A Note on Composite Pressure Head and Gravity Flow Systems

by

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Introduction

GROUND water movements can be broadly classified into confined, unconfined and semiconfined flows. In the theory of seepage, the movement in which there is no free surface is called 'Confined'. Such movements of ground water occur under hydraulic structures flowing under the influence of pressure difference between head and tail-water or water flows towards a well in a confined water bearing stratum. 'Unconfind' flows are those in which the flow region is bounded at its top by a free surface. Such flows occur in seepage through earth dams. 'Semiconfined' seepage is characterised by the fact that the seepage flow at first is contiguous with the underground contour of the structure and then breaks away from it forming a free surface. Semiconfined flow takes place under structures below which significant drop of ground water-level occurs, *e.g.*, under the storage reservoirs of hydropower plants. Semiconfined flows, are also designated as composite pressure head and gravity flows. In this paper, a method is given to compute the discharge and pressure distribution at the base of a composite flow system for two-and three-dimensional cases. The results obtained for the two-dimensional case are compared with the experimental results from a Heleshaw apparatus and an electrical analogy equipment. The pressure distribution equations given by Muskat using superposition principle are also compared.

Formulation of Theoretical Expressions

(i) THREE-DIMENSIONAL CASE

Figure 1 is a definition sketch of a composite pressure head and gravity flow system wherein h is the sand height, h_o the fluid head at the inflow surface, h_w fluid head in the well, L the distance from the centre of the well to the breakaway point from confined flow, r_w the well radius and r_e the radius of the inflow surface. The flow system could be divided into two

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regions (Figure 1). In region I there is confined flow and in region II unconfined flow. At the junction of the two regions, the pressure head is equal to the thickness of the sand height. The expression for discharge in the confined radial flow region is (1946).

$$Q_r = \frac{2\pi \bar{k} h\,(h_o - h)}{\log_e \dfrac{r_e}{L}} \qquad \text{...(1)}$$

and that for unconfined flow

$$Q_g = \frac{\pi \bar{k}(h^2 - h_w^2)}{\log_e \dfrac{L}{r_w}} \qquad \text{...(2)}$$

By equation of continuity $Q_r = Q_g = Q$ (say)

The radius L from which the ground water flow becomes gravity flow can be computed from Equations (1) and (2) as

$$\log_e L = \log_e r_e - \frac{\pi \bar{k}}{Q}\, 2h(h_o - h)$$

$$= \log_e r_w + \frac{\pi \bar{k}}{Q}(h^2 - h_w^2) \qquad \text{...(3)}$$

Equating the two expressions for L in Equation (3), we get

$$Q = \frac{\pi \bar{k}\,(2h\, h_o - h^2 - h_w^2)}{\log_e \dfrac{r_e}{r_w}} \qquad \text{...(4)}$$

From Equations (1) and (3) an expression for the resultant head h at the base in the confined flow region can be derived as

$$\bar{h} = h_o + \frac{(h_o - h)\log_e \dfrac{r}{r_e}}{\log_e \dfrac{r_e}{r_w}} + \left\{\frac{h^2 - h_w^2}{2h}\right\} \times \left\{\frac{\log_e \dfrac{r}{r_e}}{\log_e \dfrac{r_e}{r_w}}\right\} \qquad \text{...(5)}$$

$$L < r < r_e$$

Similary from Equations (2) and (3) an expression for the resultant head $\bar{h}$ in the unconfined flow region can be derived as

$$\bar{h} = \sqrt{h_w^2 - \frac{(2hh_o - h^2 - h_w^2)\log_e \dfrac{r_w}{r}}{\log_e \dfrac{r_e}{r_w}}} \qquad \text{...(6)}$$

$$r_w < r < L$$

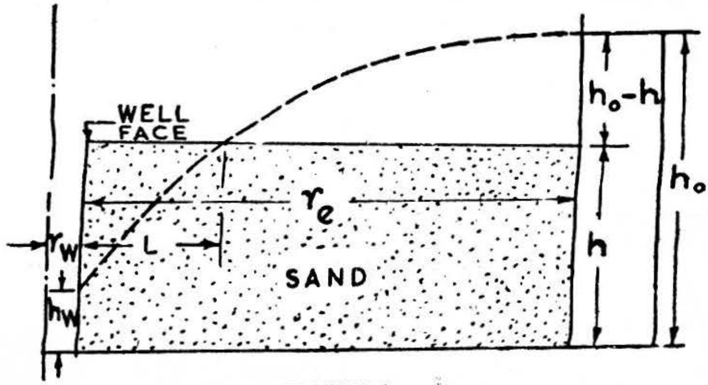

FIGURE 1.

