Design Procedure for Sand Drains for Time-Dependent Loading

by

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Introduction

THE purpose of a sand drain is to reduce the length of drainage path of the excess water several times as it is squeezed out of a soil layer during consolidation. Thus with the provision of drain wells, the rate of consolidation can be accelerated. The consolidation of a clay layer in the presence of drain wells for time-independent loading can be predicted combining the theories of Terzaghi (1943) and Barron (1948). Barron has recommended the use of the 'equal strain' solution since the results obtained by mathematical solutions between the 'free vertical strain' and 'equal vertical strain' cases are negligibly small and equal strain solution is simpler and consumes less time. Richart (1957) has also shown that for $n > 5{\cdot}0$, both the solutions were in close agreement. Subsequently Leonards (1962) and Younger (1968) have developed design procedures for sand drains. Barron, Richart and Younger have recommended a reduction factor in the well diameter to account for smearing effects. But none of these investigations have been extended for time-dependent loading.

In conventional laboratory test, the increment of load is applied instantaneously. In addition to this, the laboratory test is based on a constant load acting on the sample from the beginning. But in practice, it is seldom the case. During the erection of a structure be it a building, an earth dam or a surcharge on drain wells, for rapid consolidation, often a long time is taken in the application of load. To obtain reasonable predictions of foundation settlements of the above mentioned structures, during and after construction, the effect of loading period must be taken into account. Normally even about 90 percent of the primary consolidation can take place during the construction period itself. This percentage of consolidation depends on the period of construction, the longer is the period of construction, the more will be the percentage of consolidation. In this investigation the total consolidation is obtained by combining the solutions presented by Lumb (1963) and Schiffman (1959) for vertical and radial drainages for time-dependent loading for the equal vertical strain condition, taking into consideration the variation of coefficient of consolidation due to vertical and radial flows.

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This paper was received on 27 January 1971. It is open for discussion up to July 1972.

Review of Fundamental Theory

A theoretical solution for a gradually applied load increasing linearly with time up to the completion of construction for vertical drainage, was persented by Lumb (1963). Lumb's solution for one-dimensional consideration is

$$U_v = \frac{1}{T_{vo}}\left\{T_v - \tfrac{1}{3} + \left(\frac{32}{\pi^4}\right) e^{-\frac{\pi^2 T_v}{4}}\right\} T_v \leqslant T_{vo} \qquad \ldots(1a)$$

$$U_v = 1 - \left(\frac{1}{T_{vo}}\right)\left(\frac{32}{\pi^4}\right) e^{-\frac{\pi^2 T_v}{4}} \left\{ e^{\frac{\pi^2 T_{vo}}{4}} - 1 \right\} \ldots T_v > T_{vo} \qquad \ldots(1b)$$

$$T_v = \frac{C_{vv} \cdot t}{H^2} \qquad \ldots(1c)$$

$$T_{vo} = \frac{C_{vv} \cdot t_0}{H^2} \qquad \ldots(1d)$$

in which

U_v = degree of consolidation for vertical drainage and vertical consolidation,

T_v = time factor for vertical drainage and vertical consolidation,

T_{v_0} = time factor at the end of construction for vertical drainage and vertical consolidation,

t_0 = time at the end of construction,

H = vertical drainage path distance, and

C_{vv} = coefficient of consolidation for vertical drainage and vertical consolidation.

At the end of construction period Equations (1*a*) and (1*b*) converge to

$$U_v = 1 - \frac{1}{3T_{vo}} + \frac{32}{\pi^4 T_{vo}} \cdot e^{-\frac{\pi^2 T_{v_o}}{4}} \qquad \ldots(2)$$

Solutions for degree of consolidation for radial flow and time-dependent loading applied to equal-strain case was developed by Schiffman (1959). The permeability of soil was kept constant during the process of consolidation by Schiffman, to achieve a closed form solution.

The excess pore pressure both during and after construction is given by

$$\frac{\bar{u}}{\bar{u}_0} = \frac{1}{8T_{Ro}}\left[F(n,s) + \frac{G(n,s)}{x}\right]\left[1 - e^{-\frac{8T_R}{F(n,s)}}\right] \ldots T_R \leqslant T_{Ro} \qquad \ldots(3a)$$

$$\frac{\bar{u}}{\bar{u}_0} = \frac{1}{8T_{Ro}}\left[F(n, s) + \frac{G(n, s)}{x}\right]\left[1 - e^{-\frac{8T_{Ro}}{F(n, s)}}\right] e^{-\frac{8(T_R - T_{Ro})}{F(n, s)}} \quad ..T_R \geqslant T_{Ro} \qquad ..(3b)$$

$$F_1(n, s) = \frac{n^2}{n^2 - s^2} l_n \left(\frac{n}{s}\right) + \frac{s^2 - 3n^2}{4n^2} \qquad ..(3c)$$

