumptions :

(*i*) The depth of soil is infinite ;

(*ii*) The soil media is isotropic.

It is rather difficult to have the above conditions in the field. In the present investigations, an effort is made to study the effect of heterogeneous media, each layer being isotropic in nature but having different permeabilities, on seepage below the apron by electrical analogy method. The uplift pressure has been studied below a flat apron with cutoff at downstream end.

Figure 1 shows the problem and different variables :

(*i*) base width of apron '*b*'

(*ii*) depth of layer of pervious media '*D*'

(*iii*) depth of downstream cutoff '*d*'

(*iv*) permeability ratio k_1/k_2.

The two layers are taken of equal depth. A total of 64 tests were conducted, incorporating various combinations of above variables.

The curves giving variation of pressure distribution with b/D, d/D, and k_1/k_2 ratios have been presented.

The pressures at the junction of floor and pile, and the tip of the pile are important from design point of view. Hence these design curves have also been given.

Experimental Set-up

The electrical analogy model for this study consisted of several components, such as, electrical analogy tray, electrical circuit, the conducting medium and the analogy boundaries.

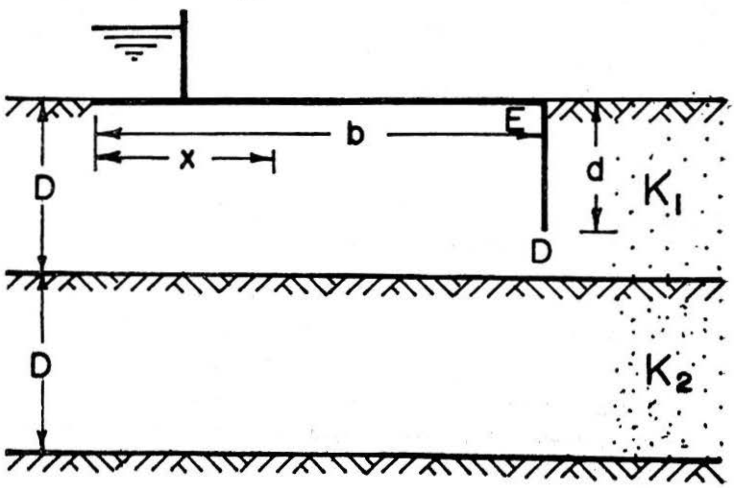

FIGURE 1 : Problem under investigation and different variables.

As the conducting medium was an electrolyte, a water-tight tray of size 120 cm × 150 cm × 6·25 cm was constructed with 6 mm glass plate bottom fitted over a graph paper so as to read the coordinates of desired point. The sides of the tray were made of 62·5 mm × 3 mm perspex sheet fixed with solution to timber sides. The junction of glass plate and perspex sides was made water-tight by araldite. Draining of conducting medium was facilitated through number of drain holes provided on sides.

Figure 2 shows a circuit diagram, tray and model. A step-down transformer was used to supply a 20 v/50 cycles current. The boundary equipotential strips were made of 62·5 mm × 2 mm copper strips and the

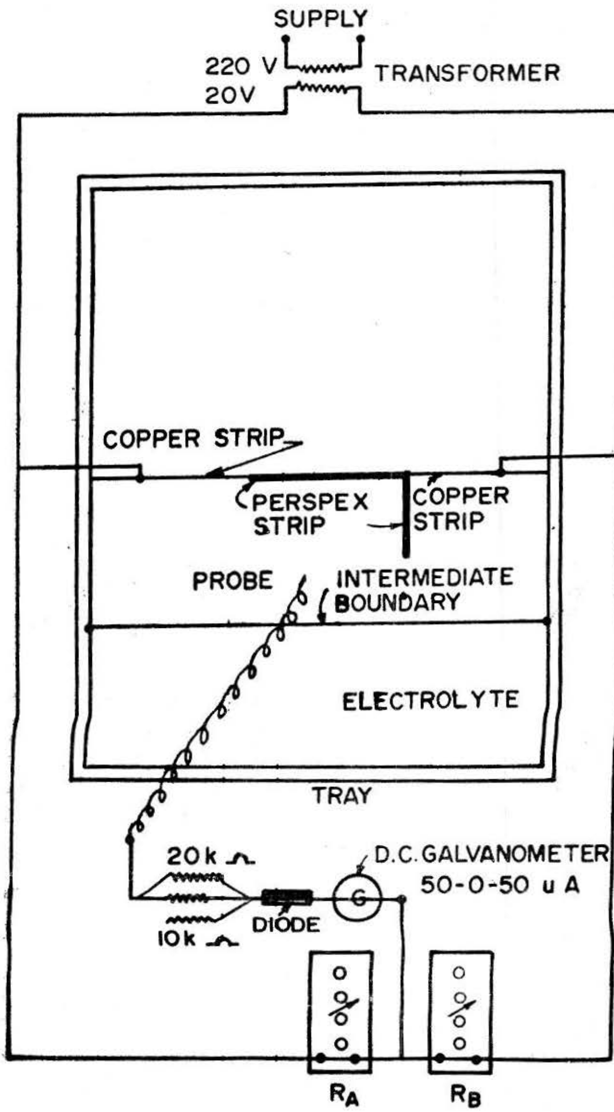

FIGURE 2 : Circuit diagram.

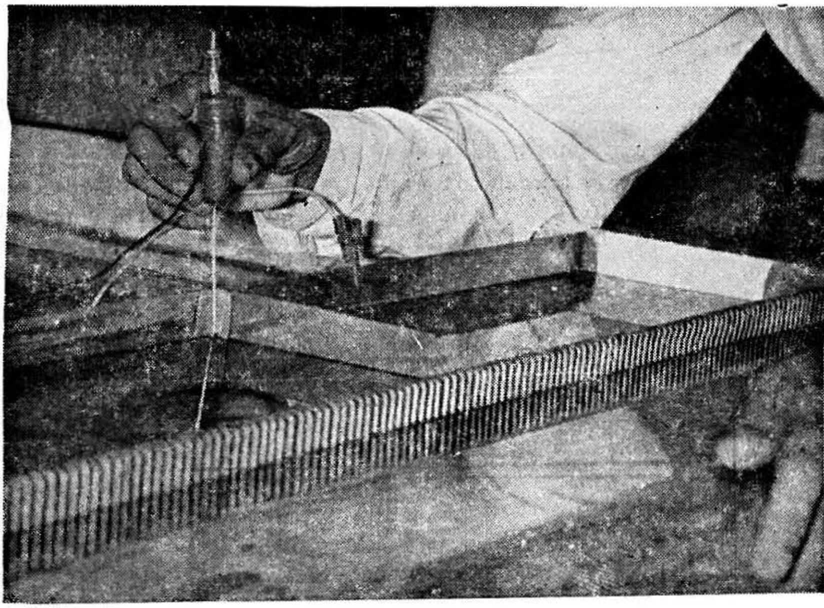

FIGURE 2 (*a*) : Photograph showing model and inner boundary.

boundary flow lines were simulated by 62·5 mm × 3 mm perspex strips. These were secured in position by plasticene which is a non-conducting adhesive. An alternating current of 20 v/50 cycles was fed across the copper strips while a D.C. Galvanometer of sensitivity 50-0-50μ *A* was supplied with D.C. current through a diode. Two resistances of 10 $K\Omega$ and 20 $K\Omega$ introduced between probe and diode gave different sensitivity of null point. Two decade resistance boxes, giving wide variety of resistances (Table I) by operating four knobs, were used to provide a wide variety of fixed potentials in a tank.

Different permeabilities of two layers were represented by different conductivities of liquid. Various concentrations of copper sulphate solution in distilled water were employed. In all four permeability ratios of 1, 4, 20 and 50 were tried. The relative conductivities of liquids were ascertained by a conductivity meter. The constant liquid depth of 45 mm was maintained throughout the tests.