(ii) TWO-DIMENSIONAL CASE

A definition sketch of a two-dimensional composite pressure head and gravity flow system is presented in Figure 2. In this sketch, h_a is the sand height, H the fluid head at the inflow surface, h_w the fluid head in the well, L the distance from the inflow surface to the discharge face and L_c the distance from the inflow surface to the breakaway point from the confined flow. As before the flow can be divided into two regions—confined flow in region I and unconfined flow in region II. Considering the confined aquifer the discharge per unit length is—

$$Q_1 = kh_a \frac{H-h_a}{L_c} \qquad \text{...(7)}$$

For the unconfined aquifer the discharge per unit length is—

$$Q_2 = \frac{k}{2} \frac{h_a^2 - h_w^2}{L - L_c} \qquad \text{...(8)}$$

By principle of continuity, $Q_1 = Q_2 = Q$ (say). Hence from Equations (7) and (8)

$$L_c = \frac{2Lh_a\,(H-h_a)}{2h_a\,(H-h_a)+(h_a^2-h_w^2)} \qquad \text{...(9)}$$

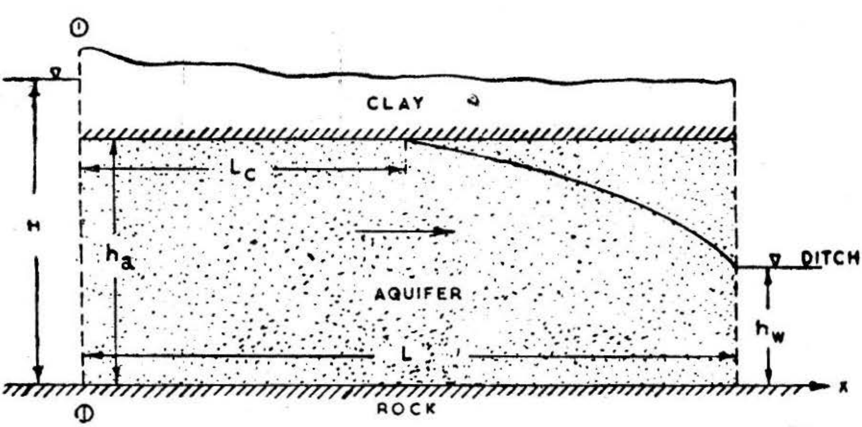

FIGURE 2.

Substituting the expression for L_c in Equation (7), we get

$$Q=\frac{k}{2L}(2Hh_a-h_a^2-h_w^2) \qquad \ldots(10)$$

At any distance x_1 from the inflow face

$$Q=k.h_a.\frac{H-h_1}{x_1} \qquad \ldots(11)$$

Hence the pressure head h_1 in the confined flow region

$$=H-\frac{Q}{kh_a}x_1, \quad 0<x_1<L_c \qquad \ldots(12)$$

Similarly in the unconfined flow region at a distance x_2 from the inflow face

$$Q=\frac{k}{2}\left(\frac{h_2^2-h_w^2}{L-x_2}\right) \qquad \ldots(13)$$

Hence pressure head

$$h_2=\sqrt{\frac{2Q}{k}(L-x_2)+h_w^2} \qquad \ldots(14)$$

This problem has been treated analytically by Muskat using the principle of superposition. In this method, instead of considering the flow region as confined and unconfined, the whole flow region is considered as a unit and the total head is divided into heads causing confined and unconfined flow respectively. His equations are as follows :

Two-Dimensional

Discharge for two-dimensional case

$$Q=Q_1+Q_2$$

$$=\frac{k}{2L}(2Hh_a-h_a^2-h_w^2) \qquad \ldots(15)$$

Three-Dimensional

Discharge for three-dimensional case

$$Q=Q_r+Q_g$$

$$=\frac{\pi k(2hh_o-h^2-h_w^2}{\log_e \frac{r_e}{r_w}} \qquad \ldots(16)$$

Total pressure head $\bar{h}$ for two-dimensional case

$$=H-\left[\frac{H-h_a}{L}+h_a-h_a^2-\frac{h_a^2-h_w^2}{L}.x\right] \qquad \ldots(17)$$

