Experimental and Theoretical Investigations of the Influence of Rough Rigid Layer on Settlement of Pile Foundations

by

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Introduction

IN the recent years one of the most important development in foundation engineering is in the field of driven and bored piles which are extensively used for transmitting heavy structural loads to strata of high bearing capacity. As the permissible load is linked with the deformation of foundations, it is necessary to estimate properly the settlement of such piles. Various factors such as the soil type, interaction between the soil and pile, nature of load transfer around the pile shaft govern the settlements of friction pile. There are very frequently encountered situations when the piles are located over the strata where the load is transferred through friction but rigid layer is found at some distance below the pile tips. Due to driving limitations it is also not always possible to rest all piles on rigid strata. In such situations, design engineers are interested in understanding the exact behaviour of the piles and pile groups. Therefore, in this paper theoretical expressions for displacements of single piles are developed, for distribution of load around the pile shaft in uniform and triangular manner in presence of rigid layer at any depth below the pile tip, by using (*i*) Steinbrenner's approximation and (*ii*) Mirror Image method.

Displacement, due to force acting in the interior of semi-infinite soil mass can be obtained by Mindlin's solution (1936). E D'Appolonia & J.P. Romualdi (1963) determined the load transfer in H piles and stated that within elastic range, deformation and load transfer both are the functions of depth and movement of the pile. M.A. Biot (1935) reported that the pressure distribution due to a concentrated load is given by the well-known Boussinesq's solution (1959) or Mindlin's solution (1936) depending on the load position. The load settlement characteristics of a single pile subjected to static vertical load is determined by Keshavan Nair (1967). A theoretical expression for centrally loaded pile groups in cohesionless soil is derived by Meyerhoff (1959). H.G. Poulos (1967) developed in analysis of the settlement of pile groups. From equilibrium analysis of axial

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loading, V.G. Berzantzev (1961) has given a formula for determining load bearing capacity of a pile group. H.G. Poulos and N.S. Mattes (1969) analysed the behaviour of single axially loaded end bearing pile in an ideal elastic soil.

The problem of analysing the behaviour of piles and pile groups needs careful attention due to its wide practical applications. The authors have therefore, conducted tests on model single pile and group of piles in granular soil in semi-infinite and finite medium (1970) at Indian Institute of Technology, Bombay. In this paper the results of experimental investigations on model piles and pile groups are presented and discussed Theoretical analysis for the displacement of single pile in semi-infinite and finite medium is also developed.

Laboratory Investigations

Tests were conducted on models in a special experimental set-up (Figure 1) and the important features of the same are given below.

EXPERIMENTAL SET-UP

Tests were carried out in mild steel tank of size 1·25 m × 1·25 m × 1·25 m. The load is applied through a lever arrangement.

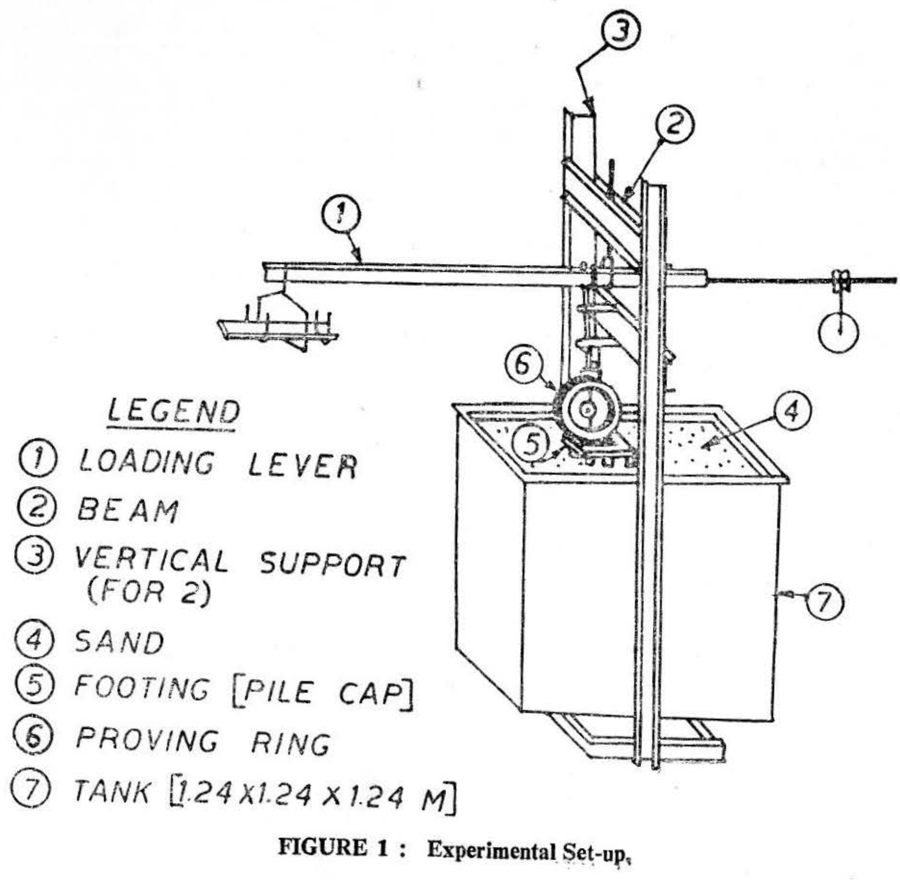

FIGURE 1 : Experimental Set-up.

PILE MODELS

Models were prepared with aluminium pile with following components :

(*a*) *Pile Shaft*

Model piles were made out of 25·4 mm diameter aluminium pile with 3·2 mm. thickness. It was threaded externally at both the ends for a length of 9·6 mm. The length of the shaft was varied from 22·86 cm to 53·34 cm in steps of 15·2 cm.

(*b*) *Pile Cap*

Pile caps were made of M.S. Plate of 12·7 mm thickness so as to be sufficiently rigid. Pile caps were threaded on one side internally at the required spacing to a depth of about 9·6 mm, so that pile could be easily fixed. On the other side of the plate a groove was made to accommodate the proving ring ball so that the load could be applied at the centre of the pile group without any eccentricity. Pile cap sizes were varied from 12·7 cm $\times$ 12·7 cm, 15·2 cm $\times$ 15·2 cm to 20·4 cm $\times$ 20·4 cm for $(2 \times 2 = 4)$ pile groups with different spacings of piles.