An additional inner boundary between layers of different permeability was required in comparison with the problem having single layer. The inner boundary had to provide a water-proof seal between the two liquids and had to conduct electricity freely from one layer to another, and at the same time each point on the boundary had to maintain a unique potential value. The boundary was constructed with 62·5 mm × 3 mm perspex strip having 3 mm dia. notches, 6 mm c/c, at the top so as to house easily *U*-Shaped 3 mm copper clips with their ends facing downward [Figure 2 (*a*)]. The boundary was secured in position by plasticene clay.

TABLE I

Table giving the details of resistances in resistance box.

Knob No.	Range of variation in 10 steps	Value of variation of each step
1	0—10 ohms	1 ohm
2	0—100 ohms	10 ohms
3	0—1,000 ohms	100 ohms
4	0—10,000 ohms	1,000 ohms

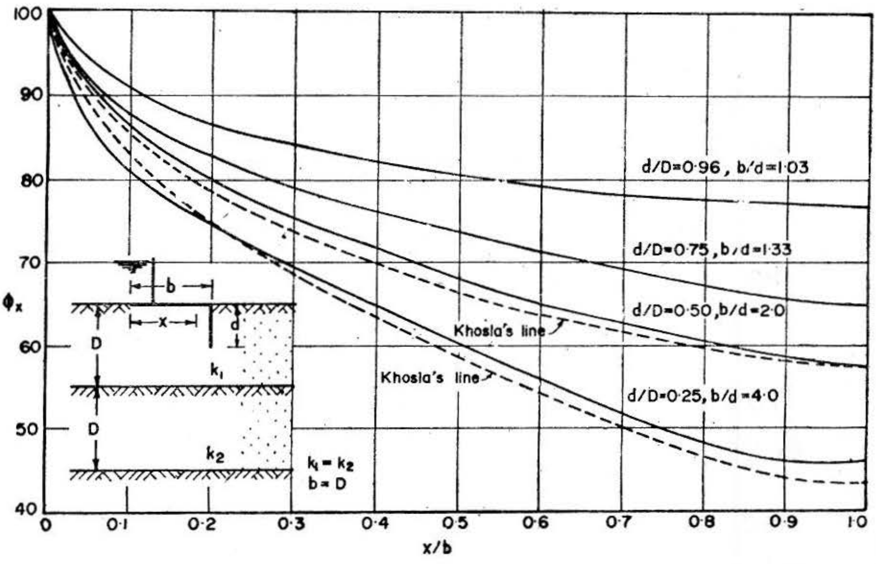

FIGURE 3 : ***Variation of φ_x for various values of d/D and b/d, b/D=1 and $k_1/k_2=1$ fixed.***

Model Dimensions

The principal parameters of the problem under investigation are b, D, d, k_1 and k_2. In all 64 tests are conducted after varying the above parameters (Phatak, 1970). These tests conducted for the case of downstream cutoff were divided into four series each having different k_1/k_2 ratio as 1, 4, 20 and 50. Each series contained four b/D ratios 1, 2, 4 and 8, and each b/D ratio was studied for four d/D ratios as 0·25, 0·50, 0·75 and practically 1.

Observations

The percentage potential at various points was observed by varying the resistance in the resistance boxes so as to get null point. The percentage potential fed through the set is given by

$$\phi = \frac{R_B}{R_A + R_B}$$

where, ϕ = uplift pressure at the point expressed as percentage of the head causing seepage.

R_B was fixed at values as 1, 10, *etc.* and R_A was varied to have a null point. With values of R_B as 1 or 10, ϕ was easily found as the reciprocal of total resistance $R_A + R_B$. Table II gives the typical observations taken.

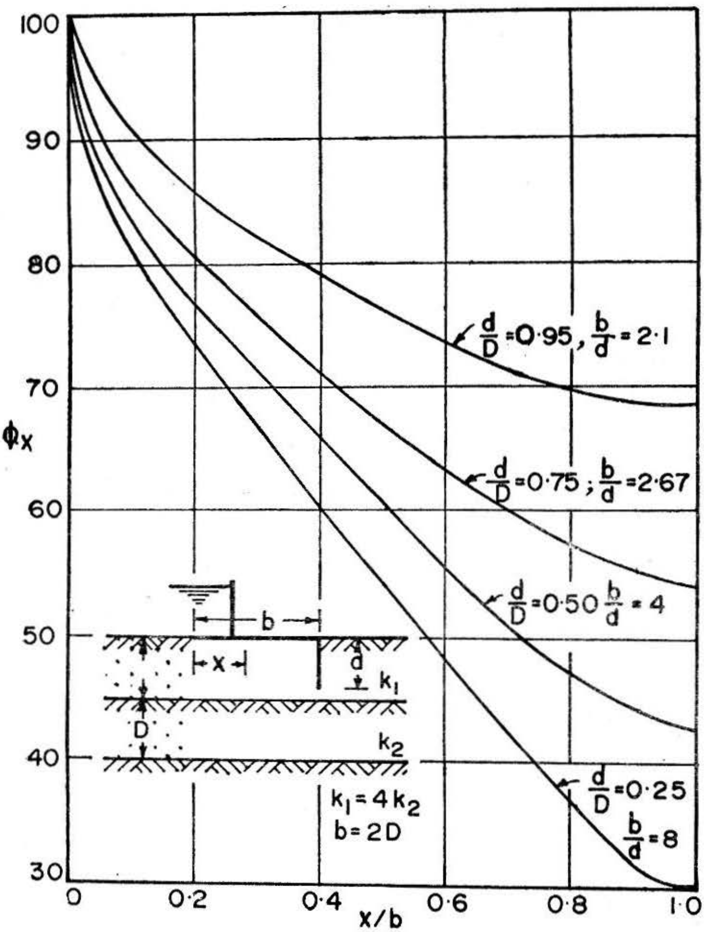

FIGURE 4 : Variation of ϕ_x for various values of d/D and b/d, b/D=2 and k_1/k_2=4 fixed.

TABLE II

Typical observation table for b=60, D=30, d=28·5 and k_1/k_2=50.

Apron					Upstream face					Down stream face				
x	R_B	R_A	$R_A + R_B$	ϕ%	Y	R_B	R_A	$R_A + R_B$	ϕ%	Y_1	R_B	R_A	$R_A + R_B$	ϕ%
6	10	0·8	10·8	92·6	4	10	3·6	13·6	75·5	2	1	60·0	61·0	1·6
12	10	1·2	11·2	89·3	8	10	3·7	13·7	72·9	4	1	36·0	37·0	2·7
18	10	1·6	11·6	86·2	12	10	3·9	13·9	71·9	6	1	30·0	31·0	3·2
24	10	1·9	11·9	84·0	16	10	4·2	14·2	70·4	8	1	22·0	23·0	4·3
30	10	2·3	12·2	81·3	20	10	4·7	14·7	68·0	10	1	18·0	19·0	5·3
36	10	2·7	12·7	78·7	24	10	5·7	15·7	63·7	12	1	15·0	16·0	6·3
42	10	3·0	13·0	76·9	28.5 tip	10	14·8	24·8	40·3	14	1	12·3	13·3	7·5
48	10	3·3	13·3	75·2						16	1	10·3	11·3	8·8
54	10	3·5	13·5	74·0						18	1	8·8	9·8	10·2
60	10	3·6	13·6	73·5						20	1	7·6	8·6	11·6
										22	1	6·3	7·3	13·7
										24	1	5·1	6·1	16·6
										26	1	4·0	5·0	20·0

Analysis

The analysis of the test results was aimed at finding the effect of b, D, d, and k_1/k_2 on the pressure distribution for the case of an apron with downstream cutoff in the following way.