Total pressure head $\bar{h}$ for three-dimensional case

$$=\frac{h_o-h}{\log_e \frac{r_e}{r_w}}\log_e \frac{r}{r_w}+$$

$$\sqrt{h^2-\frac{(h^2-h_w^2)}{\log_e \frac{r_e}{r_w}}\log_e \frac{r_e}{r}} \qquad \ldots(18)$$

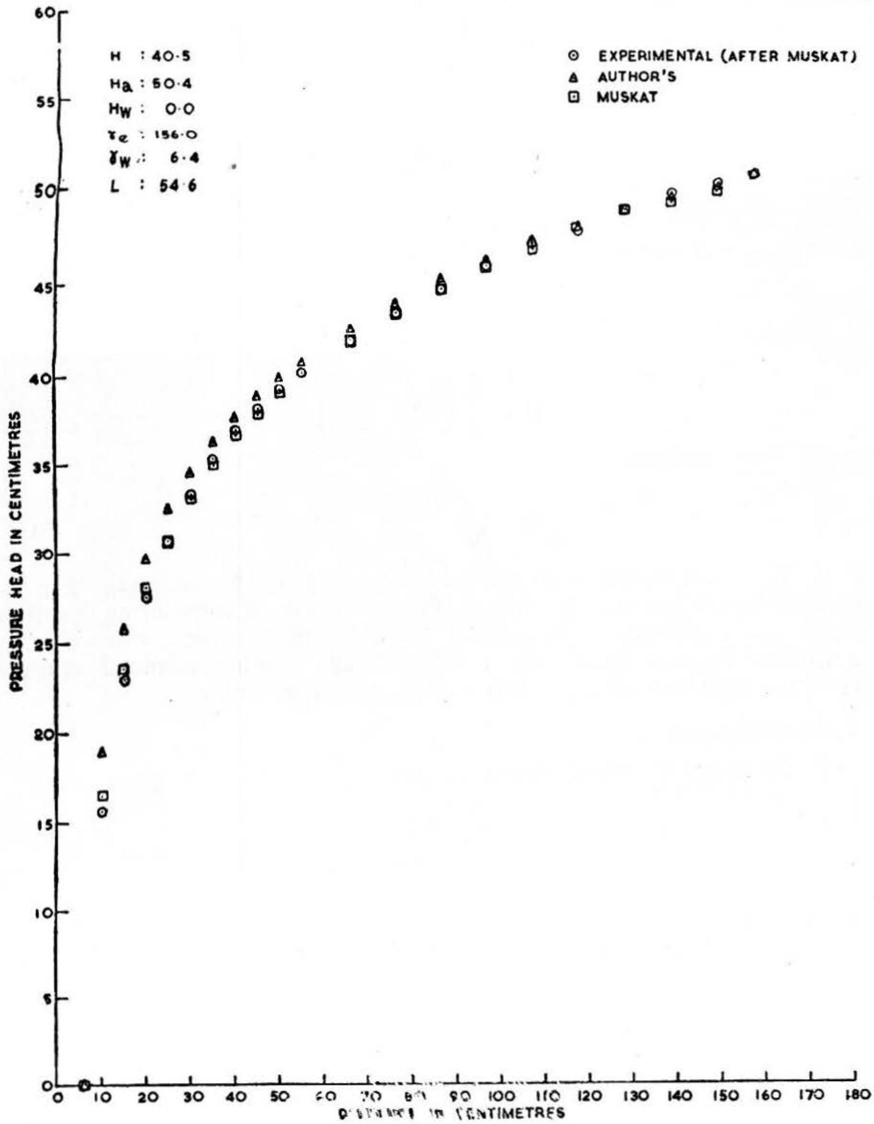

FIGURE 3 : Plot of distance versus pressure head for three-dimensional case.

Experimental Observations

Equipment : Experiments were conducted in a Heleshaw model and an electrical analogy apparatus to verify the expressions derived for the two dimensional case. The details of the equipment are reported elsewhere (1962 and 1970).

In experiments in the Heleshaw apparatus, lubricating oil SAE 40 was used as the model liquid having a viscosity of 1·722 poise. The depth of the aquifer was kept as 25 cm. Heads varying from 29·32 cm at the inlet and 15-17 cm at the outlet were maintained. The total length of the aquifer was kept as 150 cm with the confined aquifer varying from 70-83 cm depending on the upstream and downstream conditions. The pressure distribution in the unconfined aquifer was noted by measuring the height from the bottom of the aquifer to the free surface.