$$F_2(n, s) = \frac{n^2 - s^2}{n^2} l_n(s) \qquad ..(3d)$$

$$G(n, s) = \frac{1 - s^2 (1 - 2 \ l_n s)}{2n^2} \qquad ..(3e)$$

$$F(n, s) = F_1(n, s) + \theta F_2(n, s) \qquad ..(3f)$$

$$x = C_h''/C_h \qquad ..(3g)$$

$$\theta = K_r/K_r'' \qquad ..(3h)$$

$$T_R = \frac{C_V \ . t}{4d^2} \qquad ..(3i)$$

$$T_{R_o} = \frac{C_{VR} \cdot t_0}{4d^2} \qquad ..(3j)$$

in which

$\bar{u}$ = average excess pore pressure,

$\bar{u}_0$ = average initial excess pore pressure,

T_R = time factor for radial drainage and vertical consolidation,

T_{Ro} = time factor at the end of construction for radial drainage and vertical consolidation,

C_{VR} = coefficient of consolidation for radial drainage and vertical consolidation,

$F_1(n, s)$; $F_2(n, s)$ & $G(n, s)$ = *geometric drain parameters*,

K_r = radial coefficient of permeability (undisturbed),

K''_r = radial coefficient of permeability (remolded),

C_h = radial coefficient of consolidation (undisturbed),

C''_h = radial coefficient of consolidation (remolded),

n = ratio of drain spacing to drain diameter = $\frac{d}{a}$,

s = smear factor,

d = radius of influence of drain, and

a = radius of drain.

The geometric parameters F_1, F_2 and G are presented by Schiffman in the form of tables and charts. For no smear case, s becomes unity. At the end of construction period for no smear case Equations (3*a*) and (3*b*) reduce to

$$\frac{\bar{u}}{\bar{u}_0}=\frac{1}{8T_{Ro}}\left[F(n)\right]\left[1-e^{-\frac{8T_{Ro}}{F(n)}}\right] \quad ..(4)$$

Sand drains may be placed either in a triangular or square pattern. If the drains are placed in a triangular pattern the zone of influence becomes hexagonal and can be approximated by a circle. A typical layout of the sand drain pattern and dimensioned sections of the drain is shown in Figure 1.

The degree of consolidation for three-dimensional drainage, U is given by

$$U=1-(1-U_v)\,(1-U_R\ ) \quad ..(5)$$

where U_R = degree of consolidation for radial drainage and vertical consolidation.

Formulation of Design Equations

In case of time-dependent loading, the sand drain configuration is designed such that an assumed percentage of consolidation is achieved at

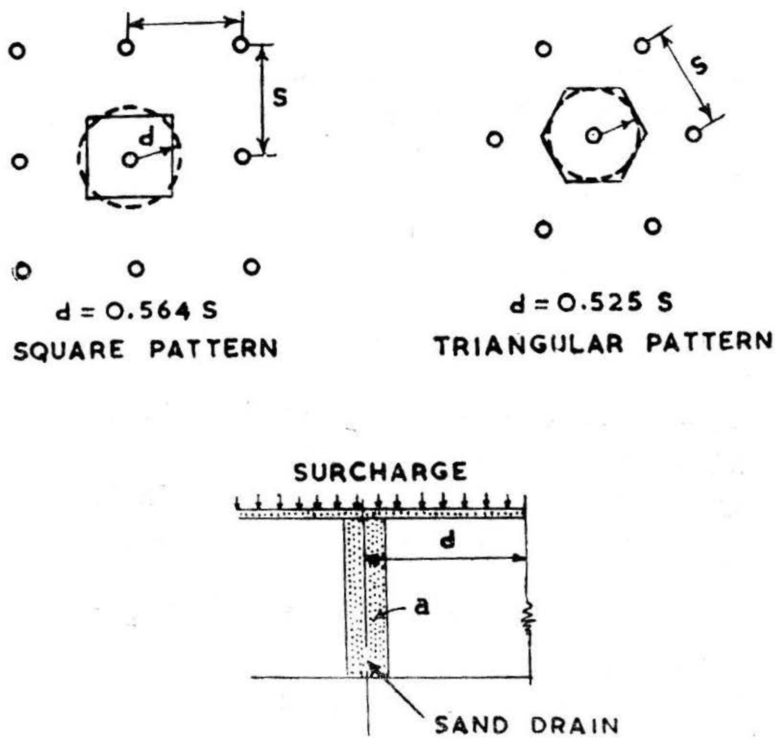

FIGURE 1 : Sand drain pattern.

the end of construction period. Specifications in any sand drain project may in general insist 90 percent of primary consolidation to be achieved at the end of construction period. Substituting Equations (2) and (4) in Equation (5), the degree of consolidation for three-dimensional flow, U, can be obtained.

$$U=1-\left[\frac{1}{3T_{vo}}-\frac{32}{\pi^4 T_{vo}}\; e^{-\frac{\pi^2 T_{vo}}{4}}\right]\left[\frac{F(n)}{8T_{Ro}}\left\{1-e^{-\frac{8T_{Ro}}{F(n)}}\right\}\right] \quad ..(6)$$

Let $\qquad V=\dfrac{1}{3T_{vo}}-\dfrac{32}{{}'T_{vo}}\,.\,e^{-\frac{\pi^2 T_{vo}}{4}}$

and V is a constant for a particular value of T_{vo} .