(*a*) *Influence of d/D, b/D, b/d and k_1/k_2 on ϕ_x* :

Variation of ϕ_x, the pressure at any point along the floor, was studied with x/b, d/D and b/d ratios for fixed values of b/D and k_1/k_2. Figures 3, 4, 5 and 6 show the variation of ϕ_x with x/b, d/D and b/d for fixed values of $b/D=1$ and $k_1/k_2=1$; $b/D=2$ and $k_1/k_2=4$; $b/D=4$ and $k_1/k_2=20$; $b/D=8$ and $k_1/k_2=50$.

Variation of ϕ_x with x/b and $\frac{k_1}{k_2}$ was studied both for fixed values of b/D, d/D and b/d. Figures 7, 8 & 9 show the influence of $\frac{k_1}{k_2}$ on ϕ_x with $b/D=1$, $d/D=0{\cdot}75$; $b/D=2$, $d/D=0{\cdot}75$; $b/D=4$, $d/D=0{\cdot}90$.

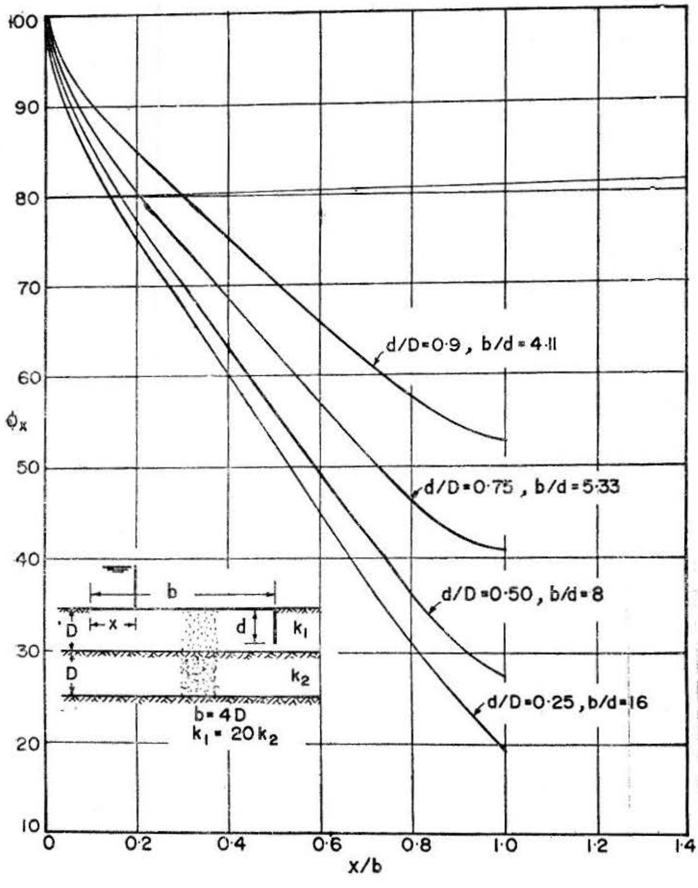

FIGURE 5 : Variation of ϕ_x for various values of d/D and b/d, b/D=4 and k_1/k_2 =20 fixed.

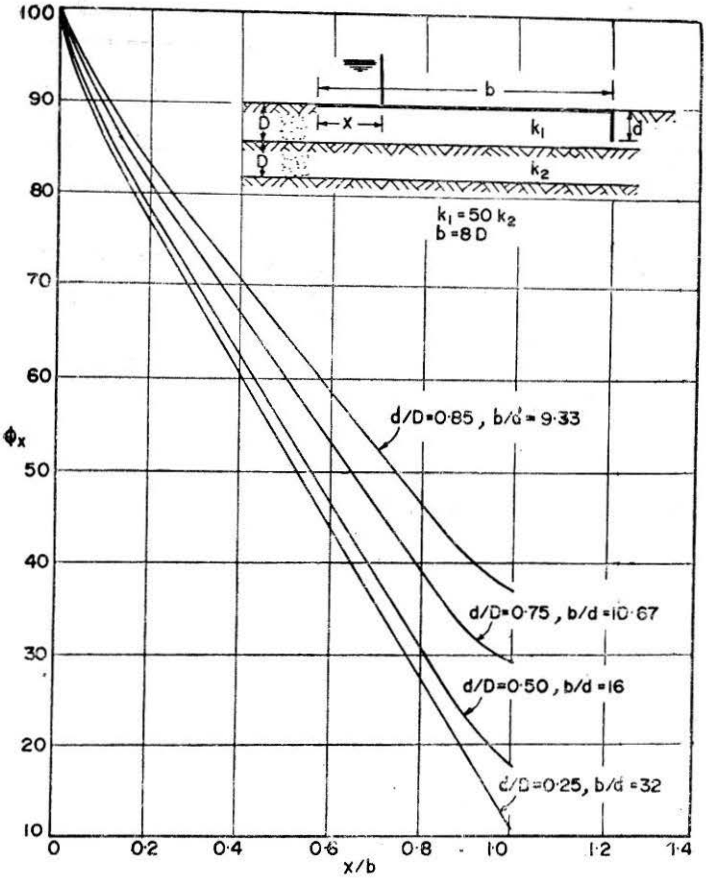

FIGURE 6 : Variation of ϕ_x for various values of d/D and b/d, b/D=8 and k_1/k_2=50 fixed.

Variation of ϕ_x with x/b and b/D for fixed values of d/D, b/d and $\frac{k_1}{k_2}$ *is shown in Figure 10 with* $\frac{k_1}{k_2}=50$ *and* $d/D=0{\cdot}75$.

(*b*) *Influence of* d/D, b/D, b/d *and* $\frac{k_1}{k_2}$ *on* ϕ_D *and* ϕ_E :

Influence of b/D and $\frac{k_1}{k_2}$ on ϕ_D and ϕ_E was studied by drawing graph of ϕ_D and ϕ_E against b/D for different values of $\frac{k_1}{k_2}$. Figures 11 & 12 show the influence of b/D and $\frac{k_1}{k_2}$ on ϕ_D for fixed values of $d/D=0{\cdot}50$ and 0·96 respectively. Figures 13 & 14 show the influence of b/D and $\frac{k_1}{k_2}$ on ϕ_E for fixed values of $d/D=0{\cdot}50$ and 0·75 respectively.

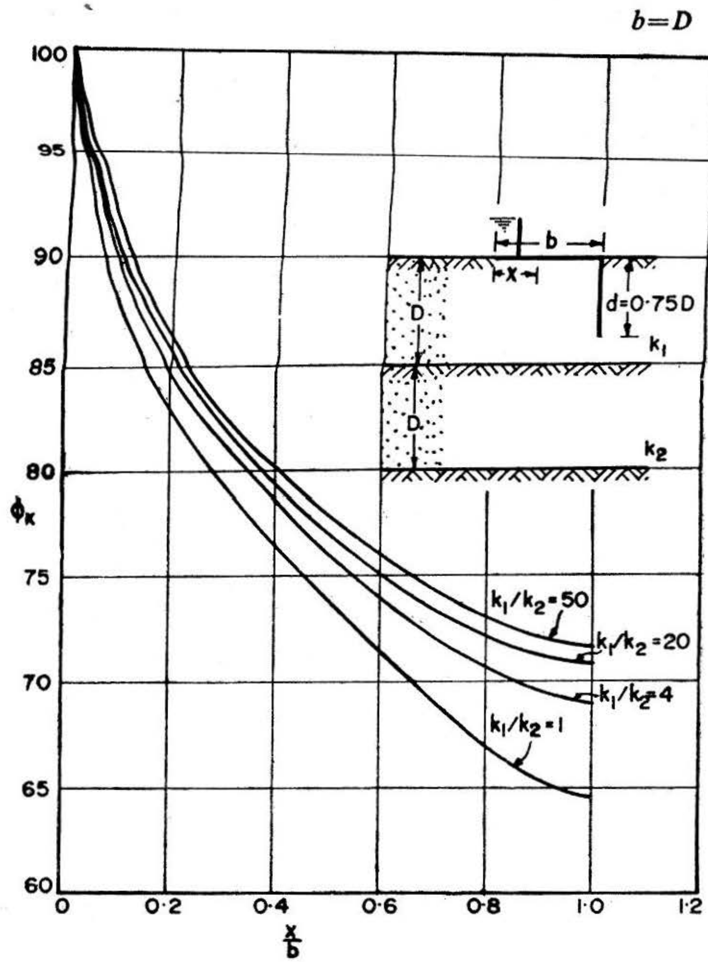

FIGURE 7 : Variation of ϕ_x for various values of k_1/k_2 b/D=1, d/D=0·75 and b/d=1·33 fixed.