(*c*) *Conical Tip*

Conical tips of the piles were made of brass rod threaded internally to a depth of 25·4 mm so that it could be fixed easily on any pile.

PROPERTIES OF SOIL

A locally available sand called 'Mumbra Sand' passing I.S.S. No. 240 (B.S.S. No. 7) sieve was used. Properties of which are given in Table I.

COMPACTION

For compaction, the pile group was held in required position by a pile group holder. The pile group was so held in position that the loading was central. Sand in tank was filled in layers of 8·89 cm thickness, so that the thickness of the compacted layer became 7·62 cm. The compaction was done with four blows of modified proctor hammer on an area 15·2 cm $\times$ 15·2 cm in two passes and two blows per pass. This gave total compactive energy of about $4{\cdot}01 \times 10^3$ kg/m^3. For all the experimental investigations pile cap was kept 7·62 cm above the soil, *i.e.*, 'free-standing' case.

Firstly compaction of soil was completed up to the level of pile tips so as to provide the required thickness of soil mass above the rigid layer. Afterwards the remaining depth of soil corresponding to the embedment length was compacted.

TEST PROCEDURE

After compaction, a proving ring was placed in position on the rigid pile cap under the loading lever. Dial gauges (range 50·0 mm least count 0·01 mm) were fixed on the rigid pile cap, so as to note the downward settlement of the group. Loading was continued till the settlement of the group at the ground level reached 2 cm or more.

Details of tests performed are given in Table II.

Presentation and Discussion of Test Results

Tests on model piles with different shaft lengths were conducted to study the load settlement behaviour of single pile and group of piles in presence of rough rigid layer. Load settlement curves for different lengths of embedment of single piles (free standing) for different depths of rigid layer were drawn. Typical results for $L/d=21$ are shown in Figure 2 and are tabulated for deformations 2·54 mm and 5·0 mm corresponding to 10 percent and 20 percent of pile diameter respectively in Table III.

Load settlement curves for $(2\times2=4)$ group of 'free standing' piles for various lengths of embedment, spacings and different depths of rigid layer were plotted. Typical results for $L/d=15$ and for spacings $2{\cdot}5d$ and $4d$ are shown in Figures 3 and 4. Results are tabulated in Table IV for 2·54 mm (10 percent of pile diameter) deformation.

The bearing capacity of group of piles in semi-infinite and finite cases are compared with that of single pile in semi-infinite medium and are expressed in the form of non-dimensional term "Group Efficiency" which is defined as :

$$\text{Group Efficiency (\%)} = \frac{\text{Group load at particular deformation} \times 100}{\text{Single pile load for the same deformation} \times \text{Number of piles in a group.}}$$

Typical plot for group efficiency *versus* L/d ratio at 2·54 mm deformation for 'free standing' square group spaced at $3d$ is shown in Figure 5 and results are tabulated in Table V.

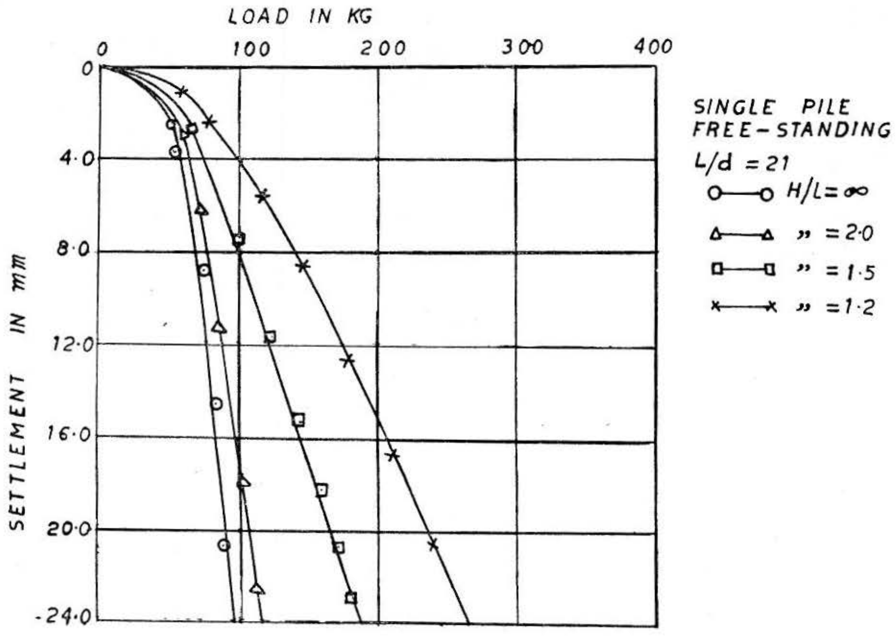

FIGURE 2 : Load versus settlement curves.

TABLE I

Properties of soil.

Sl. No.	Description	Value
1.	Average density	1·90 gm/cc $\pm$ 2%
2.	Moisture content	Air dry
3.	Relative density	0·945
4.	Angle of Internal friction (ϕ)	$40° \pm 1°$
5.	Cohesion (c)	0·035 kg/cm^2 $\pm$ 0·005
6.	Textural composition	

Particulars	Size	Percentage	
Gravel	> 2 mm	9·5	
Sand: Course	2·0—0·6 mm	53·5	90·4
Sand: Medium	0·6—0·2 mm	32·5	
Sand: Fine	0 2—0·06 mm	4·4	
Silt & Clay	< 0·06 mm	0·1	

Effective grain size (D_{10})	0·28 mm
Uniformity coefficient (D_{60}/D_{10})	3·27

The soil is classified as 'sand' according to I.S. Classification.