The values of V corresponding to various values of T_{vo} as presented by Lumb are shown in Table I.

TABLE I

Values of V corresponding to various end of construction periods.

T_{vo}	0·01	0·02	0·05	0·10	0·20	0·30	0·40	0·50
V	0·939	0·885	0·830	0·755	0·659	0·573	0·515	0·465
T_{vo}	0·60	0·80	1·0	1·2	1·4	1·6	1·8	2·0
V	0·424	0·353	0·299	0·260	0·223	0·198	0·179	0·163

Substituting V in Equation (6)

$$U=1-V\left[\frac{F(n)}{8T_{Ro}}\right]\left[1-e^{-\frac{8T_{Ro}}{F(n)}}\right] \quad ..(7)$$

Equation (8) can be rearranged as follows :

$$\frac{1}{\lambda^2}\left[1-e^{-\frac{2\epsilon\lambda^2 T_{vo}}{F(n)}}\right]=\frac{2(1-U)\varepsilon T_{vo}}{V\,.\,F(n)} \quad ..(9)$$

Where $\qquad \varepsilon=\dfrac{C_{VR}}{C_{VV}}$

and $\qquad \lambda=\dfrac{H}{d}$.

Equation (9) is a transcendental equation and no general method exists for finding their roots in terms of the parameters involved. For time-dependent loading with different horizontal and vertical coefficients of permeability (consolidation) for no smear case and for $U=90$ percent, Equation (9) reduces to

$$\frac{1}{\lambda^2}\left[1-e^{-\frac{2\epsilon\lambda^2 T_{vo}}{F(n)}}\right]=\frac{0{\cdot}2\epsilon T_{vo}}{VF(n)} \quad ..(10)$$

Design Procedure

Substantial strength-gain may occur in case of loose marine clays and other types of poor foundation soils, when 70 to 80 percent of primary consolidation is attained. The rate of strength-gain may *be different for different types of soil.* During this investigation, the design charts are prepared assuming that substantial strength-gain occurs at 90 percent primary consolidation. For any other percentage of consolidation also, similar curves can be drawn making use of Equation (9). The percentage consolidation can be obtained if final and allowable settlements are known. The final settlement, ρ_f, can be obtained using the following formula

$$\rho_f=m_v \cdot \triangle\sigma' \cdot H \quad ..(11)$$

where m_v = coefficient of volume change, and

$\triangle\sigma'$ = effective pressure increment at time t.

Both m_v and $\triangle\sigma'$ are assumed to be constant over the depth. The allowable settlement depends upon the type of problem and it will be specified in any design project. It is well-known that the soil permeability decreases during the consolidation process and the coefficient of vertical consolidation resulting from radial flow C_{VR} differes from the coefficient of vertical consolidation resulting from vertical flow C_{VV}. Taking these factors into consideration, design charts are drawn making use of Equation (10) for time-dependent loading and no smear case for different ratios of $\epsilon=1, 2, 3, 4, 5$ and 10.

For different values of Tvo, the values of $V=(1-U_v)$ are taken from the values presented by Lumb in the form of tables. By substituting the values of T_{v0}, V & $F(n)$ in Equation (10), λ can be calculated for different values of ϵ. For preparing the design charts and tables, the use of IBM 7044 computer was made since a large number of possibilities required consideration to cover the wide range of practical interest. These design charts (Figure 2 through 8) are drawn for seven different values of n ranging from 5·0 to 100. In any sand drain project, it is required to find out the drain configuration for the given soil parameters and time to achieve the required average degree of consolidation. The diameter of the sand drain 'a' is known from the boring equipment available and after applying the appropriate reduction factor by the designer to account for the smearing effects while driving the bore-hole. If the requirement is that 90 percent of U should occur in time t, calculate T_{VV} for this condition, enter the graph and arrive at the ratio $\lambda=H/d$ for a known value of ϵ. Different values of λ can be obtained for each of the values of $n=5, 7{\cdot}5, 10, 15, 20, 40$, and 100. From those values of λ, calculate the values of d and hence obtain the values of n. The design criterion is that the value