Variations of ϕ_D with $\sqrt{\frac{k_1}{k_2}}$ and d/D for fixed values of b/D equal to 4 and 8 are shown in Figures 15 & 16 while that of ϕ_E with $\sqrt{\frac{k_1}{k_2}}$ and d/D for fixed values of b/D are shown in Figures 17 & 18.

Figure 19 shows variation of ϕ_D with α and $\frac{k_1}{k_2}$ for $b/D=4$ while Figure 20 shows the variation of ϕ_E with α and $\frac{k_1}{k_2}$ for $b/D=4$. As the limiting values of α for different b/D ratios are fixed, the above curves are plotted for values of α greater than 4 for $b/D=4$. The design curves for ϕ_D and ϕ_E, for other values of b/D are given elsewhere (Phatak 1970).

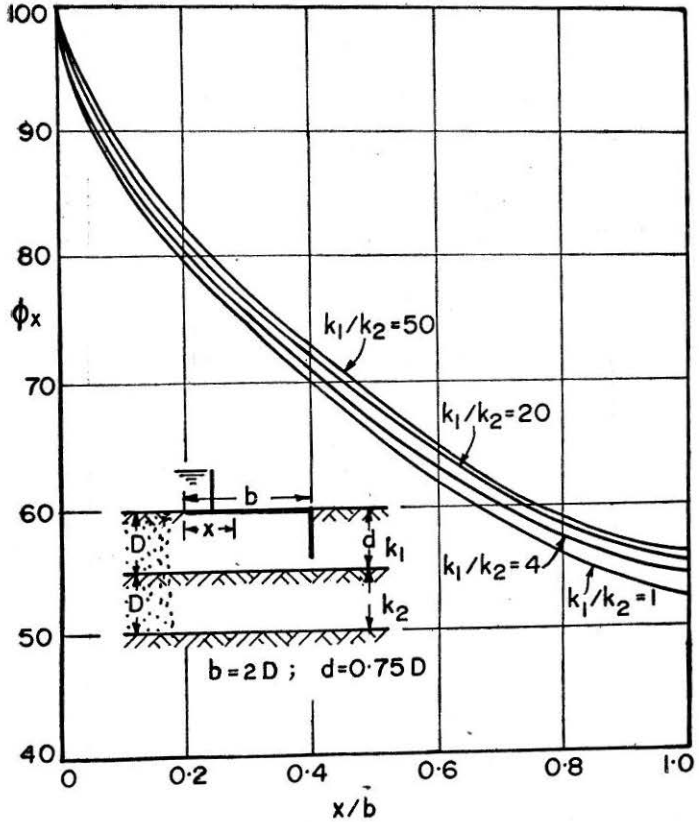

FIGURE 8 : **Variation of ϕ_x for various values of k_1/k_2, $d/D=2$, $d/D=0{\cdot}75$ and $b/d=2{\cdot}67$ fixed.**

On comparison of the experimental values with the values obtained by the theoretical solutions of known cases, it was observed that the experimental values are in close agreement with the theoretical values.

Discussion of Results

The following inferences are drawn from the test observations on the electrical analogy models for apron with downstream cutoff.

(*i*) For a given *b/D* ratio and permeability ratio, pressure at any point increases as the ratio of depth of pile to depth of pervious media, *i.e.*, penetration ratio, increases. However, the variation of pressure decreases with increasing penetration ratio.

(*ii*) At a given point, pressure at any point increases as permeability ratio increases, for a given *b/D* ratio and penetration ratio. However, this increase in pressure is more for increase of permeability ratio from 1 to 4 than from 20 to 50.

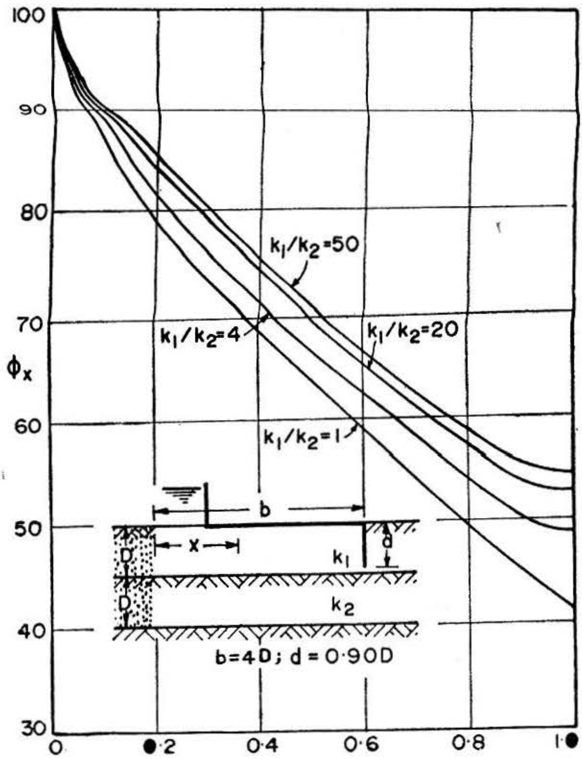

FIGURE 9 : Variation of ϕ_x for various values of k_1/k_2, $b/D=4$, $d/D=0{\cdot}90$, $b/d=4{\cdot}11$ fixed.

(*iii*) For a given penetration ratio and permeability ratio, the pressure at any point on the floor decreases as b/D ratio increases. However, variation of pressure with x/b increases with increase in b/D ratio.

(*iv*) For a given penetration ratio both ϕ_D and ϕ_E decrease with increase in b/D ratio.

(*v*) For a given penetration ratio, both ϕ_D and ϕ_E vary with $\frac{k_1}{k_2}$. But for penetration ratio equal to or greater than 0·75, ϕ_D, for a given b/D ratio increases with increase in permeability ratio, while for penetration ratio less than 0·75, ϕ_D decreases with increase in permeability ratio.

(*vi*) Effect of permeability ratio on change of ϕ_D for fixed b/D is more for higher penetration ratio than that for lower one.

(*vii*) For a given b/D ratio, ϕ_D decreases with increase in $\sqrt{\frac{k_1}{k_2}}$ for penetration ratio equal to or less than 0·5 while it increases with increase in $\sqrt{\frac{k_1}{k_2}}$ for penetration ratio greater than 0·75 and b/D ratio greater than 1. For $b/D=1$, ϕ_D decreases with $\sqrt{\frac{k_1}{k_2}}$ for all penetration ratios.

(*viii*) For a given b/D ratio, ϕ_E decreases with increase in $\sqrt{\frac{k_1}{k_2}}$ for $d/D \leqslant 0{\cdot}5$ while it increases for $d/D \geqslant 0{\cdot}5$.

