A Field Test for Permeability Ratio

by

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Introduction

THERE is considerable evidence in the fields of geology, soil mechanics and ground water hydrology to indicate that majority of natural soils are anisotropic with respect to their permeability characteristics. Since most soils are formed by the transportation and deposition of sediments in water, they usually exhibit an approximately horizontal stratification wherein the permeability in the plane parallel to the stratification tends to be considerably higher than that in the perpendicular direction. Such anisotropy is referred to as cross-anisotropy or transverse isotropy. Although the opposite situation (where the permeability in the vertical direction is greater than that in the horizontal plane) will also exist occasionally, it is relatively rare.

In a number of engineering investigations it is desirable to know the ratio of horizontal to vertical permeability; for example, this information is required to estimate flow beneath dams and to design cut-off structures to inhibit seepage. The two methods commonly used to determine the permeability are: (1) laboratory testing of soil samples, and (2) field pumping tests. Due to difficulties associated with obtaining a representative soil specimen and conducting a meaningful permeability test, laboratory methods often yield misleading results of permeability ratios. Accordingly, pumping tests that involve the measurement of water levels in the vicinity of a well which is pumped at a known rate, are used frequently to determine the permeability characteristics. In general, the pumping well fully penetrates the aquifer under consideration. However, transverse isotropy is of no significance in the case of fully penetrating wells in an artesian aquifer, wherein the flow is only radial. A partially penetrating well presents a two-dimensional problem in cylindrical coordinates and is, therefore, well-suited for the study of flow in horizontal (or radial) and vertical directions.

Previous Work

The problem of a partially penetrating well has been solved in detail by Hantush (1961). Stallman (1963) used electrical analogy to obtain flow pattern around partially penetrating wells. In connection with seepage and dewatering studies for the locks and dams planned for Arkansas River in U.S.A., Mansur and Dietrich (1965) described a very-well designed pumping test. In addition to determining the permeability ratio by trial and error, they used results of electrical analogy to obtain the theoretical response of the system. The method has the disadvantage that it may be

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This paper was received on 14 December 1971. It is open for discussion up to October 1972.

used only for analysis of tests for which steady state conditions have been reached. Also, the analogue results are often not available for the specified field situation to be analyzed. More recently Weeks (1969) approached the problem by the use of an equation modified from Hantush (1961) and Rao *et al* (1971) studied the transient nature of the drawdown at a partially penetrating well.

Theoretical Development

The flow pattern around a partially penetrating well is shown in Figure 1. In addition to the water in the region above the depth penetrated by the well, the water in the aquifer below this depth will also move toward the well; thus, simple intuitive reasoning indicates that the resulting flow lines must have *both vertical and radial components. There is an* additional loss of potential in the neighbourhood of the well, because the streamlines converge and their length is increased over the case of the fully penetrating well. Due to combined vertical and radial flow a separate drawdown is generally obtained for each flow line and the head is calculated by averaging it along the surface of the well. Beyond a certain distance, the effect of partial penetration is negligible, and the flow lines are essentially horizontal.

The analysis presented here is applicable to wells partially penetrating a transversely isotropic artesian aquifer. It is assumed that the observation wells have the same filter length as that of the pumped well. Use is made of deviations which occur between the drawdowns in observation wells and those that would occur near a pumped well which is fully penetrating. The deviations due to partial penetration are amplified Weeks (1969) when the vertical permeability, k_z, is less than the horizontal or radial permeability, k_r. At a radial distance r from the pumped well the difference in drawdown in an anisotropic aquifer is the same as that at a distance given by

$$r' = r\sqrt{\frac{k_z}{k_r}} \qquad \ldots(1)$$

if the soil was transformed into an equivalent isotropic soil.

The drawdown s after time t in an observation well near a partially penetrating pumped well (the wells being fully screened over the entire depth of penetration, L) is given by the following equation (Rao *et al*, 1971) :

$$s = \frac{Q}{4\pi B k_r}\left[-E_i(-u) + \frac{2B^2}{\pi^2 L^2}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi L}{B}\right)^2 . W(u, b_n)\right] \ldots(2)$$

where

$$u = \frac{r^2 S}{4Bk_r t} \qquad \ldots(3)$$

$$b_n = \frac{n\pi r'}{B} \qquad \ldots(4)$$

and $W(u, b_n)$ is expressed as

$$W(u, b_n)=\int_u^\infty \frac{1}{y} exp\left(-y-\frac{b_n^2}{4y}\right)dy \quad ...(5)$$

In the above equations, B is the thickness of the aquifer, Q is the discharge rate of the pumping well, S is the coefficient of storage of the aquifer and E_i is the exponential integral function.