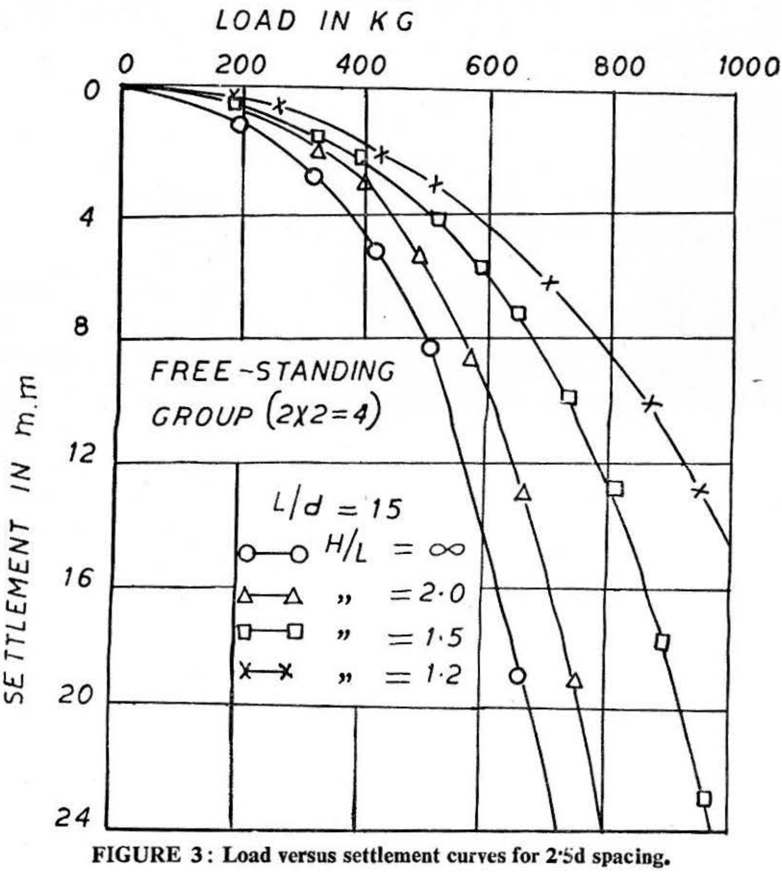

FIGURE 3: Load versus settlement curves for 2·5d spacing.

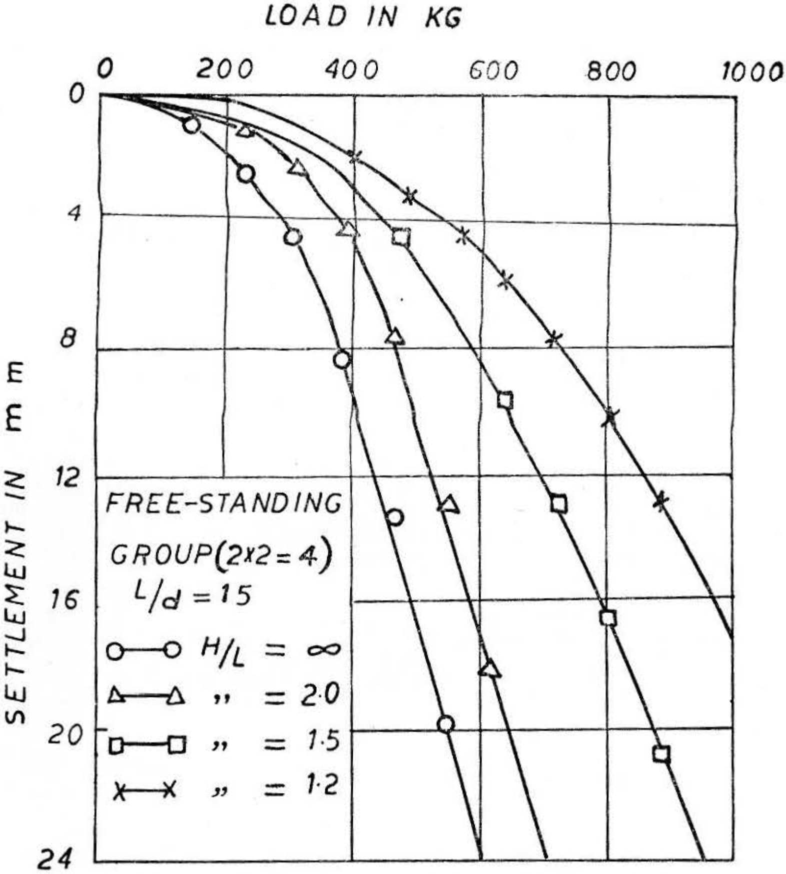

FIGURE 4 : Load versus settlement curves for 4d spacing.

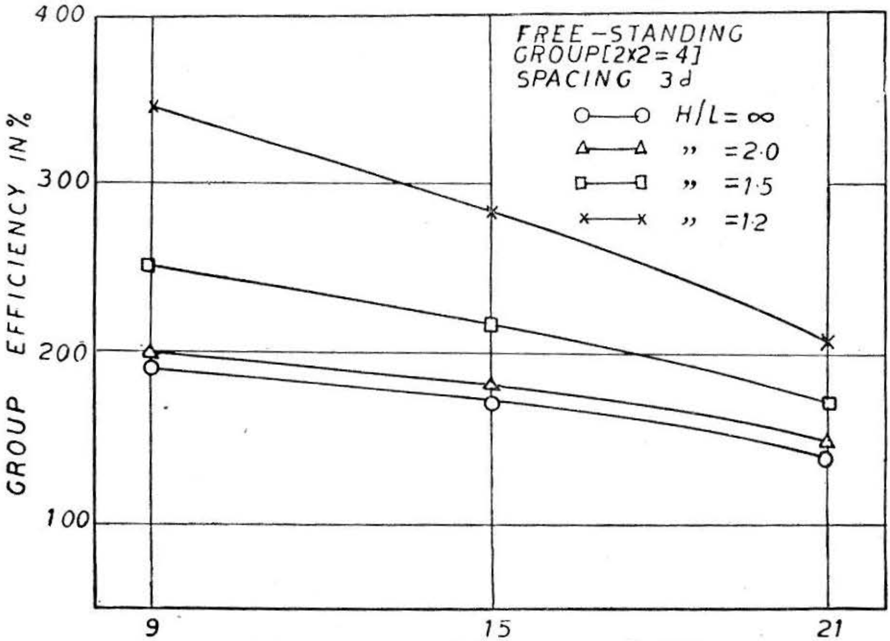

FIGURE 5 : Group efficiency versus (L/d) ratio for 2·54 mm deformation

Loads for single pile and group of piles were obtained from laboratory tests for deformation of 2·54 mm and 5·0 mm (10 and 20 percent of the pile diameter respectively). From these values group efficiencies for various conditions were worked out.