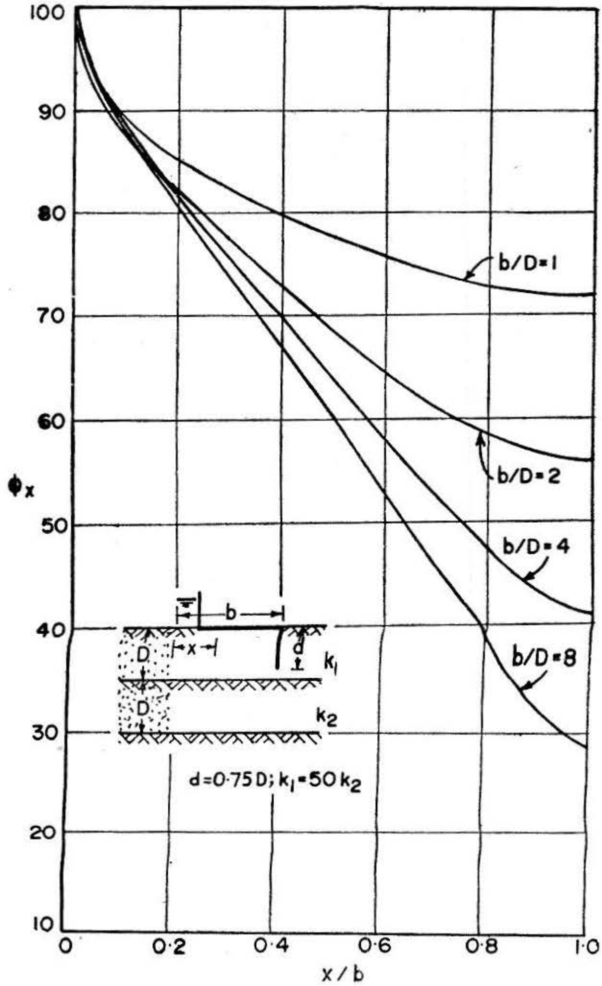

FIGURE 10 : Variation of ϕ_x for various values of b/D, $k_1/k_2=50$, d/D=0·75 fixed.

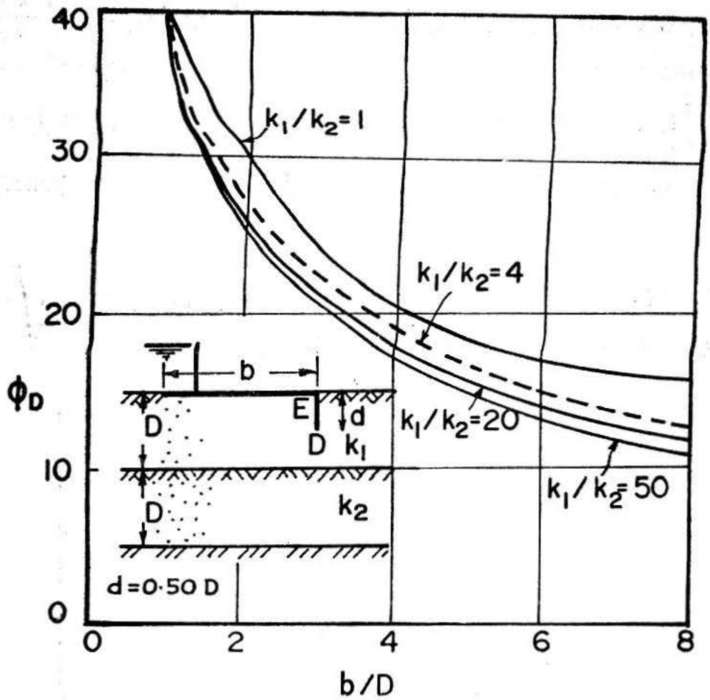

FIGURE 11 : Influence of b/D and k_1/k_2 on ϕ_D for fixed value of d/D=0·50.

(*ix*) For a given b/D ratio, ϕ_D and ϕ_E decrease with increase in b/D ratio.

(*x*) For a given penetration ratio and permeability ratio, both ϕ_D and ϕ_E decrease with increase in b/D ratio for a given α.

EXAMPLE

To find values of ϕ_D and ϕ_E for given b/D and k_1/k_2.

given :

$$\alpha = b/d = 5{\cdot}33$$
$$k_1/k_2 = 50$$
$$b/D = 4$$

from Figures 19 (*d*) & 20 (*d*)

$$\phi_D = 25{\cdot}6 \text{ percent}$$

$$\phi_E = 40{\cdot}9 \text{ percent}$$

While according to Khosla's theory

$$\alpha = 5{\cdot}33$$

$$\lambda = \frac{\sqrt{1+\alpha^2}+1}{2}$$

$$= 3{\cdot}2131$$

$$\phi_D = \frac{1}{\pi} \ \text{Cos}^{-1}\left(\frac{\lambda-1}{\lambda}\right)\times 100$$

$$= 25{\cdot}9 \text{ percent}$$

$$\phi_E = \frac{1}{\pi} \ \text{Cos}^{-1}\left(\frac{\lambda-2}{\lambda}\right)\times 100$$

$$= 37{\cdot}7 \text{ percent.}$$

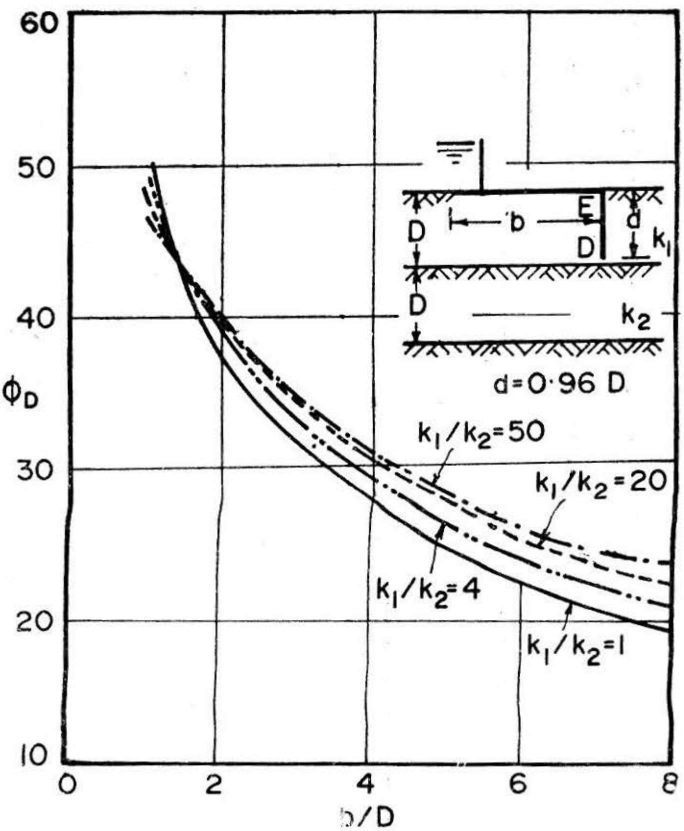

FIGURE 12 : Influence of b/D and k_1/k_2 on ϕ D for fixed value of d/D=0·96.

TABLE III

Table giving ϕ_D and ϕ_E values according to Khosla's Theory, and those experimentally observed for $b/D=4$.

$\alpha=(b/d)$	Khosla's values		$k_1/k_2=1$		$k_1/k_2=4$		$k_1/k_2=20$		$k_1/k_2=50$	
	ϕ_D	ϕ_E	ϕ_D	ϕ_E	ϕ_D	ϕ_E	ϕ_D	ϕ_E	ϕ_D	ϕ_E
4·1	29·8	42·5	26·7	43·5	28·6	48·8	29·4	52·6	30·7	54·0
5·3	25·9	37·7	25·0	35·7	24·4	40·0	25·6	40·8	25·6	40·9
8·0	21·5	31·1	20·4	29·9	20·0	28·9	19·6	28·6	18·9	28·6
16·0	16·7	22·9	15·7	22·2	16·7	20·4	13·9	19·6	13·7	19·2

Conclusions

The effect of permeability ratio on pressure distribution is evident from the test observation. Table III gives the pressure at the key points for different permeability ratios and values according to Khosla's theory. It will be evident, therefore, that the assumption of a soil to be isotropic will cause an appreciable error. The instrumental accuracy in the present investigations is $\pm$ 1 percent.

