Dynamic Study of an Earth Dam Model

by

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Introduction

EARTH Dams are increasingly being used in major river valley projects located in the seismically active areas of the country. Their stability under ground motions caused during earthquakes is of great importance. Various analytical methods (1960, 1963, 1965, 1966 and 1969) are available for analysing earth dams subjected to earthquakes. The shear beam analysis (1960, 1963 and 1965) treats the system as a one dimensional structure and is approximate. This method cannot take into account the arbitrary geometry, irregular profile and variation of material properties along the width of the dam On the contrary, the finite element method of analysis (1963 and 1968) is more versatile and can account for the above mentioned variations. This method is now increasingly being used in the dynamic analysis of dams (1966 and 1969). In the past, the design of earth dams has usually been based on pseudostatic methods assuming a certain design coefficient on an empirical basis. Very few earth dams have been built in the country, the designs of which are based on consideration of their dynamic behaviour. However, such dams have not been subjected to any major earthquake shock so as to test the adequacy of the analytical methods employed in dynamic analysis.

The behaviour of models of earth dams is also studied in the laboratory using large shock vibration tables. The base motion generated by the shock vibration tables is random in nature but generally does not have a close relationship with the ground motion expected during an earthquake at the site of the prototype dam. Conditions of similitude have been discussed here and it is seen that in actual practice, it is not possible to satisfy all the requirements of similitude. Thus, from observations of model tests, it is not possible to predict prototype behaviour. Due to impossibility of simulating all modelling conditions and also due to difficulties in reproducing earthquake type ground motions in the laboratory, experimental studies give results which are basically qualitative in nature. However, it has been suggested that tests of models of earth dams on shake tables can be used to verify the theoretical methods applied to models themselves. The model can be theoretically analysed for the base motion of the table and

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This paper was received on 20 August 1971. It is open for discussion up to December 1972.

the dynamic response thus obtained can be compared with that obtained from the model test to verify the analytical method of analysis.

Results of a vibration test of a model of a homogeneous earth dam using a shake table are reported in this paper. The model has also been analysed for the base motion of the table using finite element method. The theoretical behaviour of the model has been compared with its behaviour determined experimentally. Since there is a good agreement between the theoretical and experimental behaviour, the theoretical technique gets verified and thus can be applied for evaluating the dynamic behaviour of the prototype dams.

Theory of Model Analysis

The pertinent variables involved in the problem are :

(1) L — length of the dam (L)
(2) h — height of the dam (L)
(3) w — weight density of dam material (FL^{-3})
(4) E — modulus of elasticity of dam material (FL^{-2})
(5) t — period of vibration of the dam (t)
(6) α — acceleration applied to the dam (Lt^{-2})
(7) g — acceleration due to gravity (Lt^{-2})
(8) y — displacement of the dam (L).

The dimensions of the various variables are indicated in the brackets. The above eight variables appear in three basic dimensions and so the problem can be expressed in five dimensionless terms as follows :

$$\frac{y}{L}=f\left(\frac{h}{L},\frac{\alpha}{g},\frac{gt^2}{L},\frac{wL}{E}\right) \qquad \text{...(1)}$$

For a true structural model, for studying any quantity of interest, the remaining dimensionless terms must be the same for the model and the prototype. If the four dimensionless factors within the brackets are identical for model and the prototype, then, it will be possible to predict the behaviour of the prototype from the observed behaviour of the model and from Equation (1)

$$\left(\frac{y}{L}\right)_{\text{prototype}}=\left(\frac{y}{L}\right)_{\text{model}} \qquad \text{...(2)}$$

The requirements of similitude are, therefore, as follows :

(*i*) $\left(\frac{h}{L}\right)_m=\left(\frac{h}{L}\right)_p$; $h_m=h_p.\,q$ ...(3)

where, $q=\frac{L_m}{L_p}$ =scale ratio, m denotes the quantity pertaining to model and p denotes the quantity pertaining to prototype.