TABLE II

Details of test programme.

Discription	Spacing (s) in terms of diameter of the pile	Depth of rigid layer (H)	Length of embedment (L)
(1) Tests on single pile	—	$1{\cdot}2L$, $1{\cdot}5L$, $2{\cdot}0L$, ∞	22·86 cm 38·10 cm 53·34 cm
(2) Tests on 'free-standing' Square group ($2\times2=4$) of piles	$2{\cdot}5d$, $3{\cdot}0d$, $4{\cdot}0d$	$1{\cdot}2L$, $1{\cdot}5L$ $2{\cdot}0L$, ∞	22·86 cm 38·10 cm 53·34 cm

Note :—Reported results are the average values of at least 2 tests.

TABLE III

Load carrying capacity in kg for single 'free-standing' pile at a deformation of 2·54 mm and 5·0 mm in presence of rigid layer.

L/d	Load carrying capacity at various depths of rigid layer (H) $H/L=\infty$	$H/L=2{\cdot}0$	$H/L=1{\cdot}5$	$H/L=1{\cdot}2$
At deformation of 2·54 mm (10 percent of pile diameter)				
9	28·0	47·0	68·0	85·0
15	38·5	50·5	75·0	90·0
21	50·0	53·3	79·1	82·0
At deformation of 5·0 mm (20 percent of pile diameter)				
9	34 0	53·5	83·0	101·5
15	48·5	58·5	93·0	107·5
21	63 0	69·3	101·0	111·0

TABLE IV

Load carrying capacity in kg for group of 'free-standing' piles (2×2=4) at a deformation of 2·54 mm in presence of rigid layer.

L/d	Load carrying capacity of various depths of rigid layer (H)			
	$H/L=\infty$	$H/L=2{\cdot}0$	$H/L=1{\cdot}5$	$H/L=1{\cdot}2$
At 2·5d spacing				
9	230·0	250·0	340·0	420·0
15	307·0	350·0	400·0	450·0
21	335·0	352·5	407·5	487·5
At 3·0d spacing				
9	212·5	227·5	290·0	385·0
15	267·5	288·5	332·5	435·0
21	280·0	290·0	340·0	412·5
At 4·0d spacing				
9	205·0	235·5	305·0	395·0
15	230·0	312·5	352·5	440·0
21	280·0	305·0	355·0	445·0

EFFECT OF LENGTH OF EMBEDMENT

Referring to Figures 2 to 5 it is observed from experimental results that load carrying capacity increases with the increase in the length of embedment of the pile and shorter piles are more efficient than longer ones. The load settlement data indicate that strength mobilization continues even after deformations equal to 10 to 15 percent of pile diameter. In case of pure friction piles this much of settlement would have been adequate to develop full strength around the shaft of the pile. It may be therefore inferred that a part of the load gets transferred to the pile tips.

EFFECT OF SPACING

Referring to Figure 5 and Table V it is observed that for 'free-standing' group, group efficiency goes on decreasing with increase in spacing for semi-infinite medium. It is thus seen that the presence of adjacent piles helps in improving the development of strength in granular soil. For finite medium however, a critical spacing of 3·0d is observed where the efficiency is minimum, apparently mode of group failure changes at this spacing. Thus the presence of rigid layer not only affects the settlements of pile groups but it also considerably influences the pattern of load transfer and mechanism of failure at different spacing of piles.

EFFECT OF RIGID LAYER

Group efficiency of piles in presence of rough rigid layer for various spacings and length of embedment increases considerably. Group efficiency further increases as the rigid layer approaches the tip of the pile. Group efficiency of more than 206 percent is observed in all the cases at a deformation of 2·54 mm and at depth of rigid layer to length of embedment of pile ratio (H/L)$=1{\cdot}2$. It is also observed that at any depth of rigid layer shorter piles give higher efficiency. In general from above

TABLE V

Group efficiency for 'free-standing' pile groups ($2\times 2=4$) at a deformation 2·54 mm in presence of rigid layer.

L/d	Ratio of various depths of rigid layer to length of the pile (H/L)			
	$H/L=\infty$	$H/L=2{\cdot}0$	$H/L=1{\cdot}5$	$H/L=1{\cdot}2$
	At 2·5d spacing			
9	2·05	2·23	3·04	3·75
15	1·99	2·27	2·60	2·92
21	1·67	1·76	2·06	2·43
	At 3·0d spacing			
9	1·89	2·03	2·59	3·43
15	1·73	1·83	2·16	2·83
21	1·40	1·45	1·70	2·06
	At 4·0 d. spacing			
9	1·83	2·07	2·72	3·53
15	1·49	2·02	2·28	2·86
21	1·40	1·52	1·77	2·22

investigations it may be inferred that the load transfer changes due to presence of rigid layer. It may be further mentioned that the rigid layer will also help in transferring larger loads at the tip of the pile, thereby causing redistribution of load between the shaft and tip.

THEORETICAL ANALYSIS

Various factors such as soil type, interaction between the soil and pile, nature of load transfer around the pile shaft govern settlement of pile foundations. In the case of friction piles, the load distribution around the pile shaft can be considered to be trapezoidal which can be conveniently grouped into two parts :

(1) Uniform distribution around the pile shaft, and

(2) Triangular distribution around the pile shaft.

By suitably superimposing these two, trapezoidal distribution can be arrived at.