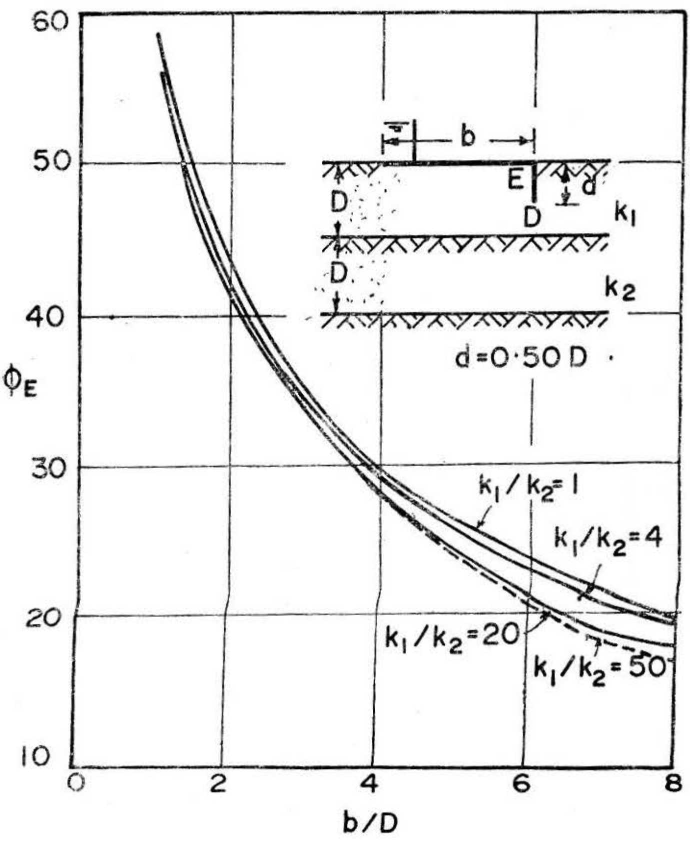

FIGURE 13 : Influence of b/D and k_1/k_2 on $\phi/_E$ for fixed value of d/D=0·50.

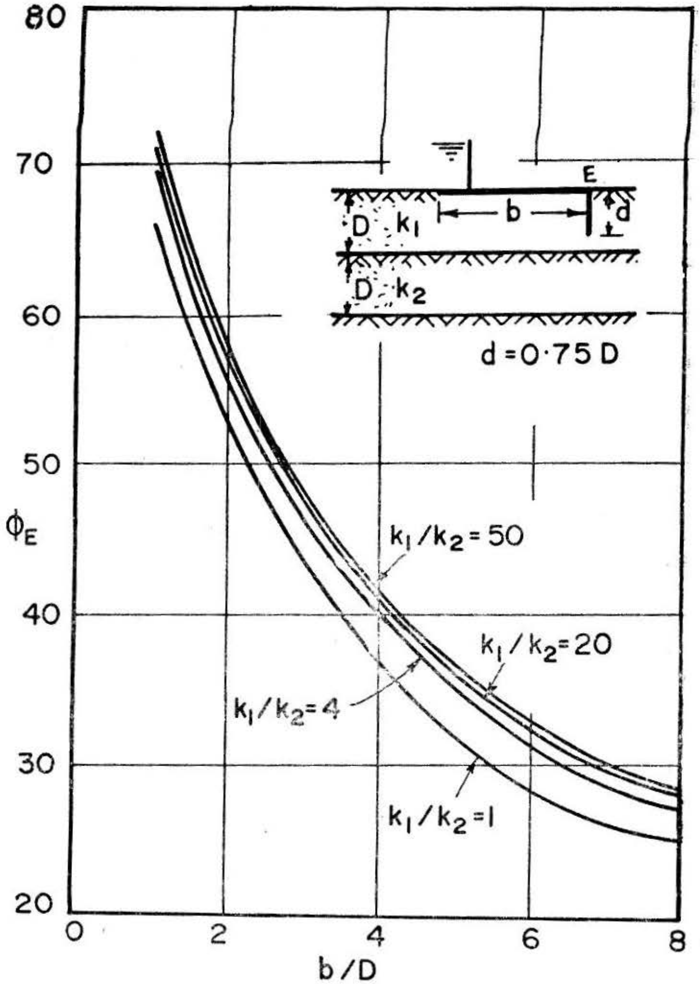

FIGURE 14 : Influence of b/D and k_1/k_2 on ϕ_E for fixed value of d/D=0·75.

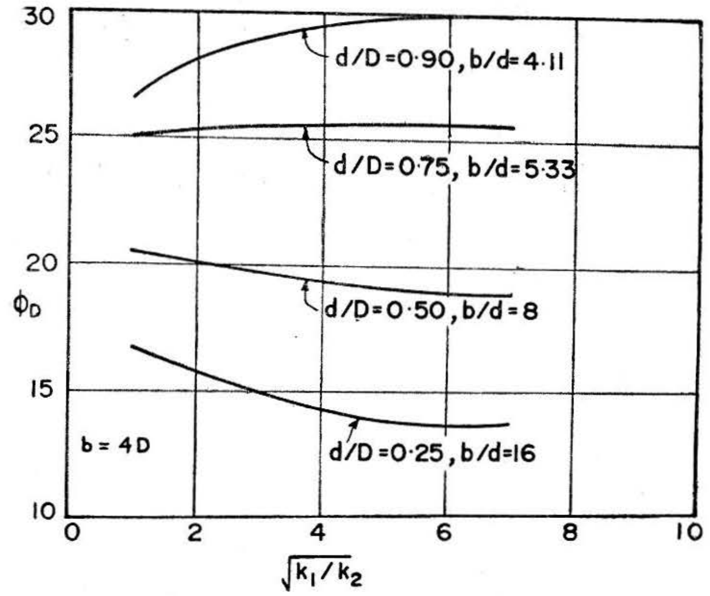

FIGURE 15 : Influence of k_1/k_2 d/D, b/D on ϕ_D for b/D=4 fixed.

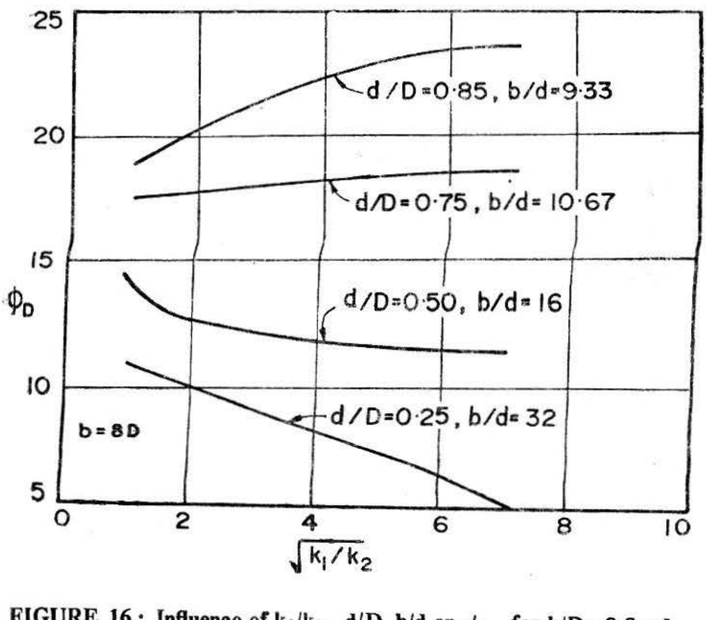

FIGURE 16 : Influence of k_1/k_2, d/D, b/d on ϕ_D for b/D=8 fixed.