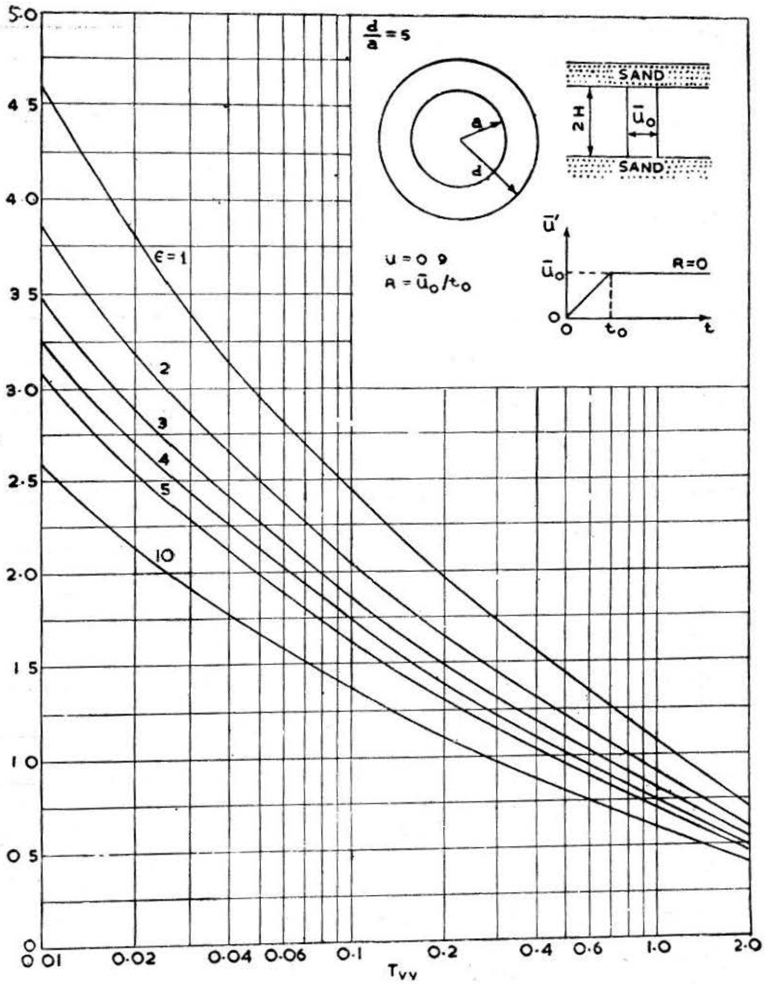

FIGURE 2 : Design Curves for Constant $\frac{C_{VR}}{C_{VV}}$ ***shown on λ (VS)*** T_{vo} **for n=5·0.**

of n obtained and value of n specified should be one. For values of n between those specified and obtained, an interpolation curve as shown in Figure 9 may be drawn and the intersection of this curve with the 45° line gives the design value of n. For square and triangular patterns the spacings between the wells can be shown respectively as follows :