Theoretical analysis is developed by suitably extending Mindlin's solution for the problem under consideration. The axial displacement due to a point force P on the pile in z direction can be written by Mindlin's solution (Figure 6) as :

$$w=\frac{P}{8\pi(1-\mu)\,2G}\left[\frac{(3-4\mu)}{R_1}+\frac{(z-c)^2}{R_1^{\,3}}+\frac{8(1-\mu)^2-(3-4\mu)}{R_2}\right.$$

$$\left.+\frac{(3-4\mu)\,(z+c)^2-2cz}{R_2^{\,3}}+\frac{6(cz)\,(z+c)^2}{R_2^{\,5}}\right] \quad ...(1)$$

Axial displacement can be exactly obtained by considering ring load acting around the shaft of the pile but such a solution is complicated. Hence in the present analysis the axial displacement in z-directions obtained is by making some simplified assumptions and by transferring the peripheral ring load as uniformly distributed load over the cross-section of the pile. The pile is replaced by an imaginary column of soil and is divided into 'n' equal elemental areas of height $h=\frac{1}{n}$. The interaction shear stresses on elemental area are actually acting around the peripheral area of the pile but as the diameter of pile is small compared to the length of the pile the resultant of interaction stresses is assumed to act at the mid cross-sectional area of the section. The solution for such a case of loading can be obtained by integrating Mindlin's equation for point load in the interior of semi-infinite, elastic, isotropic, homogeneous soil mass over the cross-sectional area of the pile which is divided into some convenient number of segments. After completion of necessary integrations of *Equation (1) above*, displacement at centre of i-th section due to load at j-th section (Figure 7) can be written as :

$$w_{ij}=\frac{2p_j}{16\pi\,(1-\mu)\,G}\left[\,I_{1ij}+I_{2ij}+I_{3ij}+I_{4ij}+I_{5ij}\,\right] \qquad \text{...(2)}$$

where,

p_j = the uniformly distributed load over j-th section.

L = length of the pile

h = length of section

n = number of sections

μ = Poisson's ratio of the soil

G = shear modulus of the soil

r = radius of the pile

$k_1=(z-e)$, $k_2=(z+c)$, $R_1^2=(k_1^2+g^2)$, $R_2^2=(k_2^2+g^2)$

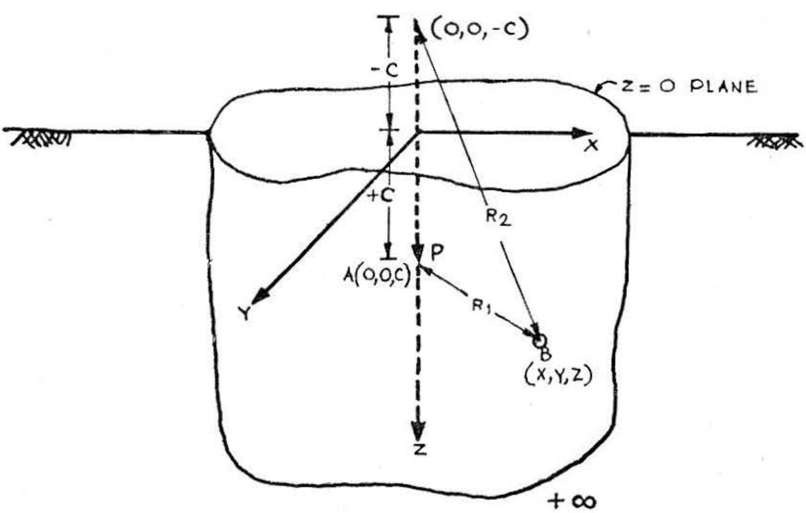

FIGURE 6 : Point force acting in semi-infinite medium.

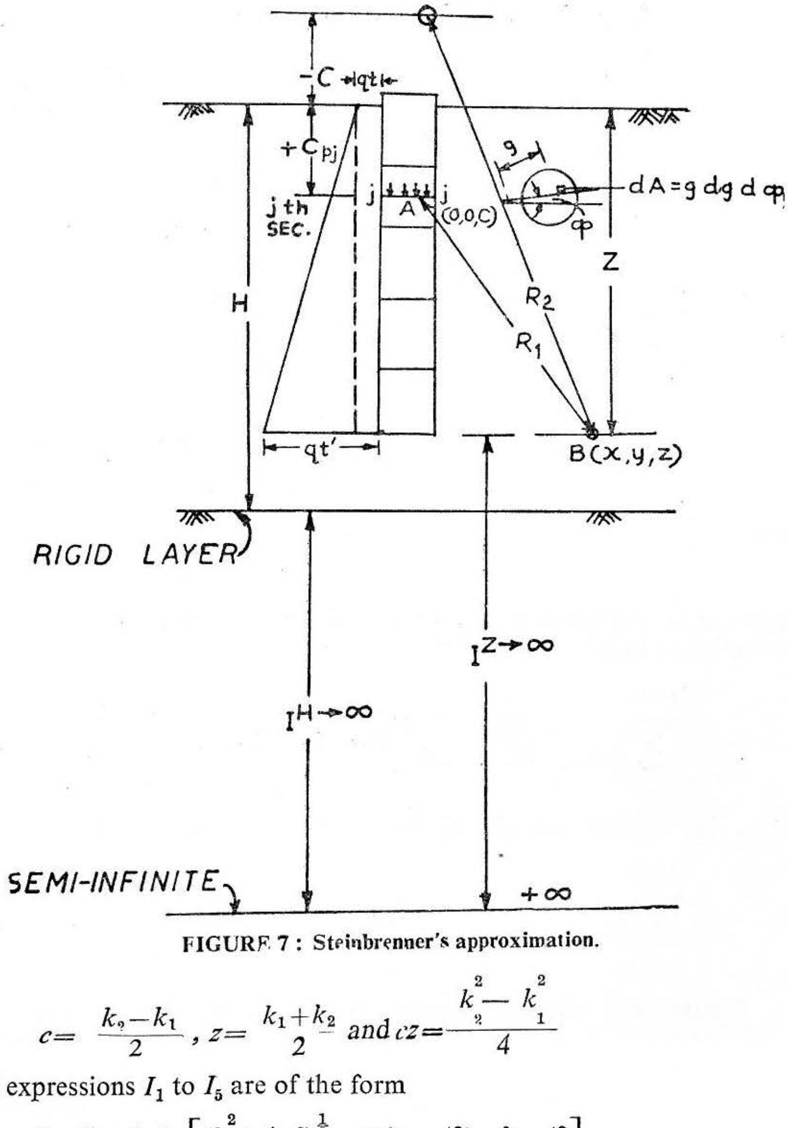

FIGURE 7 : Steinbrenner's approximation.