For $u<b_n^2/20$, the function $W(u, b_n)$ can, for all practical purposes, be replaced by $2K_o(b_n)$ where $K_o(b_n)$ is the zero order nodified Bessel function of the second kind. Thus, for $u<(k_z/k_r)(\pi_r/B)^2/20$, or for $t>BS/2k_z$ the second term in Equation (2) becomes independent of time and the drawdown is given by

$$s=\frac{Q}{4\pi Bk_r}\left[-E_i(-u)+\frac{2B^2}{\pi^2L^2}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi L}{B}\right)^2 2K_o\left(\frac{n\pi r'}{B}\right)\right] \quad ...(6)$$

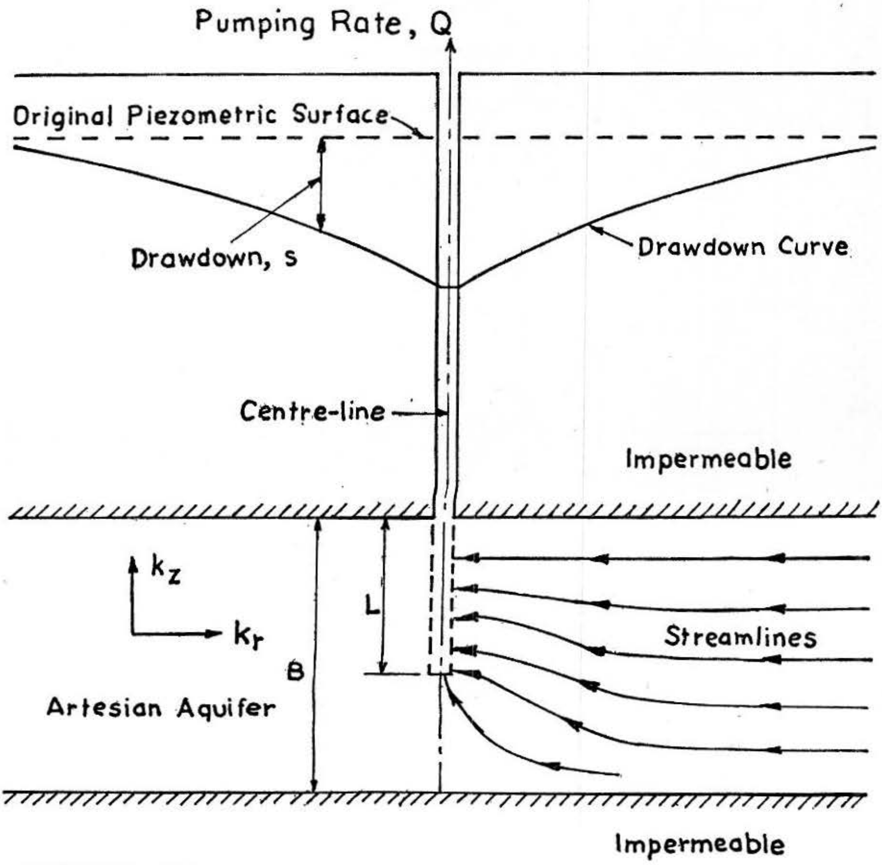

FIGURE 1 : Schematic diagram of a partially penetrating well in a transversely isotropic artesian aquifer.

From the above equation s_p, the departure in drawdown from that obtained for a fully penetrating well may be expressed as:

$$s_p = \frac{Q}{4\pi Bk_r}\left[\frac{2B^2}{\pi^2 L^2}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi L}{B}\right)^2 2K_o\left(\frac{n\pi r'}{B}\right)\right] \qquad ...(7)$$