$$d = 0{\cdot}5C4\ S \text{ and } d = 0{\cdot}525\ S \qquad \ldots(12)$$

Illustrative Examp!e

An embankment of 10 m height is to be constructed on a 8 m thick

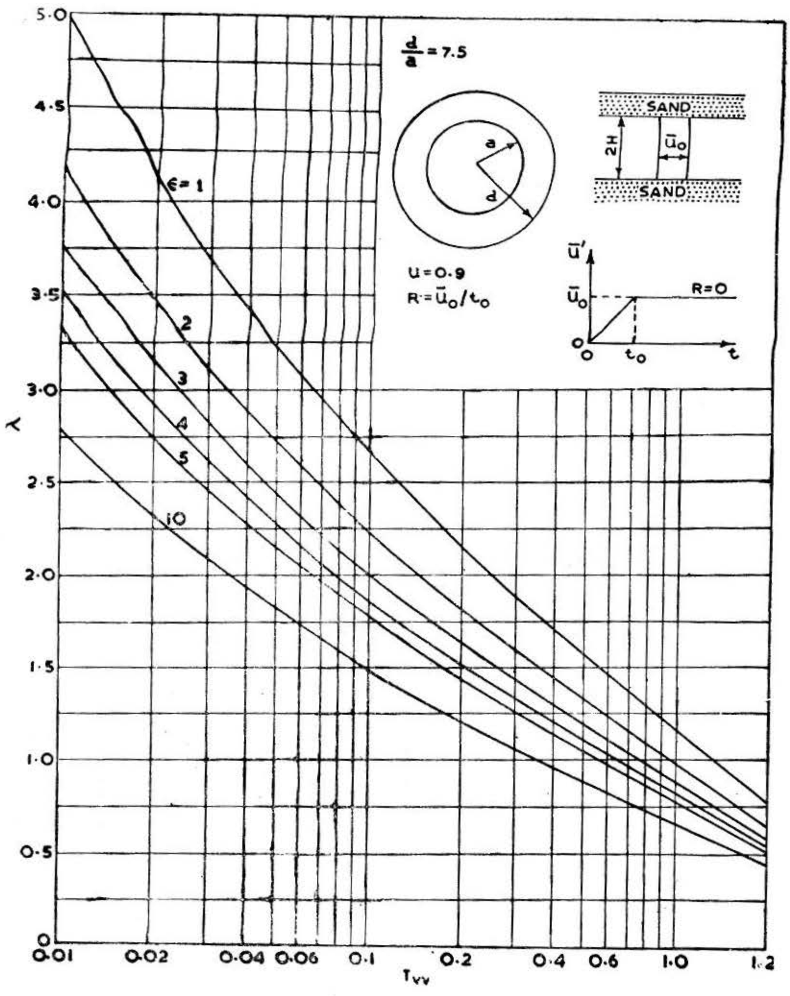

FIGURE 3 : Design Curves for Constant $\frac{C_{VR}}{C_{VV}}$ shown on λ (VS) T_{VO} for n=7·5.

layer of clay overlying rock. The embankment will increase the mean effective vertical stress in the clay, after consolidation from a value of 8 t/m^2 to 20 t/m^2. The embankment will carry a road and will be laid in 5 months. The surfacing will be laid 8 months after the commencement of the construction. Only 3 cm of settlement can be accepted after the surfacing of the road. Design a suitable sand drain installation to achieve the above requirements. The soil is having consolidation characteristics $C_{VV}=0{\cdot}2$, $C_{VR}=0{\cdot}001$ sq cm/sec and $m_v=0{\cdot}03$ sq cm/kg.

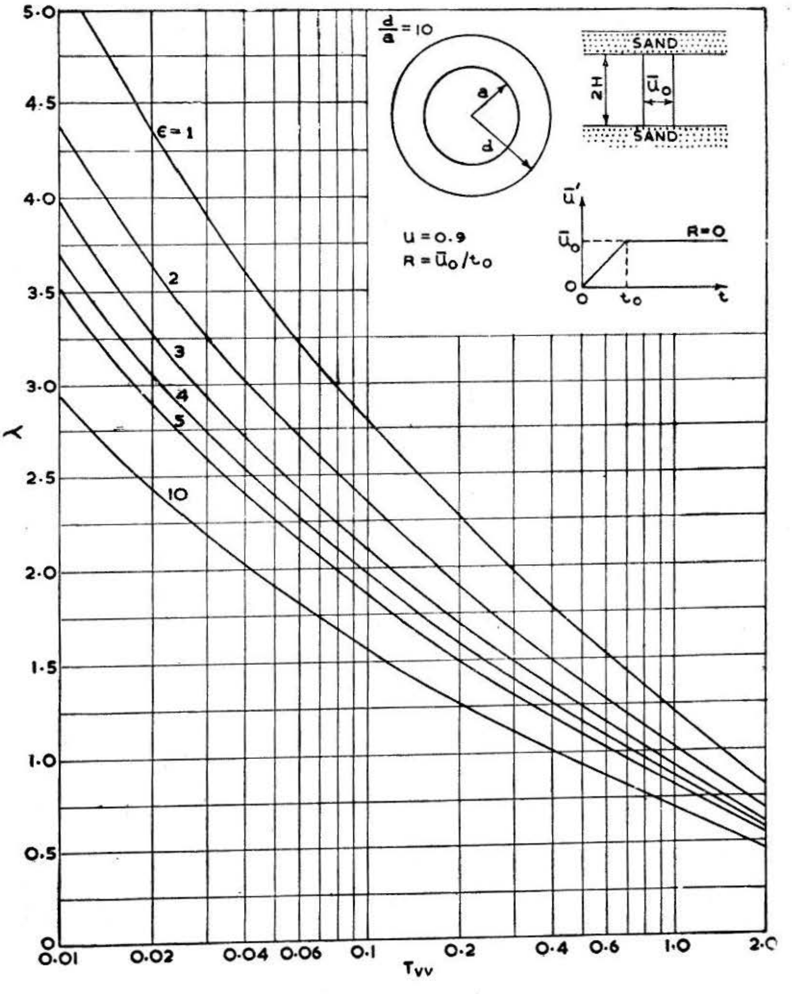

FIGURE 4 : Design Curves for Constant $\frac{C_{VR}}{C_{VV}}$ shown on λ (VS) T_{vo} for n=10.