$$c = \frac{k_2 - k_1}{2}, \; z = \frac{k_1 + k_2}{2} \text{ and } cz = \frac{k_2^2 - k_1^2}{4}$$

and expressions I_1 to I_5 are of the form

$$I_1 = (3-4\,\mu)\left[(k_1^2 + 4\,r^2)^{\frac{1}{2}} \;\; \in(c_1, \pi/2) - k_1\,\pi/2\right] \qquad \text{...(3)}$$

where, $c_1 = \left[\dfrac{(4\,r^2)}{(k_1^2 + 4r^2)}\right]^{\frac{1}{2}}$ and $\in(c_1, \pi/_2)$ is the complete elliptic integral of the second kind.

$$I_2 = \left[8(1-\mu)^2 - (3-4\mu)\right]\left[(k_2^2 + 4\,r^2)^{\frac{1}{2}} \in(c_2, \pi/2) - k_2\,\pi/2\right] \qquad \text{...(4)}$$

where, $c_2 = \left[\dfrac{(4r^2)}{(k_2^2 + 4r^2)}\right]^{\frac{1}{2}}$

$$I_3=-(k_1^2)\left[(k_1^2+4r^2)^{-\frac{1}{2}}K(c_1,\pi/2)-\pi/2k_1\right] \qquad ...(5)$$

where, $K(c_1, \pi/2)$ is the complete elliptic integral of first kind.

$$I_4=\frac{\left[(k_2^2-k_1^2)-(3-4\mu)(k_2^2)\right]}{2}$$

$$\left[(k_2^2+4r^2)^{-\frac{1}{2}}K(c_2,\pi/2)-\pi/2K_2\right] \qquad ...(6)$$

and $$I_5=\left[\left[-{}_1^2)\ \frac{(k_2^2-k_1^2)}{2}\right]\right.$$

$$\left.\left[(k_2^2+4r^2)^{-\frac{3}{2}}\pi(\pi/2,c_3,c_2)-\pi/2k_2^3\right]\right] \qquad ...(7)$$

where, $c_3=\left(\dfrac{4r^2}{k_2^2+4r^2}\right)$

Finally total displacement w_{ut} of the tip of the pile due to uniform distribution of load around the pile shaft is given by

$$w_{ut}=\sum_{j=1}^{j=n}\frac{2h}{(\pi r^2)L\ 16\pi(1-\mu)G}\left[I_{1tj}+I_{2tj}+I_{3tj}+I_{4tj}+I_{5tj}\right] \qquad ...(8)$$

and w_{tt} due to triangular of load around the pile shaft is given by

$$w_{tt}=\sum_{j=1}^{j=n}\frac{4c}{(\pi r^2)nL\ 16\pi(1-\mu)G}\left[I_{jt1t}+I_{2tjt}+I_{3tjt}+I_{4tjt}+I_{5tjt}\right] \qquad ...(9)$$

Theoretically effect of presence of rough rigid layer is determined by the following two methods for uniform and triangular distribution of load around the single pile :

(*i*) Steinbrenner's approximation, and (*ii*) Mirror Image method.

Steinbrenner's Approximation

The accuracy of this approximation has been discussed by Davis and Taylor (1968) and Poulos (1967). It has been reported that Steinbrenner's approximation gives values for the displacement of a uniformly loaded area on the surface of finite layer, which are generally within 10 percent of the correct values.

From the method by Steinbrenner, the displacement influence coefficient for any point B in a layer of depth H is given by (Figure 7) as reported by S.D. Sheth (1969)

$$I^{z\longrightarrow H}={}_I{}^{z\longrightarrow\infty}-{}_I^{-}{}^{H\longrightarrow\infty} \qquad ...(10)$$

where, $I^{z \longrightarrow \infty}$ is the influence coefficient for semi-infinite mass.

$I^{H \longrightarrow \infty}$ is the influence coefficient for a point within a semi-infinite mass at a distance H vertically below the point B.

Mirror Image Method

The solution of the problem of settlements of pile where rigid surface exists at depth H, which is assumed to be the boundary condition of zero vertical displacement can be analytically approximated by the addition of "mirror image load". The correction to be applied to the displacement at the point B (tip) due to areal load of intensity pj acting over j-th cross-section of the pile in the presence of a rigid layer is given by extended