In the electrical analogy experiments, water with 0· 1 *N* hydrochloric acid was used as the electrolyte. Brass plates were provided at the upstream and downstream ends. The seepage face was simulated by brass rods to which varying potentials were applied. The free surface was found by a trial and error procedure by considering equal drops of head available from the breakaway point to the well face.

Results and Discussions

Figure 3 shows the comparison of pressure head for a three-dimensional case. In Figure 4, the pressure head against distance is plotted as

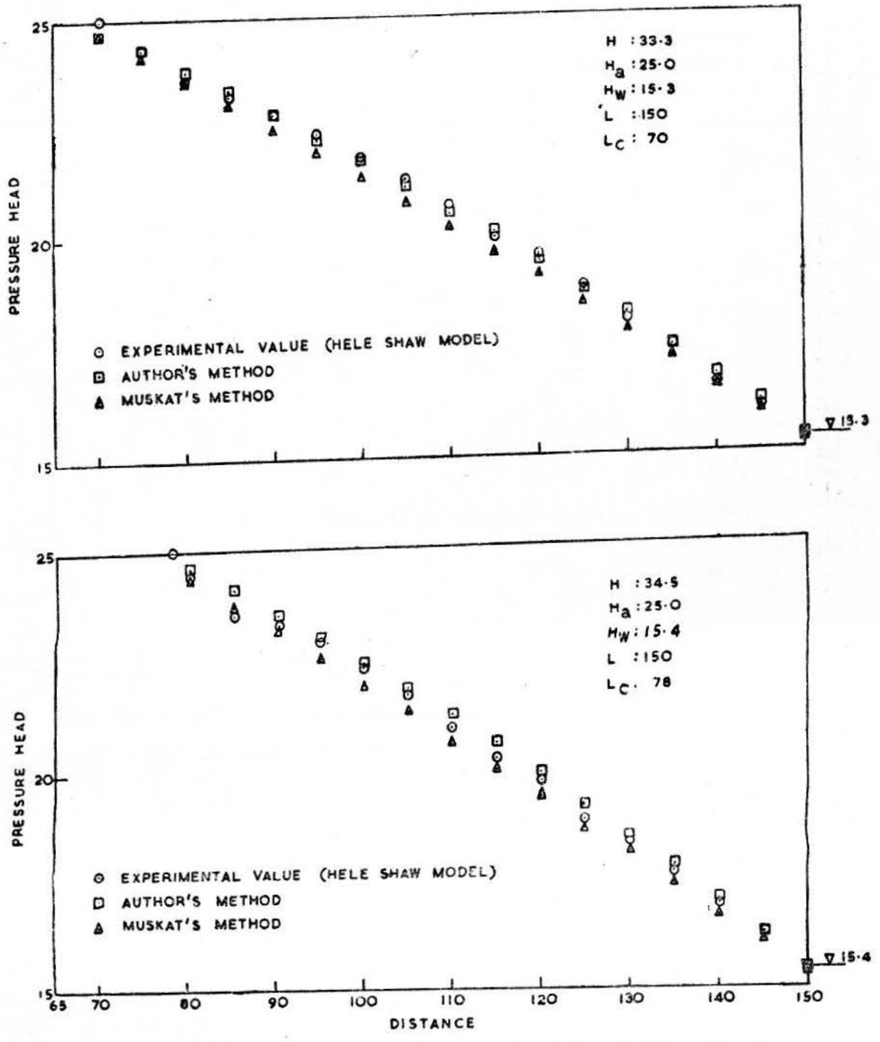

FIGURE 4 : Plot of distance versus pressure head for two-dimensional case.