From Equation (11) the final settlement is given by :—

$\rho_f = m_v . \triangle\sigma' . H$ where m_v and $\triangle\sigma'$ are assumed to be constant over the depth.

m_v =0·03 sq cm/kg=0·003 m^2/t

$\triangle\sigma' = 20-8 = 12$ t/m² and $H = 8{\cdot}0$ m

$\therefore \quad \rho_f = 3\times10^{-3}\times 12\times8 = 0{\cdot}288$ m $= 28{\cdot}8$ cm

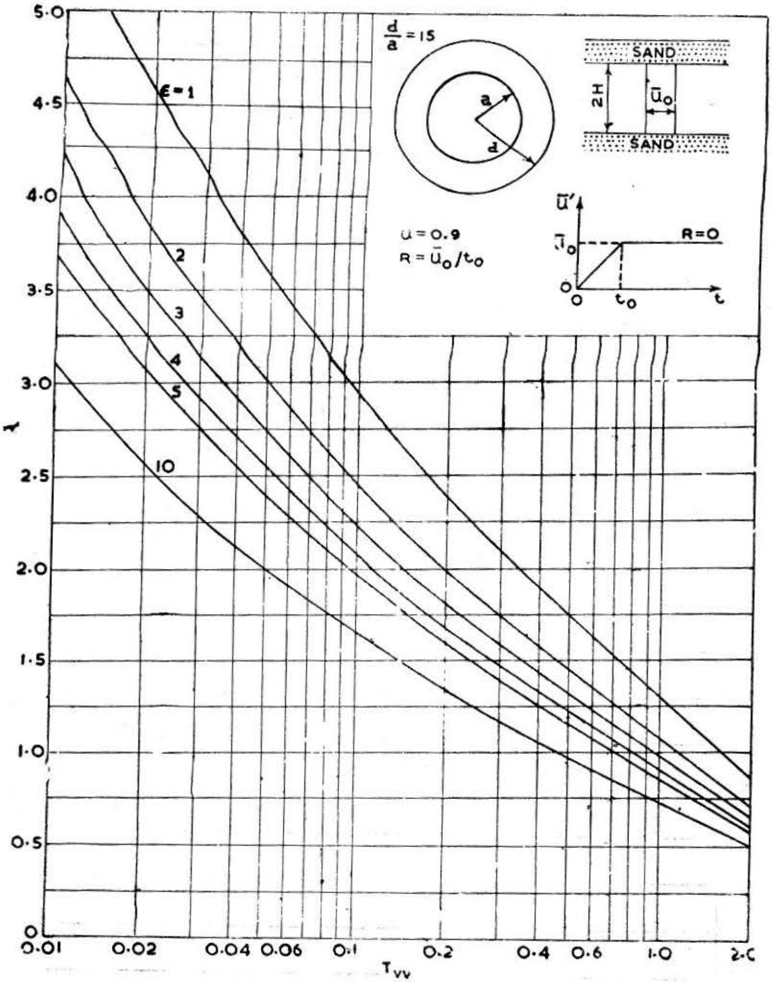

FIGURE 5 : Design Curves for Constant $\frac{C_{VR}}{C_{VV}}$ shown on λ (VS) T_{vo} for n=15.

Settlement that can be allowed after 8 months = 3 cm

∴ Settlement to be achieved at the end of 8 months

$$= 28{\cdot}8 - 3{\cdot}0$$
$$= 25{\cdot}8 \text{ cm}$$

∴ Degree of consolidation during five months period $= U = \frac{25{\cdot}8}{28{\cdot}8} \times 100 = 89{\cdot}5$ percent

= 90 percent

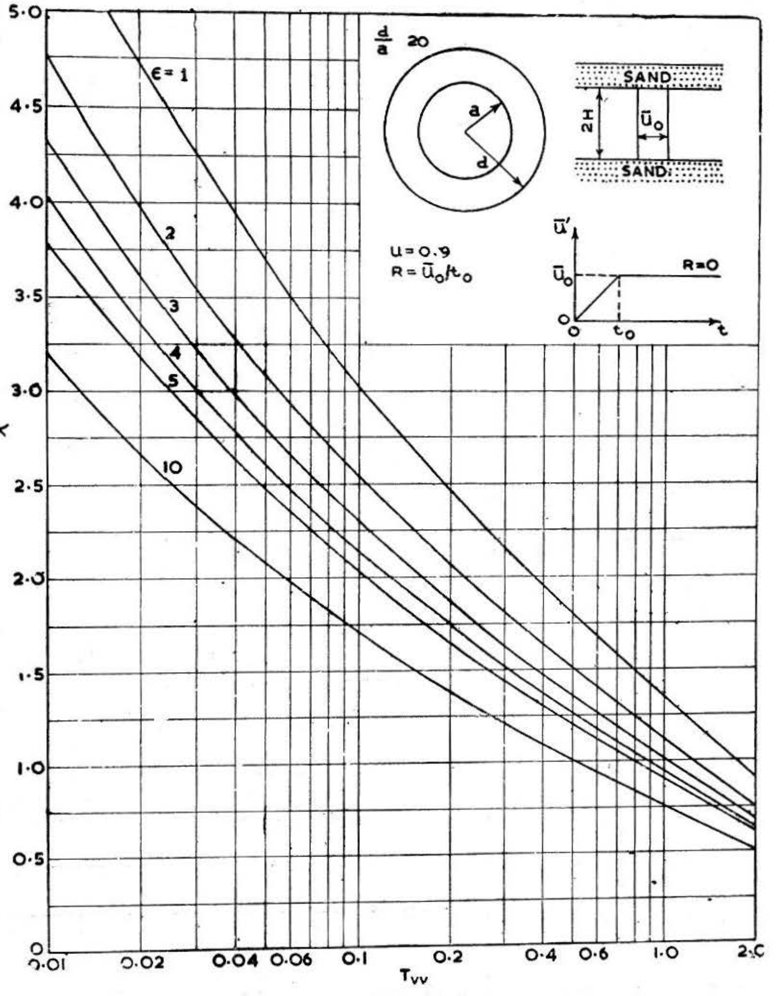

FIGURE 6 : Design Curves for Constant $\frac{C_{VR}}{C_{VV}}$ shown on λ (VS) T_{vo} for n=20.