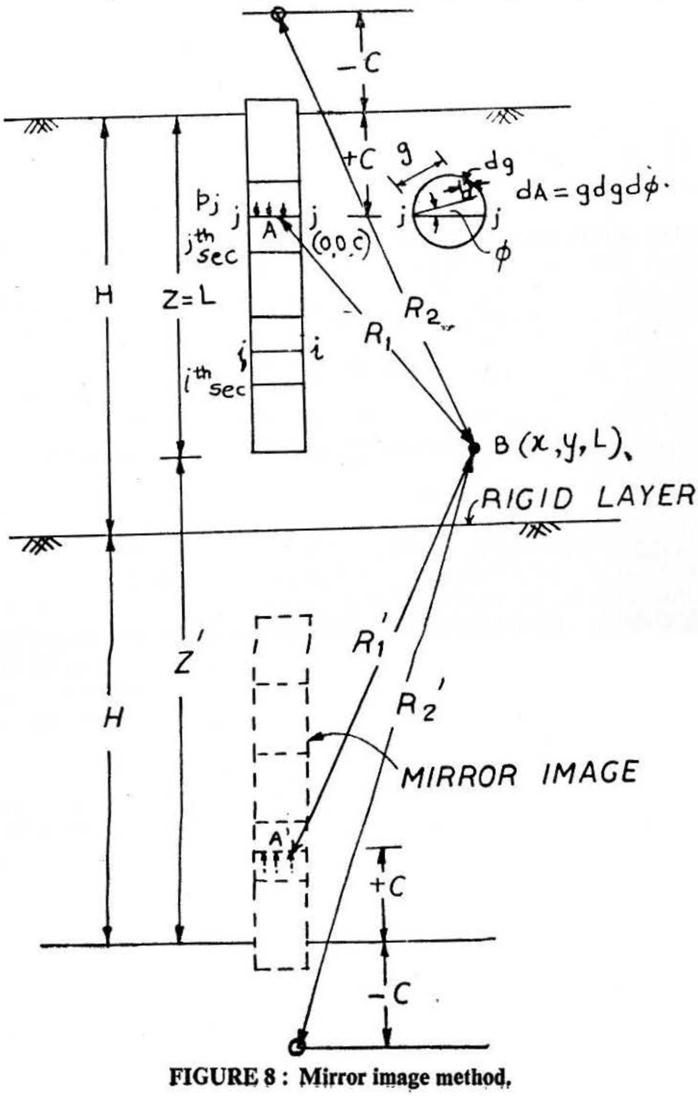

FIGURE 8 : Mirror image method.

Mindlin's solution for areal load (Figure 8) for uniform distribution of load around the pile shaft :

$$w_{tj}=-\frac{2p_j}{16\pi(1-\mu)G}\left[I_{1tj}'+I_{2tj}'+I_{3tj}'+I_{4tj}'+I_{5tj}'\right] \quad \ldots(11)$$

where, $k_1=(2H-L-c)$, $k_2=(2H-L+c)$, $z=(2H-L)$

$$c=\frac{k_2-k_1}{2},\ cz=\left(\frac{k_2-k_1}{2}\right)(2H-L),\ R_1'^2=\left(k_1^2+g^2\right), R_2'^2=\left(k_2^2+g^2\right)$$

The expressions I_1' to I_5' are *of the similar form as defined earlier* (I_1 to I_5). Similarly correction for triangular distribution of load around the pile shaft can be evaluated by substituting the proper value of c.

Thus with the help of the above Equations (8) and (9) it is possible to obtain the settlement at the tip of the pile for frictional load transferred around the pile shaft, in uniform and triangular manner respectively. The settlement values can be corrected for the influence of rigid layer by the two methods as explained above.

Presentation and Discussion of Theoretical Results

The linear elastic behaviour between load and displacement [from Equations (8, 9)] has been expressed as

$$W=K.P_s$$

where,

W=Displacement at the tip of the pile in cm.

K=Slope of the load displacement equation in cm/kg.

P_s=Load resisted by friction in kg.

Theoretical results for single piles are obtained with the help of CDC-3600 computer at Tata Institute of Fundamental Research, Bombay for uniform and triangular distribution of load around the pile by both the methods, *i.e.*, "Steinbrenner's approximation" and "Mirror Image method". Results are obtained for various lengths of embedment and physical soil properties E and μ. Typical results for $E=500$ kg/cm² and $\mu=0{\cdot}3$ are shown in Figures 9 and 10 for uniform and triangular distribution and are tabulated in Tables VI and VII.

It can be seen from Table VI that the slope (K) values for $H/L=1{\cdot}2$ and for L/d ratio equal to 15, $E=200$ kg/cm² and $\mu=0{\cdot}3$, and $0{\cdot}5$, are $0{\cdot}473\times10^{-4}$ and $0{\cdot}528\times10^{-4}$ respectively by Steinbrenner's approximation for uniform distribution of friction around the pile. Similarly $0{\cdot}641\times10^{-4}$ and $0{\cdot}716\times10^{-4}$ are the slope (K) values for above mentioned L/d ratio, E and μ values by "Mirror Image method".

It can be seen from the exprimental data and the theoretical results that the trends of settlements obtained in both the cases are similar. However, it is desirable to obtain data from field tests and compare the same with values obtained theoretically.

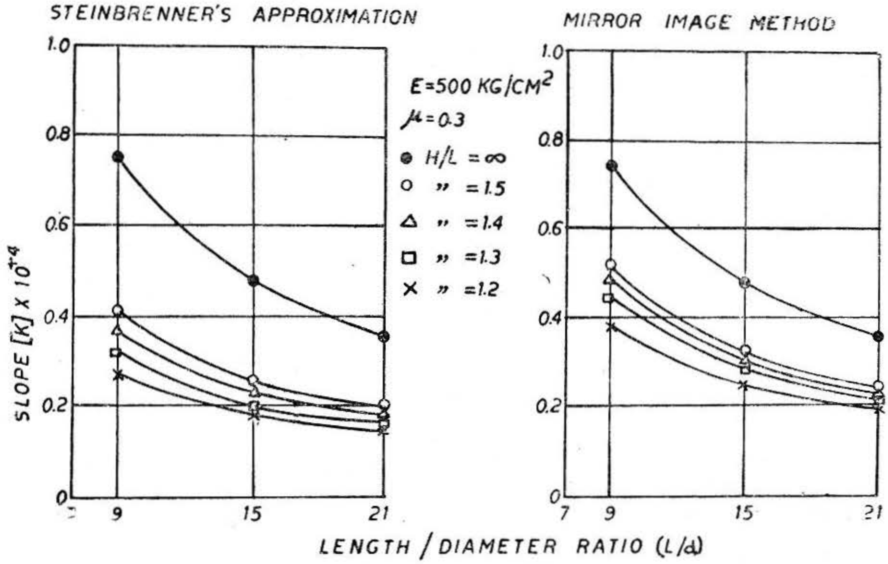

FIGURE 9 : Slope versus length/diameter ratio for uniform distribution.