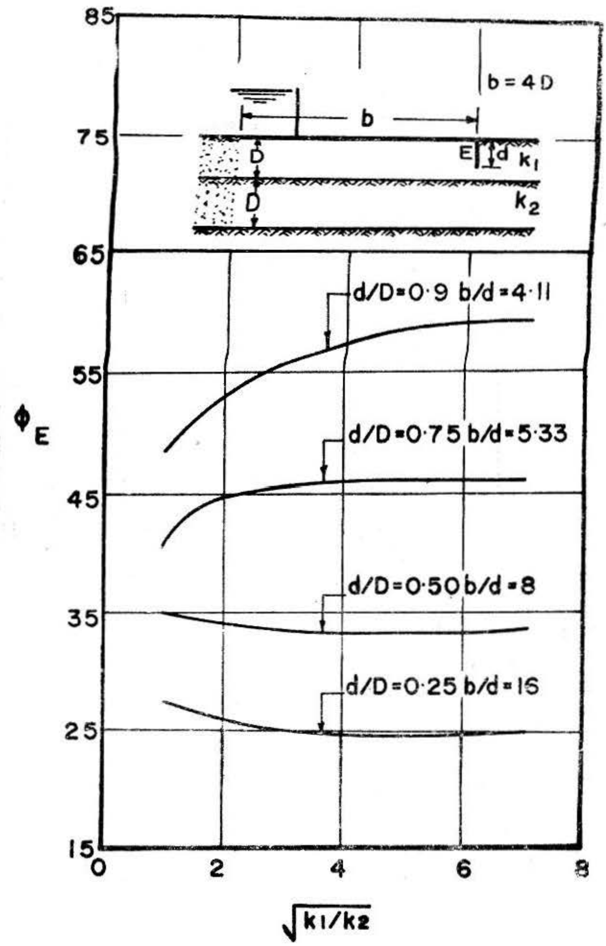

FIGURE 17 : Influence of k_1/k_2, d/D, b/d on ϕ_E for b/D=4 fixed.

List of Symbols

b	= Base width of apron
D	= Depth of layer of pervious media
d	= Depth of downstream pile
k	= Coefficient of permeability
k_1	= Coefficient of permeability of upper layer
k_2	= Coefficient of permeability of lower layer
R_A, R_B	= Resistance
x	= Distance from origin along floor
Y	= Ordinate on upstream face of pile from base

Y_1 = Ordinate on downstream face of pile from base

ϕ_x = Uplift pressure in percent at a distance x from origin

ϕ_E = Uplift pressure in percent at the junction of pile and floor, *i.e.*, point E

ϕ_D = Pressure at tip of pile, *i.e.*, point D

λ = Khosla's variable

α = Khosla's variable.

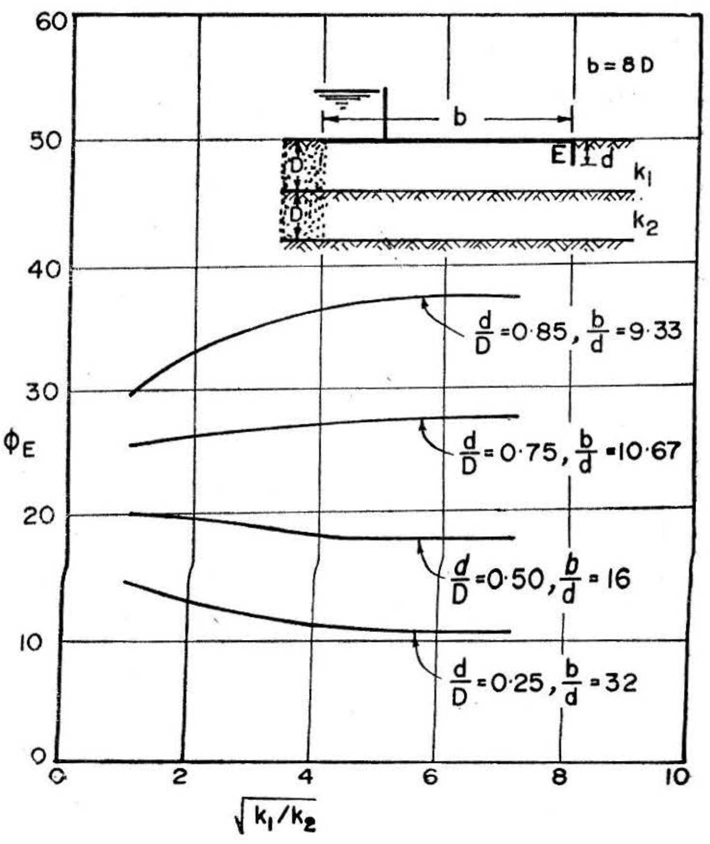

FIGURE 18 : Influence of k_1/k_2, d/D, b/d on ϕ_E for b/D = 8 fixed.

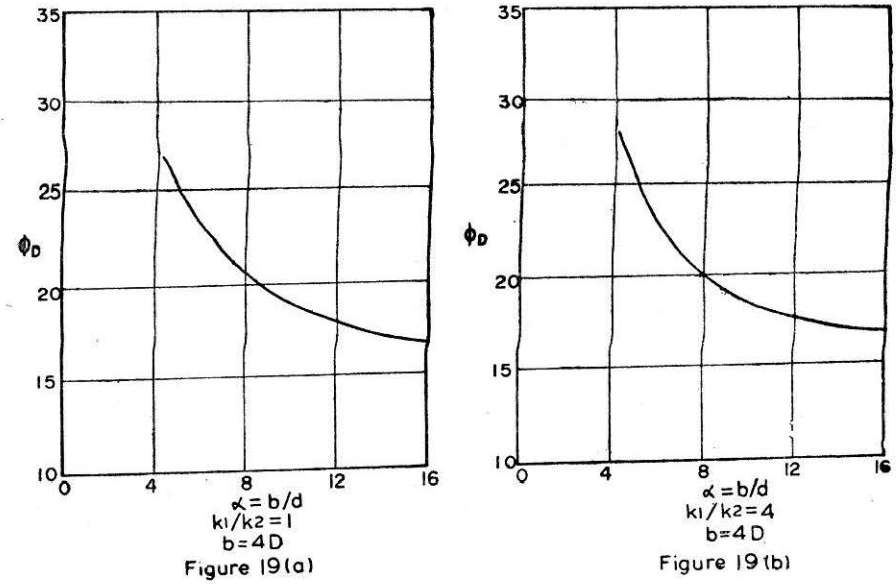

FIGURE 19 (a and b) :

(a) Variation of ϕ_D with α, i.e., b/d for $k_1/k_2=1$ and b/D=4.

(b) Variation of ϕ_D with α, i.e., b/d for $k_1/k_2=4$ and b/D=4.