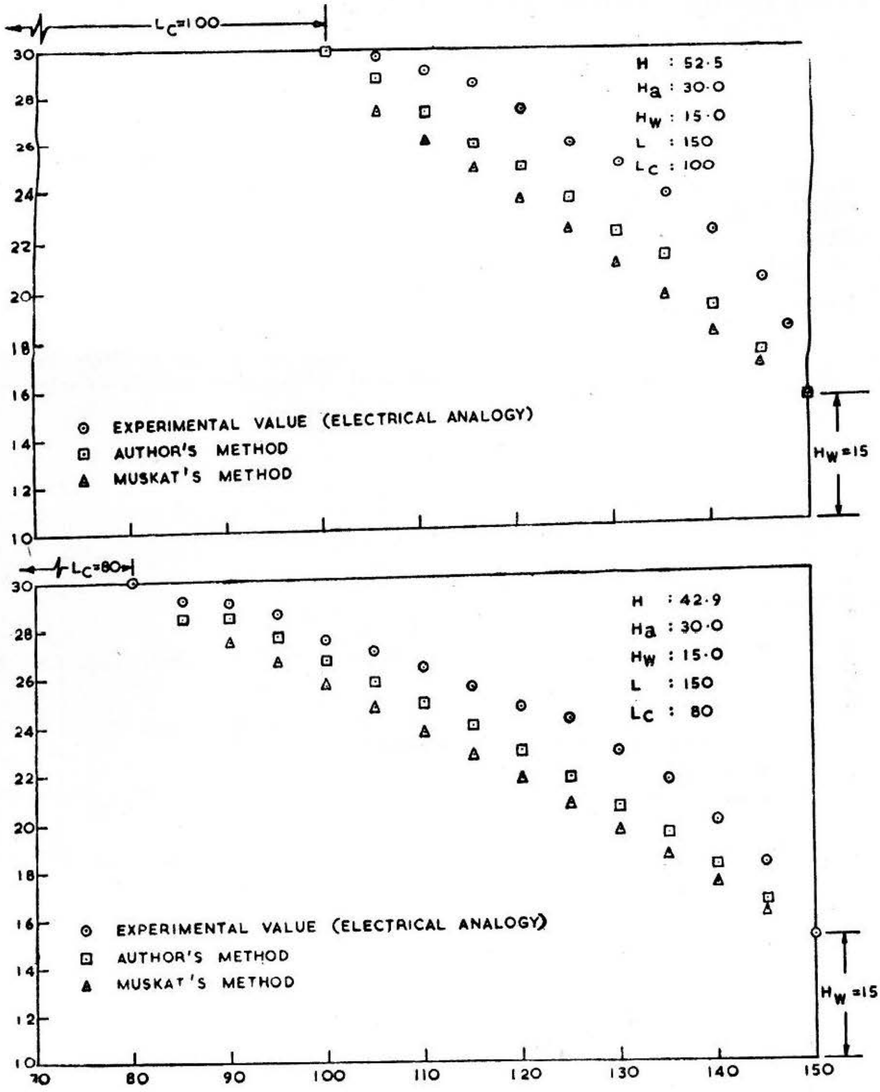

FIGURE 5 : Plot of distance versus pressure head for two-dimensional case.

derived using the authors' equations, Muskat's method and experimental results obtained from a Heleshaw apparatus. Comparison with results obtained from electrical analogy equipment are presented in Figure 5. The discharge equations obtained by the authors' and Muskat's methods are the same for both two-and three-dimensional cases [*vide* Equations (4), (10), (15) and (16)].

They are the same since they depend only on the inlet and exit potentials and not on the distribution of it inside the flow region. In fact, in the superposition principle used by Muskat, he has taken average potentials at the inlet and exit ends and has obtained the same discharge equations as the authors' by considering a composite gravity flow system as a strictly confined radial flow.

Regarding pressure distribution on the base, Figures 3, 4 and 5 show that the differences between the two methods occur only in the unconfined region and that the pressure distribution obtained by the superposition principle is always smaller than that obtained by authors' method. This may perhaps be due to the assumption in the superposition method that a part of the flow takes place throughout the sand thickness while the rest moves only in the unconfined region whereas in actuality all the flow takes place only in the unconfined region. Because of the larger area of flow assumed in the superposition method, a smaller gradient results and hence a flatter pressure distribution.

In conclusion, it may be stated that the discharge through a composite gravity flow system can be calculated either by the authors' method or by the method of superposition suggested by Muskat. Whereas the plot for the three-dimensional case (Figure 3) does not bring out clearly as to which method is more suitable for predicting the pressure distribution, the plots for the two-dimensional case (Figures 4 and 5) show that the authors' method gives values more closer to the experimental points than that of Muskat. In many problems of composite gravity flow systems, determination of the breakaway point from confined to unconfined flow is necessary for design purposes. Such a breakaway point can be determined only by the authors' method.

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