For convenience, this may be written as

$$s_p = \frac{Q}{4\pi Bk_r}F \qquad ...(8)$$

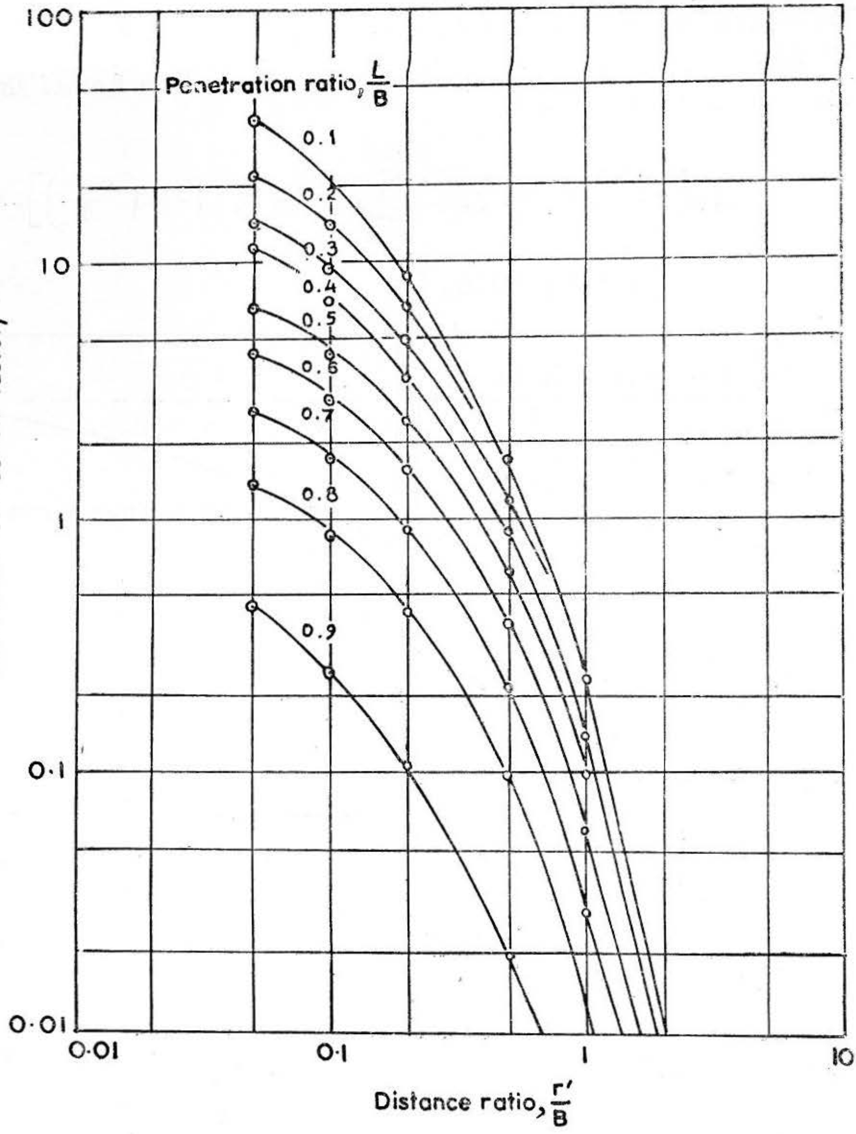

FIGURE 2 : Drawdown correction factor as a function of penetration and distance ratios.

where F is a dimensionless drawdown correction factor. A plot of the variation in F with respect to changes in the penetration ratio, L/B, and the distance ratio r'/B, is shown in Figure 2. The curves in this figure will be referred to as type curves.

Procedure

The procedure to determine the permeability ratio is outlined in the following steps :

(1) Plot the time drawdown graph for each observation well and determine the average value of the coefficient of permeability by use of Jacob's method, which is explained in Appendix A.

(2) For a selected time, plot the distance-drawdown graph, the distance being plotted on a logarithmic scale. To obtain such a plot, the drawdown measurements, of course, must have been made in all wells at essentially the same time. Draw straight line of slope $0{\cdot}366\ Q/kB$ [*vide* Equation (A. 5) of Appendix A] below the plotted drawdown values.

(3) Determine the values of s_p for each observation well by subtracting the drawdown value for straight line plot from the observed drawdown.

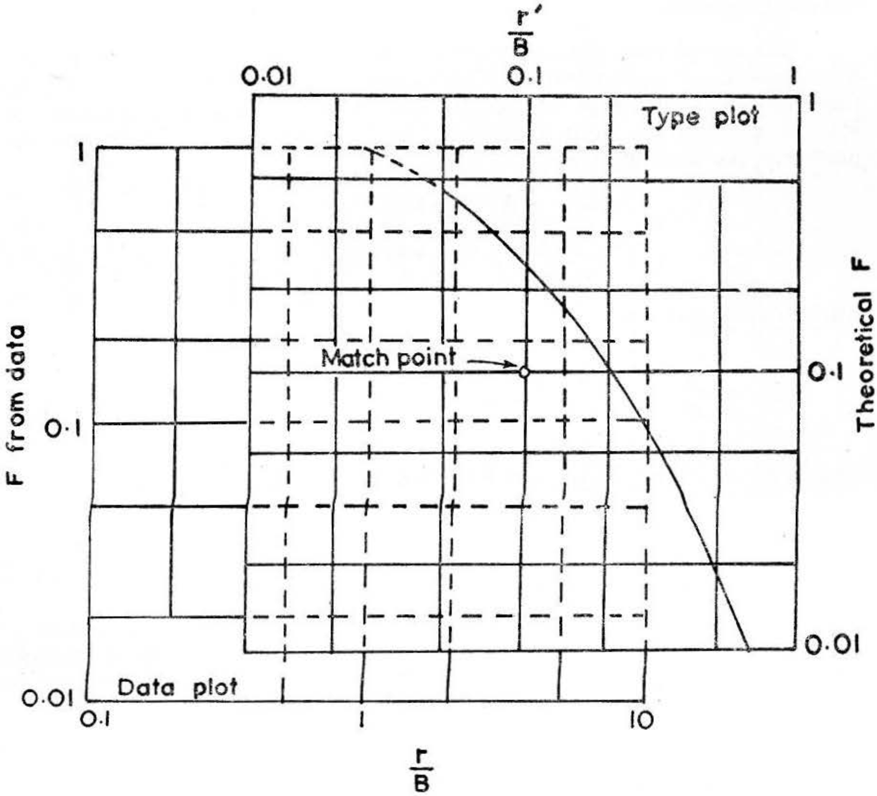

FIGURE 3 : Matching the data and type curves.