Effective time within which

this settlement is to occur $=8-\frac{5}{2}=5{\cdot}5$ months

$=167$ days

$=167\times3600\times24$ seconds.

$C_{VV}=0{\cdot}001$ sq cm/sec$=1\times10^{-7}$ m²/sec

$$\therefore \quad T_{V_0}=\frac{1\times10^{-7}\times167\times3600\times24}{8\times8}=0{\cdot}0226.$$

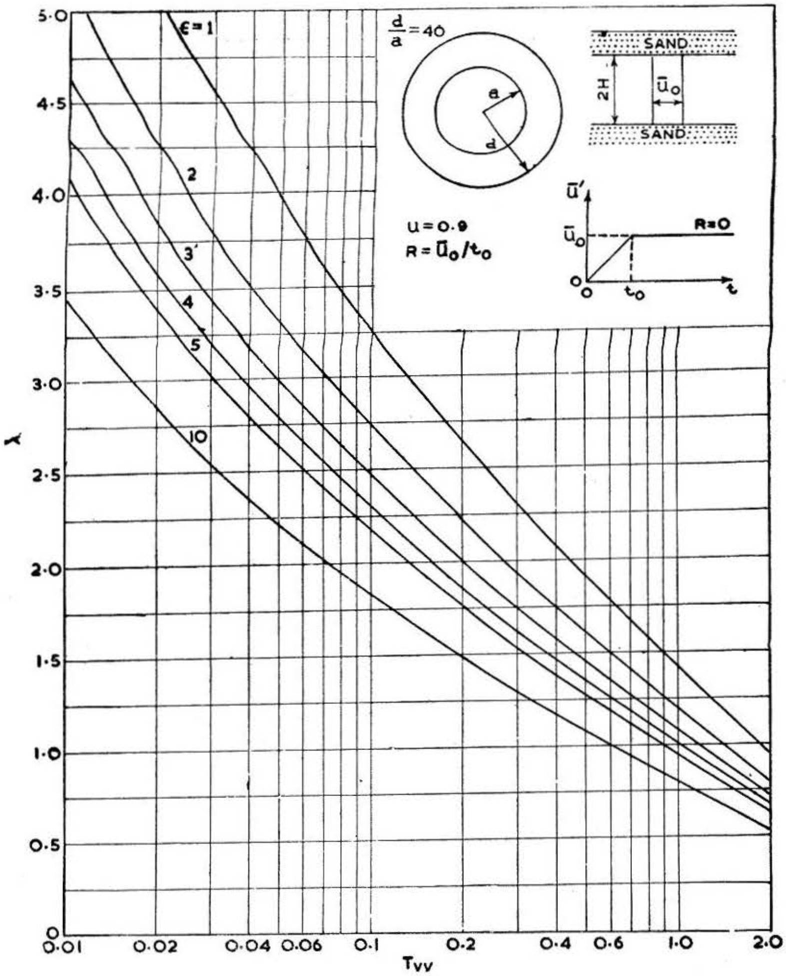

FIGURE 7 : Design Curves for Constant $\frac{C_{VR}}{C_{VV}}$ shown on λ (VS) T_{VO} for n=40.

Let the design considerations including a smear factor make the radius of drain '*a*' equal to 15 cm and let the drains be arranged in a square pattern. Plot a curve as shown in Figure 9 between the specified values of *n* and those that are obtained from design charts (Table II).

TABLE II

Values of n obtained corresponding to n specified.

Specified *n*	*d* from charts in metres	*n* obtained
5·0	3 33	22·20
10·0	2·86	19·05
15·0	2·72	18·10
20·0	2·58	17·20
40·0	2·42	16·10