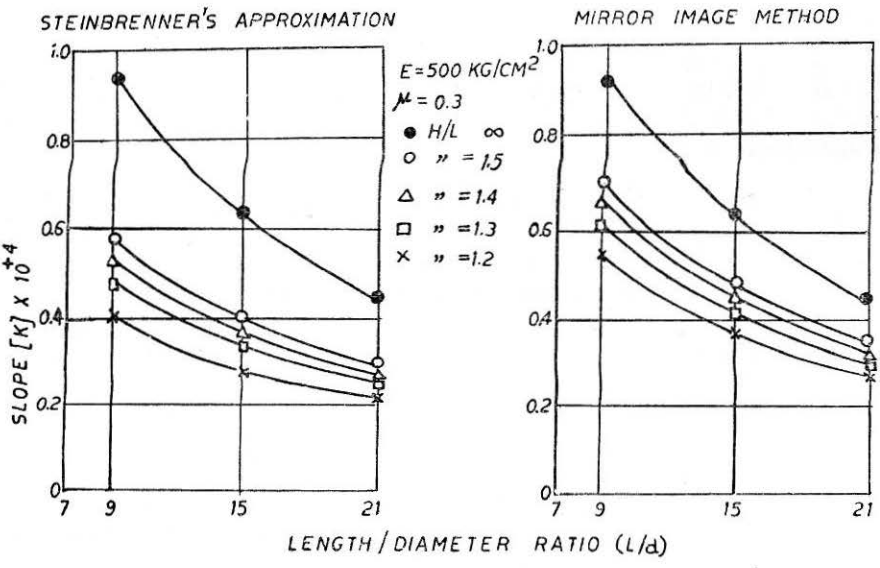

FIGURE 10 : Slope versus length/diameter ratio for triangular distribution.

TABLE VI

Effect of various depths of rigid layer (H) on slope (K) $\times 10^{+4}$ for uniform distribution of load around the pile shaft.

L/d	Ratio of depth of rigid layer to length of the pile (H/L)				
	1·2	1·3	1·4	1·5	∞
Steinbrenner's approximation					
E=200 kg/cm²					μ=0·3
9	0·675	0·833	0·948	1·036	1·902
15	0·473	0·571	0·641	0·695	1·216
21	0·357	0·427	0·478	0·517	0·889
E=200 kg/cm²					μ=0·5
9	0·734	0·910	1·037	1·135	2·052
15	0·528	0·648	0·716	0·776	1.327
21	0·402	0·482	0·538	0·581	0 975
Mirror Image Method					
E=200 kg/cm²					μ=0·3
9	0·998	1·108	1·217	1·298	1·902
15	0·641	0·738	0·804	0·853	1·216
21	0·478	0·547	0·595	0 629	0·889
E=200 kg/cm²					μ=0·5
9	1·037	1·214	1·334	1·422	2·052
15	0·716	0·823	0·896	0·949	1·327
21	0·538	0·615	0·667	0·705	0·975

Conclusions

The laboratory investigations and theoretical analyses of axially loaded model friction piles and pile groups in granular soil in presence of rigid layer lead to following conclusions.

FROM EXPERIMENTAL INVESTIGATIONS

(1) For a given value of deformation it is found that the shorter piles are able to mobilize more strength and further, group efficiency for the shorter piles is more as compared to longer once in presence of rigid layer.

(2) Load carrying capacity of single pile and pile groups increases as rigid layer approaches the tip of the pile. From this, it is inferred that the mechanics of load transfer and failure changes in presence of rigid layer. Thus in the field, even if the piles are not driven up to rigid strata, the load transfer through these would be improved in granular soil.

(3) It is observed that for 'free-standing' group ($2\times2=4$) the group efficiency goes on decreasing with increase in spacing for semi-infinite medium but in presence of rigid layer a critical spacing of $3d$ is observed where the mode of group failure apparently changes.

TABLE VII

Effect of various depths of rigid layer (H) on slope $(K)\times10^{+4}$ for triangular distribution of load around the pile shaft.

L/d	Ratio of depth of rigid layer to length of the pile (H/L) 1·2	1·3	1·4	1·5	∞
Steinbrenner's approximation					
$E=200$ kg/cm²					$\mu=0·3$
9	1·001	1·212	1·358	1·666	2·296
15	0·720	0·852	0·941	1·008	1·568
21	0·548	0·644	0·708	0·756	1·156
$E=200$ kg/cm²					$\mu=0·5$
9	1·088	1·325	1·489	1·611	2·613
15	0·806	0·956	1·057	1·132	1·736
21	0·621	0·729	0·802	0·856	1·288
Mirror Image Method					
$E=200$ kg/cm²					$\mu=0·3$
9	1·258	1·551	1·677	1·767	2·396
15	0·941	1·059	1·135	1·190	1·568
21	0·708	0·790	0·847	0·886	1·156
$E=200$ kg/cm²					$\mu=0·5$
9	1·489	1·706	1·846	1·947	2·613
15	1·057	1.190	1·265	1·335	1·736
21	0·792	0·897	0·968	1·002	1·288

FROM THEORETICAL INVESTIGATIONS

(1) Displacement of the pile reduces considerably as rigid layer approaches the pile tip and for a particular value of deformation, although the longer piles can carry more load than shorter piles, the efficiency of load transfer for shorter piles is more.

(2) The displacement values are considerably affected by modulus of elasticity (E) of the soil. The influence of Poisson's ratio (μ) is, however, not significant.

(3) Displacements obtained by 'Mirror Image method' are high as compared to 'Steinbrenner's approximation' for particular values of E, μ, length of embedment of pile and depth of rigid layer.

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Notations

L	= Length of embedment of the pile.
d	= Diameter of the pile.
H	= Depth of rigid layer from ground surface.
w	= Axial displacement in z-direction.
z	= Depth of the point in z-direction from ground surface.
P	= Point force.
π	= 3·14159265.
n	= Number of sections.
h	= Length of the section.
μ	= Poisson's ratio of the soil.
G	= Shear modulus of the soil.
r	= Radius of the pile.
p_j	= The uniformly distributed load over j-th section.
$E(c_1, \pi/2)$	= Complete elliptic integral of second kind.
$K(c_1, \pi/2)$	= Complete elliptic integral of first kind.
$\pi(\pi/2, c_3, c_2)$	= Complete elliptic integral of third kind.
E	= Modulus of elasticity of the soil.
K	= Slope of the load displacement equation in cm/kg.
P_s	= Load resisted by friction in kg.
s	= Spacing of the pile in terms of diameter.
c	= Cohesion.
ϕ	= Angle of internal friction.

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