(*ii*) $\left(\frac{\alpha}{g}\right)_m=\left(\frac{\alpha}{g}\right)_p$; $\alpha_m=\alpha_p$ because $g_m=g_p$ ...(4)

(*iii*) $\left(\frac{gt^2}{L}\right)_m=\left(\frac{gt^2}{L}\right)_p$; $t_m=t_p.\,\sqrt{q}$ ...(5)

(*iv*) $\left(\frac{wL}{E}\right)_m=\left(\frac{wL}{E}\right)_p$; $w_m=w_p.\frac{E_m}{E_p}\ \frac{1}{q}$ ...(6)

Condition (*i*) means that the model should be geometrically similar to the prototype. Condition (*ii*) requires that same accelerations should be given to the model as are expected in the prototype. Condition (*iii*) requires that periods in model should be $\sqrt{q}$ ratio with those in prototype. Condition (*iv*) requires the relationship between unit weights. Since models are usually made smaller than the prototype, condition (*iv*) requires that the material of the model should have a low elastic modulus and high density. This condition is usually not practicable with a large scale ratio. In addition, stress strain relationship of materials of model and prototype should be identical. It is, therefore, desirable to have same material for model as well as prototype. If this is done, then condition (*iv*) is violated. Hence it is not possible to predict prototype behaviour from observations of models.

Vibration Test

The test set-up consists of a shock vibration table which is excited by means of a pendulum of adjustable weight and fall. The size of the shake table is 5 metres by 3 metres in plan and could accommodate a model 0.6 metres high. So accordingly, the height of the model was kept 0.6 metres. A model is constructed of sand on the shake table. The cross-section of the model is shown in Figure 1 and has side slopes of 2.0.

Seven acceleration pick-ups, the positions of which are marked in Figure 1 were embedded in the model at the time of construction. Out of these, one acceleration pick-up was placed at the base of the dam so as to measure the table motion. When the table is excited by the pendulum, the accelerations at different levels are recorded on pen recorders which are connected to the acceleration pick-ups through universal amplifiers.

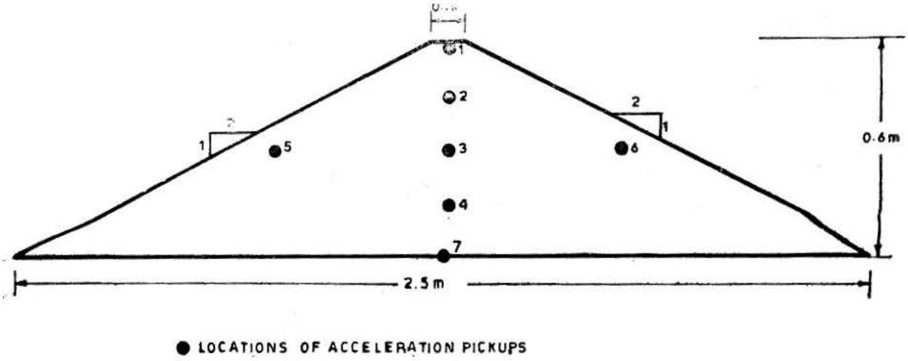

FIGURE 1 : Cross-section of earth dam model.

For the earth dam model, acceleration records at various levels were obtained. A typical record of the base motion of the table is shown in Figure 2. The peak accelerations obtained experimentally at the various points in the model are given in Table II.

Theoretical Analysis

The model has also been analysed theoretically for the base motion

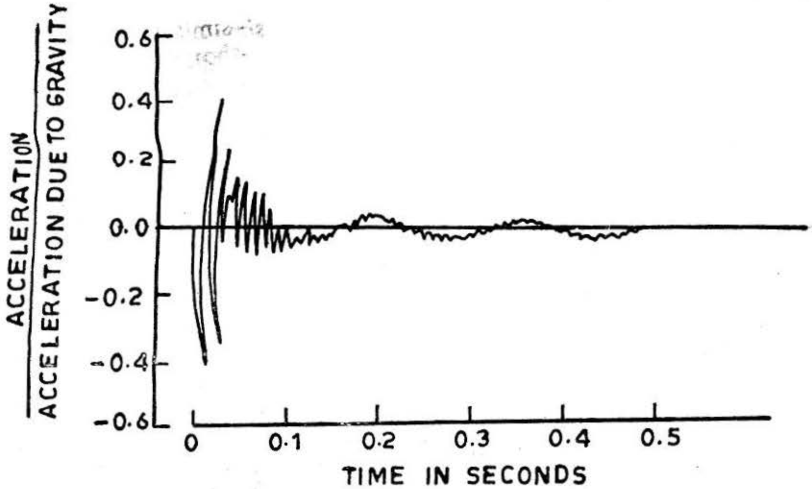

FIGURE 2 : Acceleration record of shake table motion.

of the table. Finite element technique has been used in the analysis (1963 and 1968). The earth dam model was represented by 84 degrees of freedom for the purpose of analysis. Response has been evaluated using mode superposition method (1966, 1969 and 1970). Thus it is first necessary to determine the frequencies and mode shapes of the dam model. These have been determined from the case of free undamped vibration using the inverse iteration procedure (1964, 1970). Following properties of the material of the model were obtained experimentally and used in the analysis :

Weight density $=2{,}000$ kg/m^3

Modulus of elasticity $=4.5\times10^6$ kg/m^2

These values are associated with a longitudinal wave propagation velocity of 150 m/sec. A representative value of Poisson's ratio of 0·35 for sand has been used in the analysis.