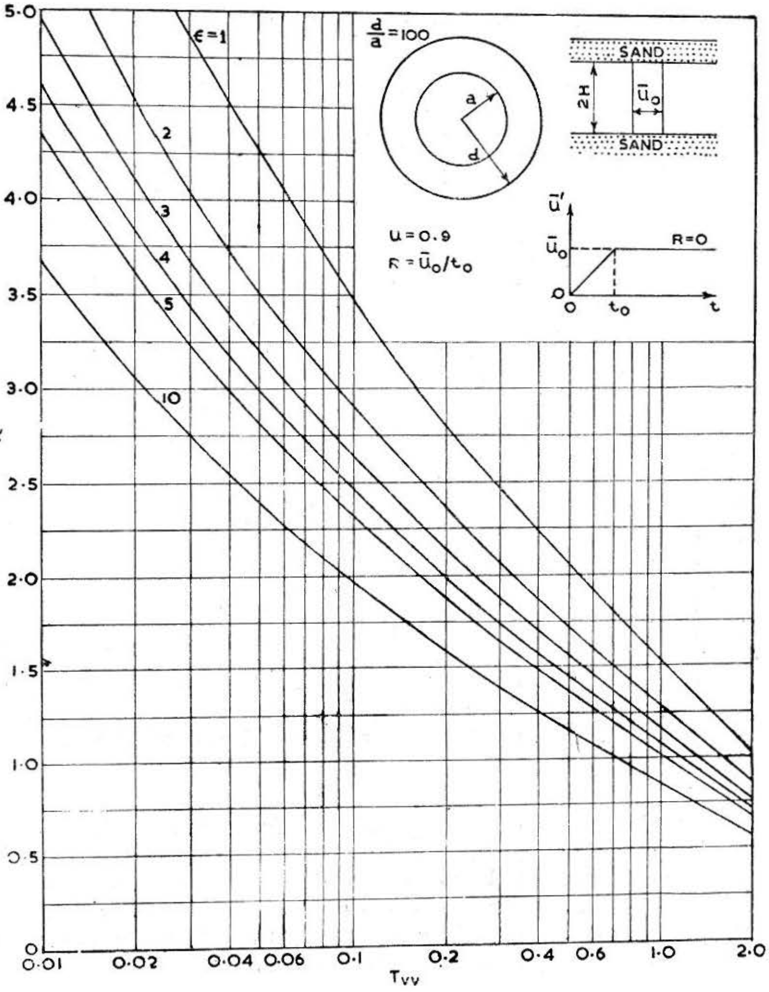

FIGURE 8 : Design Curves for Constant $\frac{C_{VR}}{C_{VV}}$ shown en λ (VS) T_{VO} for n = 100.

From Figure 9 the design value of *n* is 17·5 and the corresponding value of radius of influence of drain *d* is 2·62 m. For square pattern, the spacing *S* for the sand drains will be 4·64 m.

Doubling the value of the radius of drain '*a*', the design spacing changes to 5·26 m. The design charts have once again proved the finding of Barron and Richart that the effectiveness of a given drain well installation is considerably more dependent upon the choice of well spacing than it is upon the well diameter.

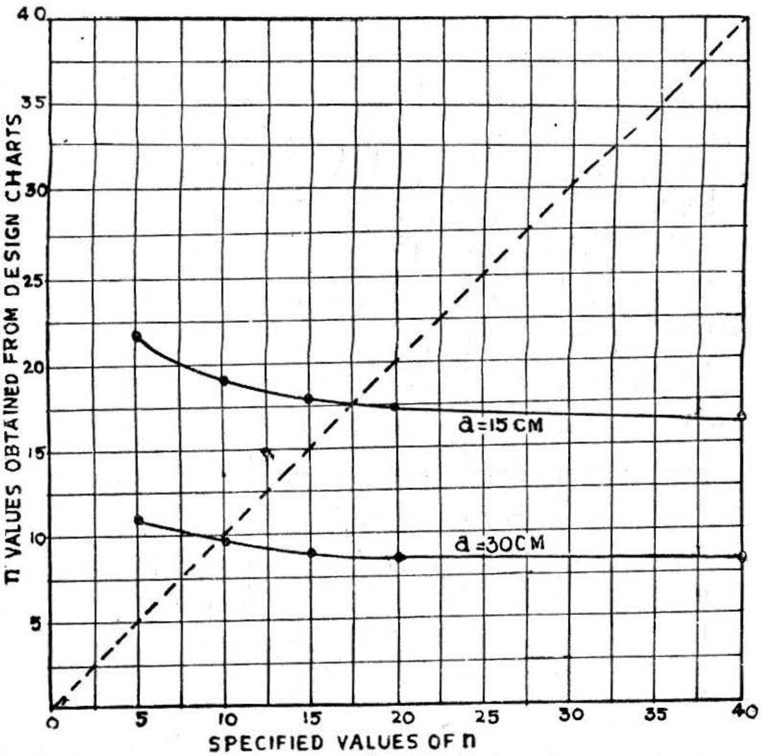

FIGURE 9 : Plot between specified and obtained values of n.

Conclusion

An effective method of design of sand drain installations has been presented for time-dependent loading for the equal vertical strain case taking into consideration the variation of coefficient of consolidation due to vertical and radial flows. Knowing the soil properties and time to achieve the required degree of consolidation, the drain configuration can be obtained by entering into the design charts.

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