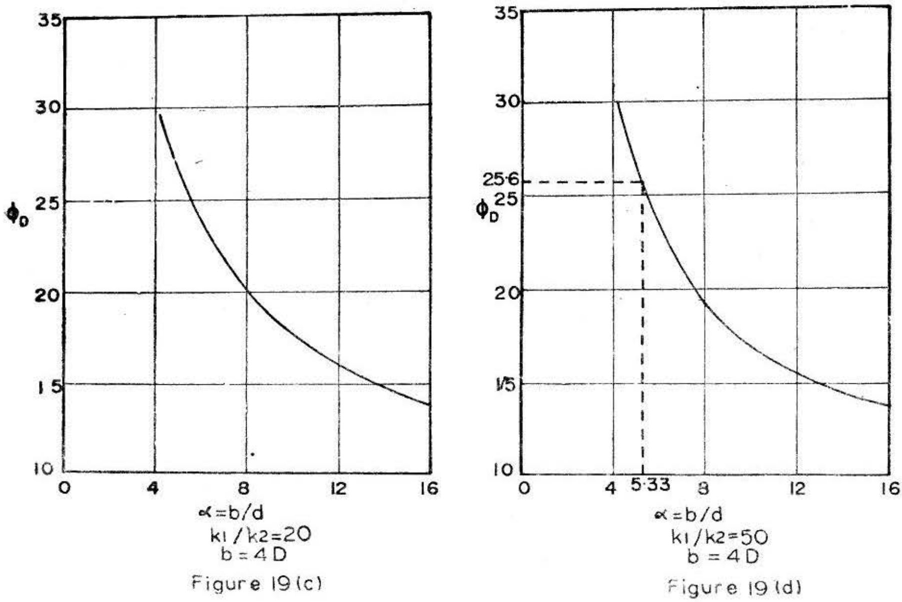

FIGURE 19 (c and d) :

(c) Variation of ϕ_D with α, i.e., b/d for $k_1/k_2=20$ and b/D=4.

(d) Variation of ϕ_D with α, i.e., b/d for $k_1/k_2=50$ and b/D=4.

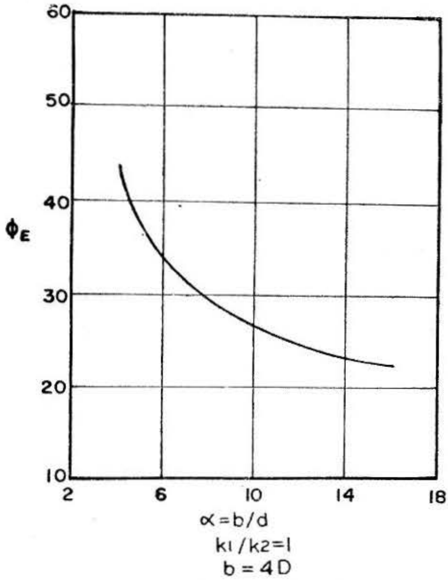

Figure 20 (a)

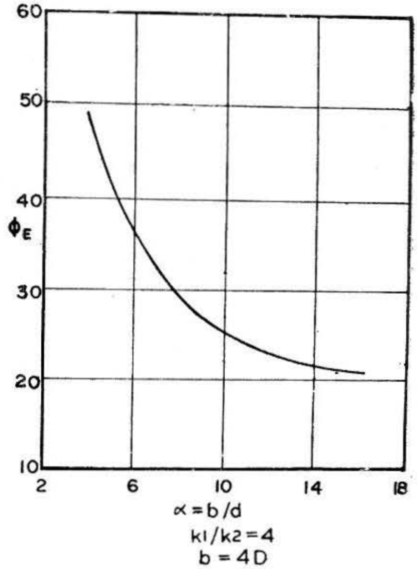

Figure 20 (b)

FIGURE 20 (a and b) :

(a) **Variation of ϕ_E with α, i.e., b/d for $k_1/k_2=1$ and b/D=4.**

(b) **Variation of ϕ_E with α, i.e., b/d for $k_1/k_2=4$ and b/D=4.**

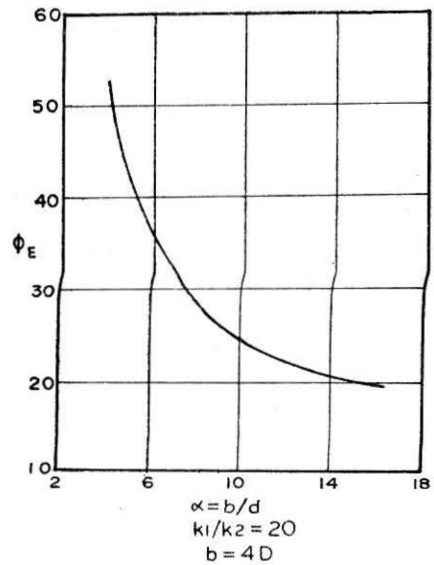

Figure 20 (c)

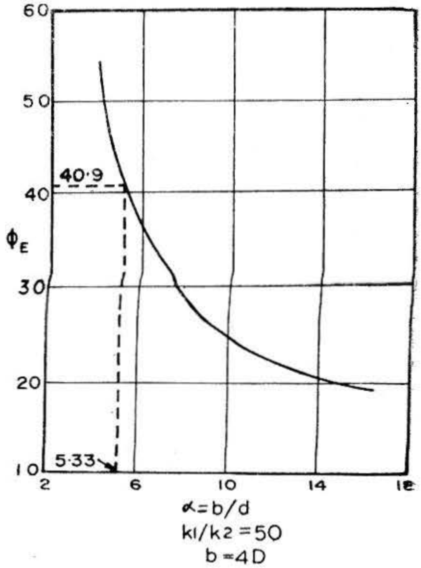

Figure 20(d)

FIGURE 20 : (c and d) :

(c) **Variation of ϕ_E with α, i.e., b/d for $k_1/k_2=20$ and b/D=4.**

(*d*) **Variation of ϕ_E with α, i.e., b/d for $k_1/k_2=50$ and b/D=4.**

References

BLIGH, W.G. (1910) : "Practical Design of Irrigation Works." Published by Constable & Co., London, Second Edition.

BOSE, N.K. (1930) : "Exponential Law of Sub-soil Flow". Punjab Engineering Congress Paper No. 140.

CLIBBORN AND BERESFORD (1902) : "Experiment on Passage of Water through Sand". Government of India Central Printing Office.

COLMAN, J.B.T. (1961) : "The Action of Water under Dams". *A.S.C.E.*, Paper No. 1356.

HAIGH, F.F. (1935) : "Design of Weirs on Sand Foundations". Punjab Engineering Congress Paper No. 182.

HARR, M.E. (1962) : "Ground Water and Seepage". Mc-Graw Hill Book Co.

HARZA, L.F. (1934) : "Uplift and Seepage under Dams on Sand". Paper No. 1920, *Proceedings A.S.C.E.*

KHOSLA, A.N. ; Bose, A.K. and Taylor, E.M. (1936) : "Design of Weirs on Permeable Foundations", Publication No. 12, Central Board of Irrigation, India.

LANE, E.W. (1935) : "Security from Under-seepage Masonry Dams on Earth Foundations". *Trans. A.S.C.E.*, No. 100.

LELIAVSKY, S. (1965) : "Design of Dams for Percolation and Erosion". Chapman and Hall.

PAVLOVSKY, N.N. (1922) : "The Theory of Ground Water beneath Hydrotechnical Structures (in Russian)" Petersburg.

PHATAK, M.B. (1970) : "Seepage below Apron with Downstream Cutoff founded on Two Layer Media". A dissertation submitted in partial fulfilment of requirements for M.E. Civil (Hydraulic Engg.) University of Jodhpur.

PUNMIA, B.C. (1969) : "Study of Seepage below a Horizontal Impervious Apron founded on Anisotropic Pervious media of Finite Depth." A dissertation submitted in partial fulfilment of the requirements for M.E. Civil (Soil Engineering), University of Jodhpur.

SALIM, M.A. (1947) : "Dams on Porous Media", *Trans. A.S.C.E.*, Vol. 112.

TODD, D.K. (1961) : "Seepage through Layered Anisotropic Porous Media", *Journal of the Hydraulic Division A.S.C.E.*, May 1961.

VAIDHINATHAN, V.I. and GURDAS RAM (1936) : "The Electric Method of Investigating the Uplift Pressure under Dams and Weirs". Research Publication, Vol. V, No. 4, Irrigation Research Institute, Lahore.