Table I gives the natural periods of vibration in the first three modes of vibration.

TABLE I

Natural periods of vibration.

Mode No.	Natural Period in Seconds
1	0·020
2	0·011
3	0·008

In the various modes of vibration, the dam vibrates in such a way that the mode shapes satisfy the following orthogonality relationship :

$$\left.\begin{array}{l}\{\phi_r\}^T [M] \{\phi_s\}=0 \\ \{\phi_r\}^T [K] \{\phi_s\}=0\end{array}\right\} \quad r \neq s \qquad \ldots(7)$$

where. $[M]$ is the mass matrix, $[K]$ the stiffness matrix, $\{\phi\}$ the mode shape, r and s are two different modes of vibration and the superscript T indicates that the matrix has been transposed.

The base motion of the table has been digitized and the response of the model for this base motion determined. The equations of motion of the idealized system of the model due to the table acceleration can be written as :

$$[M]\{\ddot{Z}\}+[C]\{\dot{Z}\}+[K]\{Z\}=-[M]\{\ddot{G}\ (t)\} \qquad \ldots(8)$$

Where, $[M]$ is the mass matrix, $[C]$ the viscous damping matrix, $[K]$ the stiffness matrix, $\{Z\}$ the vector of nodal point displacements relative to base, and $\{\ddot{G}\ (t)\}$ the vector of table acceleration and is given by

$$\{\ddot{G}\ (t)\}=\begin{Bmatrix} \ddot{x}_g(t) \\ 0 \\ \ddot{x}_g(t) \\ 0 \\ \vdots \\ \vdots \\ \ddot{x}_g(t) \\ 0 \end{Bmatrix} \qquad \ldots(9)$$

Where, $\ddot{x}_g\ (t)$ is the horizontal table acceleration and the vertical table acceleration is zero. To transform the nodal co-ordinate 'Z' to the normal co-ordinates 'Ψ', let

$$\{Z\}=[\phi]\ \{\Psi\} \qquad \ldots(10)$$

where, $[\phi]$ is a square matrix composed of model vectors as columns and is given by :

$$[\phi]=\begin{bmatrix} \phi_1^{(1)} & \phi_1^{(2)} & \cdots & \phi_1^{(n)} \\ \phi_2^{(1)} & \phi_2^{(2)} & \cdots & \phi_2^{(n)} \\ \vdots & \vdots & & \vdots \\ \phi_n^{(1)} & \phi_n^{(2)} & \cdots & \phi_n^{(n)} \end{bmatrix}$$

n is the number of degrees of freedom of the system.

Substituting Equation (10) in Equation (8) and premultiplying throughout by $[\phi]^T$,

$$[\phi]^T\ [M]\ [\phi]\ \{\ddot{\Psi}\}+[\phi]^T\ [C]\ [\phi]\ \{\dot{\Psi}\}+[\phi]^T\ [K]\ [\phi]\ \{\Psi\} = -[\phi]^T\ [M]\ \{G\ (t)\} \qquad \ldots(11)$$

Making use of the orthogonality relationships given by Equation (7), it is noted that $[\phi]^T [M] [\phi]$ and $[\phi]^T [K] [\phi]$ are diagonal matrices. If it is assumed that the transformation given by Equation (10) which diagonalises the mass and stiffness matrices, also diagonalises the damping matrix, then Equation (11) can be simplified and solved. $\{\phi\}$'s can be chosen such that $[\phi]^T [M] [\phi]=[I]$, an identity matrix, then Equation (11) can be written as :

$$\left\{\ddot{\Psi}\right\}+\begin{bmatrix}\ddots & & \\ & 2p_r\,\beta_r & \\ & & \ddots\end{bmatrix}\left\{\dot{\Psi}\right\}+\begin{bmatrix}\ddots & & \\ & p_r^2 & \\ & & \ddots\end{bmatrix}\left\{\Psi\right\}=-[\phi]^T [M] \{\ddot{G}(t)\} \quad \ldots(12)$$

$$\text{where, } \begin{bmatrix}\ddots & & \\ & 2p_r\,\beta_r & \\ & & \ddots\end{bmatrix}=[\phi]^T [C] [\phi] \,; \begin{bmatrix}\ddots & & \\ & p_r^2 & \\ & & \ddots\end{bmatrix}=[\phi]^T [K] [\phi],$$

p_r the circular natural frequency and β_r the fraction of critical damping in r^{th} mode of vibration.