(4) Use the s_p values to compute the drawdown correction factors by employing Equation (8), and prepare a graph of F *versus* r/B on logarithmic axes. This is the data curve.

(5) Match the data curve to one of the type curves, corresponding to the selected penetration ratio, and select any convenient point common to both plots. This is shown in Figure 3.

(6) For the selected point, determine the coordinate value of r/B from the data curve and the value of r'/B from the type curve. The principal permeability ratio is then given by the following equation :

$$\frac{k_r}{k_z}=\left[\frac{r/B}{r'/B}\right]^2 \qquad \ldots(9)$$

Summary

A method is presented to determine the ratio of horizontal and vertical permeabilities in an anisotropic soil. This is based on the field pumping test of a *partially penetrating well. Comparison is made with a* fully penetrating well and the solution is obtained by altering the domain of the anisotropic aquifer to that of an equivalent isotropic aquifer. Step-by-step procedure indicates the evaluation of permeability ratio. It is suggested that the proposed technique is useful in a quantitative determination of the permeability ratio, which is often guessed in many engineering projects.

Acknowledgements

The calculations necessary for the present work were performed at Vogelback Computing Centre of Northwestern University, U.S.A. Sincere thanks are extended to Professors Raymond J. Krizek and Gabor M. Karadi of Northwestern University and University of Wisconsin respectively, for their advice.

APPENDIX A

Analysis of Pumping Tests

In the case of an isotropic aquifer fully penetrated by a well, the drawdown is given by

$$s=\frac{Q}{4\pi Bk}\int_{u}^{\infty}\frac{exp\ (-u)}{u}du \qquad \ldots(\text{A. }1)$$

where, $u=Sr^2/4kBt$. When u is less than about 0·01, Jacob (1947) suggested the following simplification :

$$s=\frac{Q}{4\pi Bk}ln\ \frac{2{\cdot}25kBt}{r^2\ S} \qquad \ldots(\text{A. }2)$$

Time-drawdown curves are obtained by plotting the drawdown on arithmetic scale and time on logarithmic scale of a semi-logarithmic graph. The resulting graph should be a straight line according to Equation (A. 2).

Distance-drawdown graphs are prepared by plotting the drawdown in each of several observation wells *versus* the distance of the observation well from the pumped well. Drawdown is plotted on arithmetic scale and distance on logarithmic scale. Based on the assumption that the drawdown measurements have been made in the wells at essentially the same time, the difference in drawdown $\triangle s$, for wells at distances r_1 and r_2, may be written as :

$$\triangle s = 2{\cdot}3 \frac{Q}{4\pi Bk} \log_{10} \left(\frac{r_2}{r_1}\right)^2 \qquad \text{...(A. 3)}$$

and, if $\triangle s$ is chosen for a logarithmic cycle of distance such that

$$\text{Log}_{10} \frac{r_2}{r_1} = 1 \qquad \text{...(A. 4)}$$

the coefficient of permeability is given by

$$k = 0{\cdot}366 \frac{Q}{B \triangle s} \qquad \text{...(A. 5)}$$

References

HANTUSH, M.S. (1961). ' Drawdown Around a Partially Penetrating Well". *Proceedings of the A.S.C.E.*, Vol. 87, No. HY 4, pp. 83-98.

JACOB, C.E. (1947). "Drawdown Test to Determine Effective Radius of Artesian Well". *Transactions of the A.S C.E.*, Vol. 112, pp. 1047-1070.

MANSUR, C.I. and DIETRICH R.J. (1965). "Pumping Test to determine Permeability Ratio". *Proceedings of the A.S.C E.*, Vol. 91, No. SM 4, pp. 151-183.

RAO, D.B. *et al* (1971). "Unsteady Drawdown at a Partially Penetrating Well in a Transversely Isotropic Artesian Aquifer". *Journal of the American Water Resources Association,* in Press.

STALLMAN, R.W. (1963). "Electric Analog of Three-Dimensional Flow to Wells and its Application to Unconfined Aquifers". *United States Geological Survey Water—Supply Paper 1536-H*, Washington, pp. 205-242.

WEEKS, E.P. (1969). "Determining the Ratio of Horizontal to Vertical Permeability by Aquifer-Test Analysis". *Water Resources Research,* Vol. 3, No. 1, pp. 196-214.