From Equation (10), it can be written that

$$\{\ddot{Z}\}=[\phi]\{\ddot{\Psi}\} \quad \ldots(13)$$

$$\{\ddot{Z}\}=[\phi]\left[-[\phi]^T [M] \{\ddot{G}(t)\}-\begin{bmatrix}\ddots & & \\ & 2p_r\,\beta_r & \\ & & \ddots\end{bmatrix}\left\{\dot{\Psi}\right\}-\begin{bmatrix}\ddots & & \\ & p_r^2 & \\ & & \ddots\end{bmatrix}\left\{\Psi\right\}\right] \quad \ldots(14)$$

$$\{\ddot{Z}\}=-[\phi][\phi]^T [M] \{\ddot{G}(t)\}-[\phi]\begin{bmatrix}\ddots & & \\ & 2p_r\,\beta_r & \\ & & \ddots\end{bmatrix}\left\{\dot{\Psi}\right\}-[\phi]\begin{bmatrix}\ddots & & \\ & p_r^2 & \\ & & \ddots\end{bmatrix}\left\{\Psi\right\} \quad \ldots(15)$$

Since $\{\phi\}$'s are such that $[\phi]^T [M] [\phi]=[I]$,

it follows that $[\phi]^T [M]=[\phi]^{-1}$. (16)

Substituting in Equation (15), one obtains

$$\{\ddot{Z}\}=-\{\ddot{G}(t)\}-[\phi]\begin{bmatrix}\ddots & & \\ & 2p_r\,\beta_r & \\ & & \ddots\end{bmatrix}\left\{\dot{\Psi}\right\}-[\phi]\begin{bmatrix}\ddots & & \\ & p_r^2 & \\ & & \ddots\end{bmatrix}\left\{\Psi\right\} \quad \ldots(17)$$

$$\left\{\ddot{X}-\ddot{G}\right\}=-\{G(t)\}-[\phi]\begin{bmatrix}\ddots & & \\ & 2p_r\,\beta_r & \\ & & \ddots\end{bmatrix}\left\{\dot{\Psi}\right\}-[\phi]\begin{bmatrix}\ddots & & \\ & p_r^2 & \\ & & \ddots\end{bmatrix}\left\{\Psi\right\} \quad \ldots(18)$$

Where, $\{\ddot{X}\}$ represents the vector of absolute accelerations.

$$\{\ddot{X}\}_r=-2p_r\,\beta_r\,\{\dot{Z}_r\}-p_r^2\,\{Z_r\} \quad \ldots(19)$$

Where, $\{\ddot{X}\}_r$ represents the vector of absolute accelerations in r^{th} mode of vibration. The total absolute accelerations are obtained by summation in the various modes of vibration :

$$\{\ddot{X}\}=\sum_{r=1}^{n}\left[-2p_r\,\beta_r\,\{\dot{Z}_r\}-p_r^2\,\{Z_r\}\right] \quad \ldots(20)$$

Contribution of the first three modes of vibration has been considered. The accelerations obtained theoretically for damping values of 5, 10 and 15 percent of critical damping in each mode are given in Table II.

TABLE II*

Accelerations in Earth Dam Model.

Point No.	Experimental	Theoretical		
		5% damping	10% damping	15% damping
1	1.07	1.60	1.11	0.94
2	0.98	1.40	1.03	0.87
3	0.86	0.96	0.81	0.72
4	0.50	0.48	0.43	0.45
5	0.71	0.83	0.70	0.73
6	0.67	0.83	0.70	0.73

* Multiply the entries by acceleration due to gravity.

It may be noted from the table that for a damping of 10 percent of critical damping in all modes, the experimental and theoretical results compare with each other. Since the contribution of the first mode of vibration is the maximum, it appears that the model has a damping of about 10 percent of critical damping in the fundamental mode.

Summary and Conclusions

Vibration test on a model of a homogeneous earth dam has been carried out using a large shock vibration table. The model has also been analysed theoretically for the table motion. The theoretical and experimental results show a good comparison and so the theoretical procedure gets verified and can be applied for the analysis of prototype dams.

Acknowledgements

The authors are thankful to the Director, School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee for providing facilities. Thanks are also due to the staff of the Structural Dynamics Laboratory and the workshop for assistance in the experimental work.

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